PREPARING SUCCESSFUL TEACHERS OF MATHEMATICS

Joy Anderson Davis

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Supervisor of Dissertation:

_______________________________________
Howard Stevenson, Professor of Education

Dean, Graduate School of Education:

_______________________________________
Pamela L. Grossman, Dean and Professor

Dissertation Committee:

Howard Stevenson, Professor of Education

Frances Rust, Visiting Professor and Director of Teacher Education

Janine Remillard, Associate Professor of Education
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DEDICATION

This study is dedicated to my mother, Thelma Earline Anderson, who through her silent urgings, constant prayers and unmatched ability to see what I could become, stood by me through this long, dark, night, daily carrying me on her heart and ardently fanning the flickering embers of my dreams with her hope. It is my sincere desire that the daybreak looming on the horizon is one that we both can share.

This study is also dedicated to my dear children, Taliyah, Lauryn and Justin. Taliyah, my eldest, my spark, you have been my cheerleader. Lauryn and Justin, my loves, just knowing that I was coming home to you—to your smiles and your hugs—was enough. I love each one of you more than words can express!

Finally, this study is dedicated to my husband, Rodlin, the steadfast one. Faithful to your promise, you simply would not let me give up on this. You saw my tears but held on to faith and reminded me that God was for me. For that, I am grateful.
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endeavored to complete this dissertation. Her unmatched investment in our lives will never be forgotten.
ABSTRACT

PREPARING SUCCESSFUL TEACHERS OF MATHEMATICS

Joy Anderson Davis
Howard Stevenson

This dissertation investigates the contributions of the preservice experience to the continuing development of novice elementary school teachers’ math pedagogy and practice. Using case study methodology and descriptive narrative, this phenomenologically-oriented study focuses on novice teachers’ development along two critical knowledge constructs: subject matter knowledge and pedagogical knowledge. Emphasis is placed on the mathematical knowledge, experiences and beliefs that novice teachers bring to the preservice experience. The results of this study suggest that despite the decades old theoretical shift in K-12 mathematics education to a constructivist, meaning making pedagogy, many novice teachers’ K-12 math experiences continue to be dominated by a decontextualized, procedural approach to the discipline. Consequently, the primary contribution of preservice to the study participants’ development as math teachers involved facilitating a fundamental shift in their perspective of math and how to teach it—a shift toward a constructivist-oriented pedagogy (Cochran-Smith, 2008; Kennedy, 1999).
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEDICATION</td>
</tr>
<tr>
<td>ACKNOWLEDGMENT</td>
</tr>
<tr>
<td>ABSTRACT</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
</tr>
<tr>
<td>PREFACE</td>
</tr>
<tr>
<td>CHAPTER 1: INTRODUCTION</td>
</tr>
<tr>
<td>Statement of the Problem</td>
</tr>
<tr>
<td>Purpose of the Study</td>
</tr>
<tr>
<td>Research Questions</td>
</tr>
<tr>
<td>CHAPTER 2: LITERATURE REVIEW</td>
</tr>
<tr>
<td>The Reemergence of Content Knowledge</td>
</tr>
<tr>
<td>Subject Matter Knowledge and Instructional Approach</td>
</tr>
<tr>
<td>The Continuum of Pedagogical Approaches</td>
</tr>
<tr>
<td>Constructivism in the Mathematics Education</td>
</tr>
<tr>
<td>Theoretical Framework</td>
</tr>
<tr>
<td>Conceptual Framework</td>
</tr>
<tr>
<td>Preservice Teacher</td>
</tr>
<tr>
<td>Penn GSE Teacher Education Program (Penn TEP)</td>
</tr>
<tr>
<td>Novice Teacher</td>
</tr>
</tbody>
</table>
CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY .................................................. 28

Research Context ........................................................................................................... 28

Researcher Identity ....................................................................................................... 30

Study Participants ......................................................................................................... 31

Data Sources .................................................................................................................. 32

Why Case Study? .......................................................................................................... 32

Why a Phenomenological Orientation? ........................................................................ 35

Data Collection and Analysis ...................................................................................... 37

CHAPTER 4: DESCRIPTIVE CASES ............................................................................ 42

Introduction .................................................................................................................... 42

Teacher Gwen ............................................................................................................... 44

K-16 Math Background. ............................................................................................... 44

Learning From Preservice ............................................................................................. 45

Teaching After Preservice .............................................................................................. 50

Teacher Myles ............................................................................................................... 54

K-16 Math Background. ............................................................................................... 54

Learning from Preservice. ............................................................................................. 55

Teaching After Preservice .............................................................................................. 58

Teacher Lacey ............................................................................................................... 61

K-16 Math Background. ............................................................................................... 61

Learning From Preservice ............................................................................................. 62

Teaching After Preservice. ............................................................................................ 66

Teacher Michael .......................................................................................................... 68

K-16 Math Background. ............................................................................................... 68

Learning From Preservice ............................................................................................. 69
CHAPTER 6: CONCLUSION .......................................................................................................................... 107
Summary of Findings .................................................................................................................................... 107
Implications for Future Study .................................................................................................................................. 109
APPENDIX I: PLAN FOR DATA COLLECTION, ANALYSIS AND DEFENSE..............112
APPENDIX II: BACKGROUND SURVEY EMAIL......................................................................................113
APPENDIX III: BACKGROUND SURVEY .................................................................................................114
APPENDIX IV: SAMPLE MKT SURVEY QUESTIONS ...............................................................................118
APPENDIX V: INTERVIEW 1 [SUBJECT MATTER KNOWLEDGE] ......................................................119
APPENDIX VI: CONSTRUCTED RESPONSE LESSON AGENDA [CRLA] ...........................................122
APPENDIX VII: INTERVIEW 2 [PEDAGOGICAL APPROACH] .................................................................123
APPENDIX VIII: FACULTY/STAFF INTERVIEWS ....................................................................................126
APPENDIX IX: PARTICIPANT CONSENT FORM .......................................................................................128
BIBLIOGRAPHY .........................................................................................................................................147
LIST OF TABLES

Table 2.1 Knowledge Base for Teaching (Shulman)………………………………9
Table 3.1 Data Collection Points………………………………………………….32
Table 5.1 Results of Constructed Response Lesson Agenda (CRLA)…………….98
Table 5.2 Summary of Implementation…………………………………………100
LIST OF FIGURES

Figure 2.1 Mathematical Knowledge for Teaching ................................................ 13
Figure 2.2 Conceptual Framework Map ............................................................... 23
Figure 5.1 Organization of Emerging Themes ..................................................... 87
Figure 5.2 Novice Teacher’s Persistence in Implementation .............................. 101
PREFACE

We can transform public education in this country and finally begin to deliver an excellent education for every child . . . the first step is with how we handle teacher preparation—what happens before many teachers even step foot in the classroom. . . . Our goal is simple: We want every teacher to receive the high-quality preparation and support they need, so that every student can have the effective teachers they deserve. (US Department of Education, 2011, p. 1-2)
CHAPTER 1: INTRODUCTION

Statement of the Problem

This dissertation investigated how positively perceived preservice education influenced teachers’ knowledge of, perspectives about, and ultimately practice in teaching elementary mathematics. My research was an attempt to begin identifying and unpacking the ways that preservice education contributes to the development of novice, elementary school math teachers.

This study was timely for two reasons. First, as a nation we are engaged in massive efforts to enhance student outcomes and preparedness for college and career, especially in the STEM fields of science, technology, engineering and math. In this effort, we are largely dependent on our system of public education and public school teachers. However, public discourse around teachers and schools reflects quite negatively on the quality of US teachers and the overall health of k-12 public education. In some segments of the population, calls for reform have become calls for retreat (School Vouchers, 2017. Blame for the condition of k-12 schooling has been leveled at teachers. McLaughlin & Talbert (2006) captured the current national sentiment when they stated:

We accept society’s sober assessment about the limited and unequal capability of America’s teachers to meet the educational needs of all students . . . acknowledge the consequent need for teachers to develop new forms of teaching and strategies to meet the needs of a diverse, continually changing student population [and] provide the learning environments and student outcomes society demands. (p. 2)
Highlighting teacher quality as the critical lever for improving student outcomes implicates, by default, the organizations that train and prepare this nation’s teachers. Schools of education, which prepare up to 84% of all new teachers (Feistritzer, 2011), have been increasingly singled out as failing to adequately prepare teachers for the demands of modern education (Levine 2006; Levine 2010).

By almost any standard, many if not most of the nation’s 1,450 schools, colleges, and departments of education are doing a mediocre job of preparing teachers for the realities of the 21st century classroom. America’s university-based teacher preparation programs need revolutionary change—not evolutionary tinkering (Duncan, 2009).

In the current policy context, “teacher education programs (TEPs) are being pushed to demonstrate their ‘value-added’ and defend their very existence” (Anderson & Stillman, 2013, p.4). The wholesale rejection of university based teacher preparation may be premature if not wholly inappropriate. The findings of the most recent national study of teacher preparation, “Preparing Teachers: Building Evidence for Sound Policy” demonstrates that our understanding of the role and impact of preservice education on the developmental trajectories and knowledge of teachers is still in its infancy (National Research Council [NRC], 2010). Two of the questions the NRC highlighted needing further investigation are:

1) to determine the relative effectiveness of the components of those pathways and programs [teacher preparation programs]

2) to better describe the characteristics of teacher candidates and how those relate to program selection and the quality of the teacher workforce.
After five years and one and one half million dollars of research the NRC concluded that they were unable to speak conclusively to the impact of preservice education on the teachers who matriculate through those programs. Moreover, while the NRC identified that content knowledge, pedagogical knowledge, and student teaching were all important for preservice teachers’ [PSTs’] subsequent instructional practice and for their students’ achievement, they also acknowledged that the research base on how teacher preparation contributes to these bodies of knowledge was too limited to allow for any “specific conclusions” (p.69). Even studies designed to focus specifically on the impact of specific programmatic choices (e.g. student-teaching), tend toward the description and analysis of logistics.

Most of the research on student teaching [a core component of most preservice education programs] describes only the composition of task, emphasizing, “student teaching’s structural and logistical dimensions—for example, its location, duration, and division of labor—but not its contributions to learning among preservice teachers” (Anderson & Stillman, 2013, p.4).

Consequently, one of the most formidable challenges we face in trying to map the long-term implications of teacher education programs on student achievement (the sine qua non for policymakers) is the dearth of research on the nature and content of preservice education and the teacher learning outcomes with which specific preservice programmatic choices can be linked (Moore-Johnson & Birkeland, 2008). The ability to identify the content and structure of individual preservice programs and more importantly what pre-certification teachers learn in and through preservice education is a critical first step in the effort to improve teacher education, teacher practice and the penultimate indicator—student achievement.
Purpose of the Study

While it is not possible for a single dissertation to engage the massive task of systematically characterizing all 1,450 university-based preservice programs and the growing corps of alternative certification programs, there were components of this undertaking for which my dissertation was suitable. My work, significantly narrower in scope was focused on the preparation of elementary school teachers, and even more narrowly on the way they are prepared to teach elementary math.

My research was inspired by the work of Mary Kennedy (1999) and Marilyn Cochran-Smith (2008) who both argued that preservice education represented “a key interval in the process of learning to teach with the potential to be a site for educational change” (Cochran-Smith, 2008, p.4). The Teacher Education and Learning to Teach (TELT) studies which occurred between 1986 and 1990 were designed to determine whether, “formal teacher education programs had any value in the formation of teachers” (Kennedy, n.d.). My research revisits these studies but with a more targeted focus. First, my study investigated a single teacher preparation program. It is focused specifically on the way the program contributed to teachers’ preparation as teacher of elementary math. Thirdly, my study embraced an appreciative inquiry approach which centers on identifying and building on successes (Johnson & Leavitt, 2001).

The purpose of my research was to illuminate instances when math teacher preparation worked—instances when preservice achieved its potential for reshaping the beliefs and understandings about math instruction that preservice teachers bring to their preparation programs in ways that improve their approach to practice (Kennedy, 1999).
In this effort, I adopted a retrospective, narrative design that allowed me to highlight the voices of individual teachers while providing them the opportunity to reflect on their preservice experiences in the light of their current work as professional teachers.

This study is personally meaningful because I have long recognized the significant and pivotal role that my own preservice education played in my development as a math educator and can clearly remember specific experiences that caused a shift in my mathematical knowledge. It was not the program alone but the convergence of combination of my personal development and readiness with critical program components that precipitated the shift.

I also recognize that U.S. elementary preservice teachers’ knowledge of mathematics tends to be thin, fragmented, rule-bound and altogether insufficient for the work of teaching (Ball, 1990; Ma, 1999). Many elementary school teachers are afraid of math, don’t like math, don’t understand math, and were not successful in math during their formative years. While preservice teachers’ K-12 apprenticeship of observation is a critical determinant and predictor of subsequent teacher practice (Lortie, 1975), my perspective on math and how to teach it was transformed by supported exposure to new ways of understanding.

**Research Questions**

I chose to limit my study to a single preservice program, the M.S. Ed. in Elementary Education offered at the University of Pennsylvania Graduate School of Education Teacher Education Program (TEP). The program has a reputation for having a strong math education component. My research focuses on two core
My research questions are as follows:

1. How, and in what ways, does preservice education contribute to the development of a novice teachers’ knowledge of elementary math content?

2. How, and in what ways, does preservice education contribute to the development of novice teachers’ espoused pedagogical approach for teaching elementary math?

3. How might teachers’ preservice learning be enacted in their early work as elementary math teachers? What, if anything, persists from preservice in novice teachers’ ideology and practice as professional teachers of elementary math.
CHAPTER 2: LITERATURE REVIEW

To study teachers’ developing knowledge of mathematics and math pedagogy, I must first define math subject matter knowledge and espoused pedagogical approach. Toward that end, I have explicated these two terms in the literature review. I begin by chronicling the reemergence of content knowledge as a critical component of teacher knowledge. I then trace the development of mathematical knowledge for teaching (MKT) as a discipline-specific representation of the content knowledge required for effective instructional practice (Ball, Thames & Phelps, 2008; Hill, Schilling & Ball, 2004). I close with a discussion of the continuum of pedagogical approaches for mathematical instruction the ends of which are commonly identified as behaviorism and constructivism.

The Reemergence of Content Knowledge.

Research on teaching during most of the 20th century gave scant attention to the question of teachers’ content knowledge (Ball, 1988; Shulman 1986).1 Heavily focused on pedagogical concerns, early studies of teaching focused on notions of “good” or “effective” teaching that were devoid of disciplinary specificity (Ball, 1988). In the early 1980’s, teachers’ knowledge of subject matter reemerged as a central concern of research on teaching. This focus was stimulated in part by the research community.

Recent work emphasizes a new dimension of difference between individuals who display more or less ability in thinking and problem solving. This dimension is the

1 At the turn of the 20th century the definition of requisite teacher knowledge was heavily weighted toward content. See Shulman (1986) for a full discussion.
possession and utilization of an organized body of conceptual and procedural knowledge, and a major component of thinking is seen to be the possession of accessible and usable knowledge. (Glaser, 1984, p. 97)

Moreover, according to Resnick (1983) a “critical theme” in the cognitive science research of that period was the relationship between knowledge, thinking and performance. This research highlighted the idea of expert performance. Resnick argued that “a person's intelligent performance is not a matter of disembodied ‘processes of thinking’ but depends intimately on the kind of knowledge that the person has about the particular situation in question” (p. 477). Cognitive scientists referred to the “set of rules, definitions and strategies” known by an expert or, the “secret of an expert system’s expertise,” as the knowledge base (Wilson, Shulman & Reichert, 1987, p. 106). Those researching the work of teachers found the idea of the knowledge base intriguing and began to utilize it as a framework for thinking about teacher knowledge. In the research on teaching, the knowledge base came to be defined as, “the body of understanding, knowledge, skills, and dispositions that a teacher needs to perform effectively in a given teaching situation” (Wilson et al, 1987, p.106). Shulman’s research emphasized the importance of content or subject matter knowledge as a core component of the knowledge base for teaching. Since that time, researchers have sought to better understand the nature and composition of teachers’ subject matter knowledge (Grossman, Wilson & Shulman, 1989; Hashweh, 1987; Leinhardt & Smith, 1985; Wilson, Shulman & Reichert, 1987) and how that knowledge is obtained (Ball & McDiarmid, 1990), organized in the mind (Wilson, Shulman & Reichert, 1987; Ma, 1999), measured (Hill,
Rowan & Ball, 2005) and translated into action (Ball, Thames & Phelps, 2008; Borko, Eisenhart, Brown, Underhill, Jones & Agard, 1992; Borko & Putnam, 1996; Ma, 1999). Other researchers have considered the specific implications of Shulman’s conceptions of teachers’ content knowledge for teacher preparation and professional development (Ball, 1988; Ball & Bass, 2003; Wilson & Berne, 1999).

Shulman referred to content knowledge as the “missing paradigm” in research on teaching, arguing that “what we miss [in contemporary discussions of teacher education] are questions about the content of the lessons taught, the questions asked, and the explanations offered” (1986, p. 8). Accordingly, Shulman’s knowledge base was heavily weighted towards disciplinary content, and he argued for the central role of content knowledge in the preparation and work of teachers (1986, 1987).

Table 1.1 Knowledge Base for Teaching (Shulman)

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<thead>
<tr>
<th>Content knowledge</th>
<th>General pedagogical knowledge,</th>
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<tr>
<td>Pedagogical content knowledge</td>
<td>Knowledge of learners and their characteristics</td>
</tr>
<tr>
<td>Curriculum knowledge</td>
<td>Knowledge of educational contexts</td>
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<td>Knowledge of educational ends, purposes and values</td>
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(Adapted from Shulman, 1987)

Shulman (1986) theorized that content knowledge was a tripartite construct consisting of subject matter knowledge, curricular knowledge and pedagogical content knowledge. It was, in fact, the introduction of pedagogical content knowledge, or PCK, that revolutionized research on teaching. Shulman argued that pedagogical content
knowledge “embodied the aspects of content most germane to its teachability” (1986, p. 8). Viewed as the knowledge bridge between the disciplinary expert and the teacher, PCK embodied a “special amalgam of content and pedagogy . . . uniquely the province of teachers” (Shulman, 1987, p. 8).

Within the category of pedagogical content knowledge I [Shulman] include, for the most regularly taught topics in one’s subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—In a word, the ways of representing and formulating the subject that make it comprehensible to others…what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning. (1986, p. 9)

While PCK claims as its foundation a deep understanding of subject matter knowledge, it includes “an understanding of what it means to teach a particular topic as well as knowledge of the principles and techniques required to do so” (Wilson, Shulman & Reichert, 1987, p. 118). PCK also includes knowledge of learners: specifically, what makes certain topics easy or difficult for learners to ascertain, knowledge of how students’ preconceptions and backgrounds factor into the way they internalize certain facts and concepts, and the ability to represent disciplinary knowledge in the manner most conducive to student learning. As a result, PCK is directly linked to the instructional choices made by teachers.

**Subject Matter Knowledge and Instructional Approach.** The early research on PCK unearthed the significant interplay between subject matter knowledge and
teachers’ instructional choices. Wilson, Shulman & Reichert (1987) observed that teachers, in process of preparing and implementing a lesson, engaged in a cycle of thinking they termed pedagogic reasoning. The two core components of the cycle of pedagogic reasoning are transformation and representation (Wilson et al., 1987). Transformation is “the set of activities engaged by the teacher to move from her own comprehension of a matter, and the representations most useful for that understanding, to the variations of representations, narrative, example or association likely to initiate understanding on the part of the students” (Wilson et al., 1987, p. 113). Representations are the actual “metaphors, analogies, illustrations, examples,” etc. that are presented to students and have at least two dimensions, the fact or knowledge to be conveyed, and the mode (e.g. analogies, simulations, diagrams, etc.) utilized to convey the fact (McDiarmid & Ball, 1989). While teachers’ subject matter knowledge and PCK may be altered by the cycle of pedagogic reasoning, the initial accuracy of the representations that are presented is a direct result of teachers’ subject matter knowledge. It is from this reservoir that they will draw in efforts to develop representations for pupils. The accuracy and adequacy of teachers’ instructional representations is determined by the adequacy and accuracy of their subject matter knowledge—they create and choose representations from the subject matter knowledge they hold (McDiarmid & Ball, 1989). Here we see the primacy of subject matter knowledge in teachers’ instructional approach. Teachers’ capacity to pose questions, select tasks and make curricular choices all depend on how they themselves understand content matter. (Wilson et al., p. 1987).
The work of Deborah Ball and her colleagues has been central to the discussion of the relationship between teachers’ subject matter knowledge and their instructional practice, particularly as it relates to the teaching and learning of elementary mathematics. Ball’s work was largely influenced by the notion of PCK as the link between what teachers know and what they do. Her research is distinct in that it starts with and foregrounds the work of teachers.

We decided to focus on their work. What do teachers do, and how does what they do demand mathematical reasoning, insight, understanding, and skill? We began to try to unearth the ways in which mathematics is entailed by its regular day-to-day, moment-to-moment demands. (Ball & Bass, 2003, p.5)

While PCK remains an under-theorized construct (Ball, Thames & Phelps, 2008), the research of Ball and her colleagues has led to critical understandings about the work of teaching math that ground my research. First, the content knowledge utilized by teachers in the work of teaching is distinct from the knowledge generally held by disciplinary experts and individuals in professional fields that draw heavily on mathematical knowledge (Ball, Hill & Bass, 2005; Hill, Schilling, Ball, 2004). Second, the work of teaching math, even pedagogical decision-making, is inherently conceptual, relying heavily mathematical reasoning, “there are predictable and recurrent tasks that teachers face that are deeply entwined with mathematics and mathematical reasoning. . . each of these common tasks of teaching involves mathematical reasoning as much as it does pedagogical thinking” (Ball, Hill & Bass, 2005, p.21).

Ball’s map of the mathematical knowledge used in teaching elementary math (Mathematical Knowledge for teaching or MKT), is comprised of two categories: subject matter knowledge and pedagogical content knowledge. Ball further subdivides subject
matter knowledge into three categories: common content knowledge, specialized content knowledge and knowledge at the mathematical horizon. My research questions investigate two of these subcategories: common content knowledge (CCK) and specialized content knowledge (SCK).

Figure 1.1 Mathematical Knowledge for Teaching

![Mathematical Knowledge for Teaching Diagram](Ball, Thames & Phelps, 2008, p.403)

CCK is defined as, “the mathematical knowledge and skill used in settings other than teaching” (Ball, et al, 2008, p.399). This is the type of mathematical knowledge commonly tested in teacher qualification exams. CCK is the domain of knowledge that teachers would need to have to successfully complete the assignment they make to their students (Ball et al, 2008). It is referred to as “common” knowledge because it is not unique to the demands of teaching math (Ball et al, 2008, p. 399).

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2 Knowledge at the mathematical horizons is still an exploratory category (Ball et al., 2008).
Alternately, specialized content knowledge or SCK is mathematical knowledge that is distinct to the work of teaching (Ball, et al., 2008). Consider the following description of a teacher’s use of subject matter knowledge, particularly SCK in the work of teaching:

Teachers do not merely do problems while students watch. They must explain, listen, and examine students’ work. They must choose useful models or examples. Doing these things requires additional mathematical insight and understanding. Teachers must, for example, be able to see and size up a typical wrong answer [and analyze] the source of the error. But error analysis is not all that teachers do. Teaching also involves explaining why. . . . Teaching entails using representation. . . . It also entails subtle mathematical considerations. For example, what would be strategic numbers to use in an example? (Ball, Hill & Bass, 2005, p.17 and 20)

Ball, Hill & Bass (2005) strongly assert that while knowledge about students is important, simply knowing math (as distinct from knowing about students) in a way that allows for effective instruction is far more complex and substantial than most would think. For example, teachers must understand how to “unpack” knowledge (Fosnot, Dolk, Zolkower, Hersch, & Seignoret, nd). As one increases their mathematical knowledge through traditional learning, the knowledge becomes compressed.

Compression assists math experts in simplifying complexity to solve problems.

Teachers, on the other hand “work with mathematics as it is being learned, which requires a kind of decompression, or “unpacking,” of ideas” (Fosnot et al., p.10).

Secondly teachers’ knowledge of mathematics must have a degree of “connectedness.” The connectedness must extend across different levels of mathematics and across different domains of mathematical knowledge, “[t]eaching involves making connections across mathematical domains, helping students build links and coherence in their
knowledge” (Fosnot et al., p.10-11). For teachers to present mathematics as a coherent, connected whole, they must understand the discipline in those terms. Finally, teachers’ mathematical work includes recognition of “mathematical practices as a component of mathematical knowledge” (Fosnot et al., p.11). These mathematical practices include “reasoning, notation, use of terms and representation” (Fosnot et al., p.11). As a result, a teacher would both need to understand the content being covered, subtraction for example, and simultaneously be conscious of mathematical practices involved. “Entailed for the teacher would be both the particular mathematical ideas under discussion as well as these other elements of knowing, learning, and doing” (Fosnot et al., p. 11).

**The Continuum of Pedagogical Approaches**

Thus far, I have argued for the complex and foundational nature of subject matter knowledge (CCK and SCK) in the work of teaching math. I have also argued that subject matter knowledge is one of the primary reservoirs from which teachers draw when making instructional choices—this is outside of their knowledge of particular learners. However, for teachers to meet the demands of 21st century education, U.S. reformers have argued that in addition to becoming more knowledgeable about subject matter (NCTAF, 1996), teachers also have to reevaluate long held assumptions about what it means to teach and learn (NCTAF, 1996). Research on teaching and teacher preparation clearly points out the need for teachers of mathematics to embrace a pedagogical stance or approach that allows students to garner a deep and conceptual solid understanding of math (McLaughlin & Talbert, 1993). In Teaching for Understanding, Cohen, McLaughlin & Talbert identify the two most common pedagogical approaches, transmission-oriented
approaches and constructivist approaches, highlighting the constructivist approach as the one contemporary teachers must embrace.

This vision of practice signals a sea change in notions of teaching and learning; constructivist ideas about teachers constructing knowledge with learners replace traditional views of teacher as knowledge transmitter and behavioral engineer... assumes substantial new learning on teachers’ part: it requires change not only in what is taught but also in how it is taught... requires comprehensive and in-depth knowledge of subject matter, competence in representation and manipulation of this knowledge in instructional activities, and skill in managing classroom processes in a way that enables active student learning. (p.2, 1993, italics added)

A behaviorist approach to teaching and learning tends to highlight an imitative application of knowledge (received by transmission) in contexts where the desired outcomes are pre-determined (Greeno, Collins & Resnick, 1996). The process is typically sequential and controlled (Greeno et al., 1996). Alternately, a constructivist approach requires teachers to shift their role from that of primary knowledge source to facilitator of learning experiences (Meirink, Meijer, & Verloop, 2007), adopt a reflective stance toward practice (Ball & Cohen, 1999), and alter their expectations for student performance (Common Core State Standards, 2011). Additionally, a constructivist approach tends to focus on the role that the individual plays in his/her own learning (Spillane, 2000). Constructivists highlight the “process of conceptual growth... involving reorganization of concepts in the learner’s understanding... growth in problem solving strategies and metacognitive processes” (Greeno et al., p.16). Thus,
at the heart of the constructivist approach to teaching and learning is the understanding that learning must go beyond assimilation and include the reconceptualization of knowledge.

The process of coming to question and then gradually reconstruct prior beliefs and knowledge is lengthy and difficult. . . . intellectual development unfolds via dissonance and reconstruction, whether used by teachers with student or by teacher educators with teachers” (Nelson & Hammerman, 1996, p.20).

**Constructivism in the Mathematics Education.** While a constructivist approach to teaching and learning math does not have a single, clearly defined set of “instructional strategies,” there are common types of activities in which both teacher and students are regularly engaged (Simon & Schifter, 1991, p. 310). Students work should involve “exploring mathematical problem situations . . . looking for patterns, generating hypotheses . . . and justifying [their] . . . ideas” (Simon & Shifter, 1991, p. 311). Students are to engage in this work by utilizing multiple representations and understanding how those representations are related (Simon & Schifter, 1991). Teachers, in this context, are charged with creating “problem-solving situations” with real world applications, facilitating effective discussions, asking questions that build understanding and assist students in the deconstruction of misconceptions (Simon & Schifter, 1991; Van de Wahl & Lovin, 2005). As a whole, the class should aim to develop a mathematical community of learners (Simon & Schifter, 1991; Van de Wahl & Lovin, 2005). The admonition to engage students in these types of activities can be seen with remarkable similarity in the recently developed *Common Core State Standards*, in particular, *The Common Core State Standards for Mathematical Practice (CCSSM)*. The descriptions of the eight
mathematical practices or mathematical habits students are required to demonstrate include the following statements:

They [mathematically proficient students] make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. . . . Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation...Mathematically proficient students look closely to discern a pattern or structure. (Common Core State Standards Initiative, 2012)

Like Simon & Schifter (1991), the Common Core State Standards for Mathematics requires teachers to model (and students to demonstrate) mathematical practices and mathematical understandings that clearly reflect a constructivist approach to math education.

The Role of Struggle in Mathematical Development

Research in teaching and learning suggests that connecting struggle to a lack of ability is a common perspective that may be more pronounced in Western cultures (Stevenson and Stigler, 1992). Renowned math educator James Stigler argues,

I think that from very early ages we [in America] see struggle as an indicator that you're just not very smart.... It's a sign of low ability — people who are smart don't struggle, they just naturally get it, that's our folk theory (Spiegel, 2012).

Stigler even contends that, in many eastern cultures, not only is struggle considered an accepted part of the learning process, but that persevering in the face of struggle is a valued trait (Spiegel 2012). Stigler relates an incredible tale of one of his early research initiatives. He and his colleagues engaged US and Japanese students in an
impossible math task—one that literally had no solution. The US children worked on the problem for an average of 30 seconds before giving up while the Japanese children struggled for sixty minutes to uncover the solution at which point they were stopped and told the problem could not be solved (Speigel, 2012). The students’ beliefs about the meaning of struggle manifested in the amount of effort they are willing to expend on the task.

Dweck (2006) connects the negative perception of academic struggle to two types of mindsets: fixed mindset and growth mindset. Dweck (2006) contends these mindsets cross cultural boundaries and that individuals with a fixed mindset see their abilities as innate and therefore limited, whereas individuals with a growth mindset see their abilities as plastic or malleable—skills and abilities that when exercised have the potential to grow (Dweck, 2006). The negative perception of struggle is related to, but distinct from math anxiety. Math anxiety is defined as “a feeling of tension, apprehension, or fear that interferes with math performance (Ashcraft, 2002, p. 181). Individuals who experience math anxiety may “become nervous when engaging in math tasks, they also avoid math and math related professions” (Beilock & Maloney, 2015, p.5).

Hiebert & Grouws (2007) introduce the idea of struggle in mathematics as a natural and necessary part of developing conceptual understanding. Commonly referred to as productive struggle, it occurs when “students expend effort to make sense of mathematics, to figure something out that is not immediately apparent” (Hiebert & Grouws, 2007, p. 387). Individuals engaged in productive struggle are challenged to solve math problems “that are within reach and grappling [grapple] with key
mathematical ideas that are comprehensible but not yet well formed” (Hiebert & Grouws, 2007, p. 387). Productive struggle stands in stark contrast to the belief that struggle in mathematics is a sign of limited ability. Productive struggle also distinguishes itself from conceptions of mathematical struggle characterized by “frustration” and “despair” (Hiebert & Grouws, 2007). Teachers who have fixed mindsets about math or hold to the cultural belief that struggle is a sign of lack of ability may transmit that set of beliefs to their students (Stigler & Hiebert, 1999) serving to limit the amount of effort those students are willing to spend on what they perceive as “hard mathematics.”
Theoretical Framework

Constructivism, as a theoretical stance, has not only influenced reformers notions of effective K-12 instructional practice; it has also significantly influenced our beliefs and understandings about the way the adults learn. As such, it provided a solid foundation for this study. In a constructivist framework, both teachers and students are understood to be meaning makers who bring significant learning from their prior experiences into current learning opportunities. These experiences provide a lens through which teachers and students understand and embrace new knowledge (Ball & Cohen, 1999; Bransford, Brown & Cocking, 2000, Kennedy, 1999; Lortie, 1975). Using this framework required that I investigate and interpret teachers’ preservice math experiences in the light of their K-16 math experiences. A constructivist stance, in particular, a social constructivist stance also required that I give equal weight to each teacher’s unique interpretation of their preservice experience. As Thomas’ theorem states, “What is defined or perceived by people as real is real in its consequences” (Patton, 2002, p.96). Consequently, I focused my dissertation on first understanding the experiences of the individual teachers in the study and how they, as stakeholders, made sense of their unique preservice experiences. Only after capturing their unique experiences did I attempt to understand patterns or trends across participant responses.
Conceptual Framework

My goal in this study was to investigate the claims made by Kennedy (1999) and Cochran-Smith (2008) that preservice can serve as a place where teachers’ beliefs about teaching and learning can be substantially altered, if not transformed. In this effort, I attempted to identify cases where teachers reported their preservice experiences as positively and substantially impacting their development as teachers of elementary math. I wanted to understand the ways in which these teachers were positively impacted by preservice (e.g. what teachers learned and how they changed) and the specific preservice experiences that contributed to their development. My primary focus in this study is the impact of the teacher preparation program or preservice experience on the developing teacher.

My conceptual framework highlighted the three experience categories connected with the novice teachers’ development as math educators: K-16 schooling, preservice and professional teaching. I argue that the individual preservice teacher moves through these three experience categories and in fact, each one presents opportunities for significant learning and growth. I proposed that teachers can move through these experience categories with minimal change or substantial change and hypothesized, based on the research conducted in the TELT study (Kennedy, 1999), that the impact of preservice on novice teachers’ development would be mitigated by the degree to which the preservice program actively addressed the teachers’ initial beliefs and understandings about math and how to teach it. As my study investigated teachers’ preservice experiences retrospectively, I also realized that I would be able to capture some data on the way in
which potential changes in teachers’ beliefs and understandings about math and how to teach it might be evidenced in their practice as professional teachers.

Figure 1.2 Conceptual Framework Map

**Preservice Teacher.** The first category in my conceptual framework is the preservice teacher. Clearly preservice teachers do not enter their preparation programs as blank slates. They come with knowledge and understandings that impact the way in which they perceive preservice training. I have six categories of knowledge and understandings that preservice teachers bring to their training program: Knowledge of
Content, Knowledge of Pedagogy, Perceived Math Competence, Teaching Background, Academic Ability, and Long Term Goals. The first three categories are drawn from Ball’s (1988) seminal work, Unlearning to Teach Mathematics. The definition of these constructs as utilized in my research can be found below:

Knowledge of content represents the conceptions preservice teachers bring to their preservice experience about math content or, what math to teach. It focuses on the preservice teachers’ definition of mathematics and serves as a response to the question, what counts as math and what does not? (Ball, 1998)

Knowledge of pedagogy is focused on pedagogical knowledge or how to teach. It serves as a response to the questions, what counts as good teaching and what are the appropriate roles for students and teachers in the learning process? (Ball, 1988). As the program under study has an explicit focus on urban education, included under this heading are teachers’ understandings about what good teaching looks like within the context of urban schools.

Perceived math competence includes preservice teachers’ feelings of satisfaction or lack thereof with their own school performance in mathematics and is consequently related to their perceived ability to be successful teachers of mathematics (Ball, 1988).

The final three categories, teaching background, academic ability and long-term goals are drawn from a different line of research on teacher preparation, specifically the

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3 There is a variety of knowledge and experience that preservice teachers have that is not captured by my six categories. I have chosen these six because research suggests that these six categories are of particular import for teachers’ development of mathematical content knowledge, pedagogical approach.

4 I have made slight alterations to the category names but the content of the categories has been maintained.
work of Humphrey & Weschler (2007) and Moore-Johnson & Birkeland (2008). Both sets of researchers investigated preservice teacher learning in alternative certification programs, development and overall program satisfaction concluding that the teacher preparation program was only one of several powerful influences on the way that preservice teachers developed.

We find teacher development in alternative certification to be a function of the interaction between the program as implemented, the school context in which participants are placed, and the participants’ backgrounds and previous teaching experiences (Humphrey & Weschler, 2007, unnumbered).

Regarding personal background Humphrey & Weschler (2007) found previous experience as a classroom teacher and variation in preservice teachers’ academic ability (as measured by the selectivity of the undergraduate institutions from which they graduated) to significantly impact teacher outcomes. Moore-Johnson & Birkeland (2008) included long term career goals as a background characteristic to measure and found that preparing teachers long term goals career goals impacted their program experience. In my research the three constructs are defined thusly,

Teaching background refers to any opportunities in which preservice teachers served as academic instructors prior to their preservice training.

Academic ability is an attempt to measure general academic aptitude. Humphrey and Weschler (2007) sorted teachers by academic ability based on a proxy—namely the selectivity of their college. Their teachers attended a variety of undergraduate institutions and matriculated thru a variety of different alternative certification programs. The participants in my study, however, were all admitted to and graduated from the same highly selective graduate school of education so I will not attempt to sort my participants
by academic ability. Instead, I have classified them all as having average to above average academic ability.

The term, Long-term goals, reflects an attempt to capture preservice teachers’ intended persistence as classroom instructors. Is teaching a means to another end or an end in itself?

**Penn GSE Teacher Education Program (Penn TEP).** The second object in my conceptual framework represents the Teacher Preparation Program, specifically Penn TEP. The preservice teacher, with her background (e.g. knowledge of math, perceived math competence, etc.) intact, is embedded in a novel context, namely the Penn TEP. In my study, the Penn TEP context includes all program experiences that occur between program start and end. These include, but are not limited to coursework, especially the math methods course, student teaching fieldwork, meetings with supervisors, assignment/projects, special topics sessions and even informal discussions with other members of the cohort. In this sense, the context is bounded by time rather than type of experience. I argue that the preservice teachers’ experiences in the Penn TEP context are a reflection of the dynamic interaction that exists between the program as enacted and the background of the preservice teacher (Humphrey & Weschler, 2007).

**Novice Teacher.** The second circle represents the preservice teacher as a novice teacher one year after the completion of her preparation program. At this juncture, I am interested in understanding how the teacher’s background and preservice experiences have influenced her development as a teacher of mathematics, particularly as it relates to her math subject matter knowledge and espoused pedagogical approach.
**Math Subject Matter Knowledge.** I earlier argued that math subject matter knowledge includes not only “the mathematical knowledge and skill used in settings other than teaching,” what Ball and her colleagues refer to as common content knowledge or CCK (Ball, Thames & Phelps, p.399), but specialized content knowledge (SCK). SCK is mathematical knowledge that is distinct to the work of teaching (Ball, et al, 2008) and includes a teachers’ ability to choose useful instructional models, explain mathematical concepts and analyze student work. In my study, math subject matter knowledge includes both CCK and SCK.

**Pedagogical Approach.** Pedagogical approach refers to teachers’ beliefs and understandings about the way in which students acquire deep conceptual understandings of disciplinary knowledge and the instructional approaches most likely to engender that type of knowledge. For my research I will discuss teachers’ espoused pedagogical approach based on continuum that has transmission-oriented as one endpoint and constructivist as the opposing endpoint. I have chosen to focus on espoused pedagogical approach rather than enacted pedagogical approach because teachers may espouse a set of beliefs and practices that they are unable to actualize. This study is focused primarily on what teachers espouse rather than the potential gap between espoused pedagogical approach and enacted pedagogical approach. While understanding the gap is both important and relevant, it is outside the boundaries of this research study. For a full discussion of the various factors that impact teaching as intended and teaching as enacted see NRC (2010).
CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

My study was a multiple case study with a phenomenological orientation (Finlay, 2009). Consequently, my study was qualitative in nature and foregrounded the experiences and interpretations of the study participants.

Research Context

My study was situated in the University of Pennsylvania’s Graduate School of Education Teacher Education Program Penn TEP, which awards a Master of Science in Elementary Education (M.S.Ed). The Penn TEP embraces a professional orientation to teacher preparation, supported by the belief that “teaching is a profession, like law or medicine, [requires] a substantial amount of education before an individual can become a practitioner” (p. 13). As a result, the program requires preservice teachers to spend significant time focused on coursework and pedagogy prior to entering the classroom as full-time teachers. Toward this end, teachers in the 2012-2013 cohort spent approximately 39 hours in a course dedicated to the study of math content and a full year in a student teaching placement.

The Penn TEP also has a decidedly urban and uniquely local focus (http://tep.gse.upenn.edu/inspire/inspire_why.html). The program seeks to prepare teachers who would be successful in highly diverse, under-resourced, high-need, urban schools and supports teachers in seeking permanent positions within the local school district (Bergey, 2012, personal communication, http://tep.gse.upenn.edu/inspire/inspire_why.html).
The eleven-month program engages preservice teachers in a wide range of courses, including education foundations courses (e.g. child development, school and society, etc.) and methods courses in literacy, math, science, and social studies. The program includes nine months of supervised student teaching and embraces a collaborative, inquiry approach to learning with the goal of seeding the development of teachers who would become life-long learners. Mastery of content and pedagogy are prioritized and preservice teachers serve in a maximum of two School District of Philadelphia schools. Their program claims to have a distinct social justice focus and preservice teachers were challenged to embrace the moral imperatives associated with their work as transformative educators—

Teachers who embrace teaching as a transformative career commit to always being in transformation themselves. We are willing to inquire into our beliefs, assumptions, stances and values in light of new learning emerging from our experiences, readings, and discourse with others. We are inspired to adapt our instruction each year to what we learn about — as well as from — our students. (http://tep.gse.upenn.edu/inspire/inspire_transform.html)

The components are held together by a central theme of inquiry, and throughout the course of study four related inquiry projects are advanced

http://tep.gse.upenn.edu/inquire/inquire_elementary.html). Ideas, practices, and understandings developed in and thru each course, and the student teaching placement(s), are brought to bear in the construction of an inquiry project. These projects culminate in the development of a Master’s Portfolio that chronicles the learning of the individual preservice teachers over the course of the program. In this way, each component of the TEP preservice experience reflects the program’s guiding philosophies of inquiry, collaboration,
leadership, transformation, and equity (http://tep.gse.upenn.edu/inquire/inquire.html). In many ways the design of the TEP program embodies what we know from the research to be core competencies required for successful teachers: strong content knowledge and pedagogical content knowledge (Shulman, 1986); pedagogical training (Wilson, Floden & Ferrini-Mundy, 2002); life-long learning (Day, 1999); inquiry as stance (Cochran-Smith & Lytle, 1999); and learning and collaboration in professional communities (McLaughlin and Talbert, 2006).

**Researcher Identity**

From 2009-2012, I worked in the Penn TEP. From 2009-2011, I served as a Penn mentor, providing direct support for preservice teachers in their student teaching placements, and from 2011-2013 I co-taught the Penn TEP field seminar. I also had the opportunity to observe two complete cycles of the math methods class. I was a graduate student at Penn and received my undergraduate degree from Penn as well. As a result of these experiences I am quite familiar with the program components, requirements, and supports of Penn TEP preservice program. In many ways, as Stakes (1995) contends, I began this research initiative long ago.

There is no particular moment when data gathering begins. It begins before there is commitment to do the study: back-grounding, acquaintance with other cases, first impressions. A considerable proportion of all data is impressionistic, picked up informally as the researcher first becomes acquainted with the case. Many of these early impressions will later be refined or replaced, but the pool of data includes the earliest of observations. (p. 49)

While this level of familiarity provided me with significantly more insight (at the outset of the investigation) than a novel inquirer, it also posed a threat to reliability, or “the need for trustworthiness, accuracy, and dependability of research findings” (Lewis, 2009, p.7).
In response, I was intentional about unpacking shared cultural definitions during interviews. I embedded into the second interview an opportunity to revisit major themes expressed by the teachers, and I habitually requested further clarification. I strove to make the familiar seem unfamiliar. At the same time, I sought to use my familiarity with the teachers and the program to create a safe “conversational space” that allowed for “the co-construction of a convincing tale while allowing interaction to continue unimpeded by embarrassment or shame” (Owens, 2006).

**Study Participants**

The participants in the study are novice elementary school teachers who completed the Penn TEP in the spring of 2013. There were 39 individuals who completed the program in May of 2013. All members of the 2012-2013 elementary cohort were invited to participate in the study via a background survey. Based on the survey results and selection criteria described below, a smaller group of seven focal teachers were selected for participation in the full study.

I initially set specific criteria for teachers whom I would consider using as cases. The criteria for admission as a case included teaching elementary math after completing preservice and identifying the Penn TEP as positively and substantively contributing to their knowledge of math and/or how to teach it. However, only nine teachers responded to the background survey. Of those nine, only seven identified a willingness to participate in the full study. Despite having to use a convenience sample, all the teachers still met the initial criteria. Each teacher reported that preservice substantively
contributed to their knowledge of elementary math and how to teach it and each teacher taught math after graduation.

I also intended to choose cases that would allow for the maximization of ethnic and gender diversity. Again, despite my convenience sample, the teachers engaged in the full study represented an unusually diverse set of teachers; three of the seven were males and two of the seven represented an ethnic minority group. There were only two white females (the dominant demographic for elementary school teachers) in the study.

**Data Sources**

There were six data collection opportunities in this study.

**Figure 3.1 Data Collection Points**

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<td>1</td>
<td>Background Survey</td>
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<td>2</td>
<td>Mathematical Knowledge for Teaching Survey (MKT) - embedded in the background survey</td>
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<td>3</td>
<td>Interview 1</td>
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<td>Interview 2</td>
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<td>5</td>
<td>Constructed Response Lesson Agenda (CRLA) - embedded in Interview 2</td>
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<td>6</td>
<td>Online Master’s Portfolio</td>
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<td>7</td>
<td>Faculty/Staff Interviews</td>
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**Why Case Study?**

Creswell (2007) argued that case study is more than a strategy for participant selection; instead case study encompasses a comprehensive approach to research design. Creswell defined case study as “the study of an issue explored through one or more cases
within a bounded system” (p. 73). Case study could also reference the unit of analysis in a case study or a final report based on case study research (Creswell, 2007). In utilizing case study as a design methodology, cases (the units of analysis) are circumscribed within a particular time and place (Creswell, 2007) and the bounded context of a case is vital to the issue or phenomenon under investigation (Yin, 1994). Creswell, Hanson, Plano-Clark & Morales (2007) linked case study methodology to the investigation of questions that focus on developing a deep understanding of a phenomenon or issue through the insights provided by cases (again, where the case represents the unit of analysis). Moreover, case study has been identified as particularly well suited to studies that seek to provide a picture of a research phenomenon that is integrated and comprehensive (Feagin, Orum & Sjoberg, 1991). Finally, Creswell stated:

The type of problem best suited for this type of research [case study] is one for which it is important to understand several individuals’ common or shared experiences of a phenomenon. It would be important to understand these common experiences in order to develop practices or policies, or to develop a deeper understanding about the features of the phenomenon (Creswell, 2007, p. 60).

My research is predicated on the understanding that learning to teach is a complex, multilayered process, and requires a methodological approach that is integrated and comprehensive (Feagin et al, 1991). Additionally, the bounded context or program in which preservice teachers are trained is integral to understanding the way they develop subject matter knowledge and a pedagogical approach.

In my study, the specific phenomenon under investigation was the development of novice teachers’ math subject matter knowledge and pedagogical approach for teaching math. The bounded context of the study was the preservice teaching experience. The
case, or unit of analysis, was the individual novice teacher. I sought, as Creswell (2007) explicated, to strategically choose cases to better understand how positively perceived preservice experiences influenced novice teachers’ knowledge of math and espoused pedagogical approaches. Serendipitously, my convenience sample met that criteria. As a result, my study was an instrumental case study. Moreover, as several cases were observed, the study was a multiple case study (Yin, 1994).

Yin (1994) also argued that a critical component of case study research is a priori theoretical propositions (Yin, 1994). My study rested, theoretically, on two propositions about the way in which preservice teachers learn to teach math:

1. The actual preparation program is one of several interrelated contributors to the way preservice teachers’ knowledge of math content and pedagogy develop; teachers’ prior knowledge and math background are also significant contributors (Ball 1988; Kennedy, 1999).

2. New knowledge/experiences and conflicting knowledge/experiences represent critical sites for teacher learning and development (Kennedy, 1999).

However, these propositions are just that, propositions. While case study “benefits from the prior development of theoretical propositions to guide data collection and analysis” (Yin, 1994, p.13) it does not wholly circumscribe the domain of participants’ responses. In my study, the propositions helped to define the parameters of the study and provided a starting point for questioning. Given the dearth of research on the way in which preservice teachers develop in and through preservice, it was critically important my
approach allowed for significant latitude in the study design and in the participants’ responses; the case study method allowed for this.

I questioned novice teachers about the preservice experiences that influenced their knowledge of and approach to teaching math after they had completed two years of post-preservice teaching\(^5\). Retrospective studies, while not the most common form of case study, are valid uses of the case study methodology (Creswell, 2007). My study, in fact, benefits significantly from retrospection, as teachers’ perceptions and understandings of their preservice experiences gain or lose legitimacy in the context of their formal teaching experiences.

**Why a Phenomenological Orientation?**

According to Lester (1999), phenomenological research is designed to,

“illuminate the specific, to identify phenomena through how they are perceived by the actors in a situation…to [gather] ‘deep’ information and perceptions through inductive, qualitative methods such as interviews, discussions and participant observation, and [represent] it from the perspective of the research participant(s)” (p. 1).

There are several phenomenological traditions and contentions that persist concerning what genuinely qualifies as phenomenological research (Finlay, 2009). The Husserlian tradition calls for reduction or bracketing; essentially freeing oneself from “any contamination that presuppositions of conceptual framework or psyche might contribute” (Cogan, Internet Encyclopedia of Philosophy, n.d.). There are also several

\(^5\) At the time of the second interview some teachers had already begun their third year of teaching. One teacher, Dennis, had his second interview at the start of his fourth year of teaching.
branches of phenomenology which do not hold to the necessity of bracketing/reductions (see Smith & Osborne, 2007). Finlay (2009) concluded that,

A genuinely psychological qualitative method implicitly uses the descriptive psychological reflection so characteristic of the phenomenological approach. In such cases, it is perhaps best to view research which does not fully embrace the phenomenological project’s commitment to description, and the researcher having an open phenomenological attitude (if not actually applying specific reductions), as phenomenologically inspired or phenomenologically orientated. (p.9)

In keeping with Finlay’s (2009) classification, this study was designed to be a phenomenologically oriented because I was particularly interested in understanding the preservice experience (i.e., the lived experience) from the perspective of the novice teacher. I used the primarily methodological tool of phenomenological research—the semi-structured, in-depth interview—as my primary research tool (Smith & Osborn, 2007). I also focused the proposed interview questions on the specific and significant preservice experiences of novice teachers and, equally important, their understanding of those experiences after having taught math professionally. I did not attempt to engage in bracketing and would argue that at some level bracketing is impossible (Finlay, 2009).

Finally, while I hoped to identify some commonalities that were associated with novice teachers who rate their preservice experiences as highly significant, my primary research imperative is to understand the individual experiences of each case and the relationship between the novice teachers’ background and the preservice training program. Finlay (2009) supports that an individual phenomenological study can potentially answer both questions. She argues that idiographic phenomenological research can also be nomothetic, in that “it may well identify general structures of experience” (p.10).
Data Collection and Analysis

The background survey was designed to capture information about the entire cohort’s perception of the influence of preservice on their mathematical knowledge and pedagogical approach. It was divided into two parts. The first part included questions about novice teachers’ math experiences prior to preservice and asked novice teachers to rate the significance of their preservice experience as it related to learning to teach math. The first part of the survey also included demographic questions (see Appendix III). The second portion of the survey was an assessment of teachers’ common and specialized math content knowledge (MKT) (see Appendix IV). The MKT survey questions were drawn from the 2008 Numbers and Operations scale.

In April of 2015 I drafted an email requesting that all the graduates of the 2012-2013 cohort complete the background survey. The email was sent to the coordinator for Penn TEP. No responses were generated. I sent several follow up emails via the program coordinator requesting teachers’ participation but received no responses. In June of 2015, I then reached out directly to the members of the 2012-2013 cohort via email. The responses were minimal. Finally, I sent individualized emails to each member of the cohort. In total, nine teachers (20-25% of the total cohort) completed the background survey, seven of whom expressed a willingness to participate in the full study. The background surveys were collected and analyzed for patterns in learning (see Appendix V).

The seven teachers who expressed a willingness to participate in the full study were contacted by email and scheduled for a first interview. Interview one began with a
discussion of the mathematical knowledge for teaching survey (MKT) and generally moved to a discussion of teachers K-16 math experiences. The remainder of the questions for interview one were focused primarily on RQ1, “How, and in what ways, does preservice education contribute to the development of a novice teachers’ knowledge of elementary math content?” Before the interview the teachers were asked to review their online masters’ portfolio and provide me with a link to the portfolio. I used the online portfolios, when accessible, to develop individualized probes for the interviews. I also asked teachers to review their portfolios before the interview. The interviews were conducted by phone and professionally transcribed.

After the first interview was completed each teacher was sent a request asking for their participation in the second interview. Interview two focused primarily on RQ2 which asks, “How, and in what ways does preservice education contribute to the development of novice teachers’ espoused pedagogical approach to teaching elementary math?” Embedded in interview two was a constructed response lesson agenda or CRLA. The CRLA, as its name suggests was an open-ended lesson plan agenda that gave teachers an opportunity to design a lesson (a list of ordered activities in which students

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6 “Constructed-response questions are assessment items that ask students to apply knowledge, skills, and critical thinking abilities to real-world, standards-driven performance tasks. Sometimes called “open-response” items, constructed-response questions are so named because there is often more than one way to correctly answer the question, and they require students to “construct” or develop their own answers without the benefit of any suggestions or choices.” (Tankersley, K., 2007). Copyright © 2007 by Association for Supervision and Curriculum Development. Retrieved 2.17.17 http://www.ascd.org/publications/books/107022/chapters/Constructed-Response-Connecting-Performance-and-Assessment.aspx

7 I refer to the tool as a lesson plan agenda because the teachers were not required to include the level of detail that might be required in a standard lesson plan. Instead they were asked to identify the activities in which students would be engaged, the order of those activities and the corresponding role of the teacher.
and teachers would engage) around a predetermined common core mathematical standard. The standard provided to all the teachers was, CCSSM 3.0A.A2. which states,

Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8. (http://www.corestandards.org/Math/Content/3/OA/)

Teachers were given the opportunity to choose a different standard if the one provided proved to be so unfamiliar that it would impede the teachers’ ability to design a lesson that reflected their espoused pedagogical approach. All the teachers chose to design lessons around the standard provided.

Prior to engaging in interview two, the teachers were required to send a completed electronic version of the CRLA to me via email (see Appendix X). During the interview the teachers used their completed CRLA to guide their description of their lesson. While the teachers described their lessons, I created a list of the pedagogies and practices they identified. I attempted to use their language whenever possible. After the teachers finished describing their lessons, I read through the list of pedagogies and practices verifying the accuracy of each pedagogy or practice recorded and providing the teachers with an opportunity to clarify or further expound on any point (see Appendix XI). The lesson description portion of the CRLA was not coded, instead a table matrix was created to identify similarities in instructional approaches of the study participants.
In keeping with the phenomenological orientation of my research I initially engaged in open coding of interview one, as suggested by interpretive phenomenological analysis (Smith & Osborn, 2007). The process begins with an open textual analysis that was designed to capture my initial feelings about, understandings of and connections to the interview data (Smith & Osborne, 2007). I read many of the interviews several times, initially making notes about anything I noticed. In the second round of coding, I organized the data from the interviews into large buckets based on the questions for which the data seemed to be a response, regularly asking myself, “what is this sentence about” (Ryan & Bernard, 2003, p.91). I found this approach to provide me with a greater insight into the teachers’ background and preservice experiences. I color coded the data by case and created individual word documents based experiences categories (e.g., K-16 background), questions and potential themes. This enabled me to more look for patterns both within and across cases. I then used the “constant comparison method” to analyze large chunks of related text (Glaser & Strauss, 1967; Ryan & Bernard, 2003). Using the data from the individual interviews, I wrote case studies that reflected each teachers’ journey from preservice to professional teaching, highlighting the experience categories identified in my conceptual framework: K-16 schooling, preservice, professional teaching. This was an iterative process and I frequently returned to the data to confirm my individual cases synopses. As is true of all phenomenological research, it is important to stay close to participants’ transcribed interviews…and in my case, narratives.
I then looked across the cases to identify emerging themes; these themes were emerging throughout and I kept regular notes and memos about patterns that were emerging. After identifying and writing about the emerging themes I recoded my interview data using NVIVO. This was an internal validity check. Based on my recoding, a sub-theme related to urban schooling emerged. In describing the new sub-theme, I found that specific information about participants would be divulged that could be used to identify the participants. I contacted each of those three participants and requested their permission to discuss the details of the theme even though it might reveal their identity. Two of the three teachers agreed and I amended by findings based on their response. Finally, I sent the completed case write ups to five of the seven study participants for an external validity check. Two teachers responded with either a minor clarification or update and their individual cases were revised accordingly.

8 Crystal only completed interview one so she does not have a case study write up. Daniel’s second interview was delayed due to demands on his time rendering me unable to send his case write up in a timely fashion.
CHAPTER 4: DESCRIPTIVE CASES

Introduction

As previously stated this study was designed as a retrospective case study focused on the ways that preservice contributes to elementary school teachers’ knowledge of math content and how to teach it. Engaging teachers two to three years post-graduation allowed teachers to interpret their preservice experiences in the light of their current work as professional teachers. The teachers were given space to tell the story of their math learning and in that process of reflection to editorialize or imbue meaning and understanding colored by the lenses of time and new experiences. While a retrospective lens may not always provide the most objective story, its power resides in the degree to which it accurately represents each teacher’s current understanding of his or her experiences. In that sense, the teachers’ interpretation of their lived experience is most important for understanding how their lived experiences have impacted and continue to impact their present work as teachers. As Thomas’ theorem states, “What is defined or perceived by people as real is real in its consequences” (Patton, 2002, p.96).

The data for chapter four is drawn from three sources: the background survey, interview one (including the MKT) and interview two (including the CRLA) and represents the six full study participants who completed both in-depth interviews. Six of the seven full study participants completed both the first and second in-depth interviews; three males and three females. T. Crystal was unable to complete the second interview and does not have a case summary. The findings are organized into the three experience categories identified in the contextual framework: K-16 background, preservice, and
professional teaching. The findings described in experience category two, learning from
preservice, are a direct response to RQ1 and RQ2 and focus on what teachers learned
about math content and pedagogy from preservice. The teachers have been assigned the
following pseudonyms: Teacher Isa, Teacher Gwen, Teacher Lacey, Teacher Dennis,
Teacher Myles and Teacher Michael.
Teacher Gwen

**K-16 Math Background.** “I had really great math classes throughout my life. . . and I really do like math” (Gwen).

Gwen spent her formative years attending a Catholic elementary school where she excelled and gained confidence in her mathematical ability. In retrospect, she recognizes that her early strength in math was directly related to her ability to commit required facts to memory.

I was good at math at that school because it was just like, “Okay who can do their times test the fastest?” . . . I think that at least gave me the confidence that, oh yeah, I’m awesome at math, even if I was really just regular. (Gwen)

Gwen continued to excel in math through her “drill-and-skill style” middle school. It was, however, at her all-girls high school where she met and seemingly conquered her first mathematical struggle. The high school teacher she had for grades 9, 10 and 12 coached a monthly math competition called Math League⁹ and required that all the freshman in Algebra II participate. “We would take these tests once a month and get—best case scenario—like one out of six, because it was so, so, so, hard . . . it was humiliating (Gwen).”

The Math League coach encouraged all the students to persist, and promised them that if they did, the work would not only become doable, but easy. After persevering in

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⁹ Math League is an organization that seeks to promote student learning in mathematics through competitive contests. For more information see: http://www.mathleague.com/index.php/about-the-math-league/statement-of-purpose
the Math League for two years, Gwen found that her teacher’s reassuring words had proved true.

Me and a few other girls stuck with it, and, yeah, sure enough, by junior year we were crushing it, and it was awesome. We felt so impressed with ourselves because those questions are . . . kind of deep level understanding questions. (Gwen)

Gwen went on to take AP Calculus. She was “on the track” and her parents would not have been satisfied with anything less (Gwen).

My parents are pretty not flexible. You will take the hardest thing that you are allowed to take at school. My mom tried to convince me that if I put that I wanted to be a math major on all of my college applications, I would be more likely to get in. It was part of that whole plan. (Gwen)

Despite having the strong encouragement of her parents to make math a priority, overcoming a genuine math struggle to find success in Math League, and then getting a five on the AP Calculus test, Gwen did not choose to major in math. In fact, after two semesters of math at Preparation University, Gwen decided to major in Government and Spanish. Her rationale:

The college course I took was Stat, which I think was very different from everything else I’d done. I just felt really under pressure in that class. Maybe if I had taken whatever comes after Calc and Calc II, maybe that would have been different, but Stats was a disaster and it was enough for me to say, “I’m never going into this math building at Preparation University ever again.” (Gwen)

**Learning From Preservice.**

I feel like almost everything I know about teaching math is from preservice. I don’t think I’ve learned a whole lot since I’ve left Penn about how to teach math. Maybe some nitty-gritty of different curriculum, but the big picture things, I definitely attribute to Penn. (Gwen)
Gwen entered the preservice program at Penn confident about having a strong math background, knowledge of elementary math, and ability to learn to teach it. Now in her third year of teaching, she identifies the preservice program at Penn as the most influential contributor to her beliefs and understandings about teaching math. Since completing the program, Gwen has faced numerous challenges in teaching math, but none for which she felt unprepared.

Nothing has been thrown at me in terms of math that I have felt unprepared to do because of Penn. Even in the CGI (Cognitively Guided Instruction) crash course, the part that people . . . were super stressed about were these videos . . . we watched—I don’t know—40 of them in Charlotte’s class. It was kids solving problems and you had to code what type of problem it was . . . keep track of the kids’ methods, kind of broken down into a code. I’d done that a bazillion [sic] times before. And I feel like that was definitely the most rigorous math PD I’d been to since I’ve been a teacher. . . . Nothing has come up in a conversation about teaching math that has thrown me for a loop, or I felt that I was not able to be a participant in. (Gwen)

Preservice was a pivotal transition for Gwen. The experience helped her to construct a set of beliefs about teaching and also challenged her image of good math instruction. One of Gwen’s fundamental beliefs about teaching and learning is centered on knowing students and developing an instructional plan that uses their knowledge as a starting point for continued learning. This is a belief which she saw underscored throughout the Penn preservice program, “But then—more big picture—Penn . . . really taught me the importance of meeting kids where they are.” (Gwen)

This constructivist view of meeting students where they are by ensuring that instruction is aligned to their demonstrated expertise has become a part of Gwen’s overall instructional approach. Her belief was strongly challenged during her time at Grenich
Elementary School, so she opted to find a new teaching placement, rather than be forced to teach in a manner which she had identified as conflicting with that belief.

Gwen claims that the TEP program at Penn produced a change in her image of good math instruction. Prior to the preservice program, Gwen’s image of good math instruction was singularly focused on teacher-driven direct instruction—providing students with explicit step-by-step directives on a procedure aimed at solving a problem.

Before [preservice], I thought it [good math teaching] was . . . explicitly teaching how to do something . . . being really explicit, like, okay, do this, and then this, and then this, and that’s how you do it. And not really caring about the why . . . I never even heard of an invented algorithm before preservice. (Gwen)

In lesson plans that she created specifically for this research project, Gwen connected her thinking about instruction directly to her learning at Penn. Though required in her school, direct instruction constituted only five to ten minutes of the entire lesson. The lesson began with students working to solve a novel problem before being instructed formally on the concept or given a procedure to follow. Gwen called this portion of her lesson, “inquiry” and argued that starting a lesson with students working on a task increased engagement. She claimed that she learned this practice at Penn. Her lesson also included repeated opportunities for student discussion, as she believes that students learn from attending to the approaches of their peers. Like beginning with a novel problem, Gwen connected this idea to her preservice experience.

The concept of more than one way to do something, I kind of . . . knew that, but it was explicitly said so much and shown so much from the way different teachers were solving problems in our math methods course, that was kind of driven home to me. (Gwen)
In addition to impacting Gwen’s beliefs, the preservice experience also provided her with practical instructional skills. Among those she cited was the ability to unpack mathematical ideas to effectively communicate with students.

Just the breaking it down—I know all the end stuff. I know how to get answers, and I know how to get to the end. But going back four steps and thinking about it on that lower level . . . I definitely needed Penn to teach me how to teach math, for sure. (Gwen)

Gwen found this approach to be particularly necessary in geometry, a subject she did not remember studying as an elementary school student. Her first recollection of geometry was in high school. “Yeah, again it wasn’t like I didn’t have the mathematical knowledge… I feel like I accessed geometry from a higher point instead of . . . from a lower point which . . . was different for me.” (Gwen)

While Gwen sees the impact of preservice on her math instruction as primarily pedagogical, she also notes the ways in which preservice increased her knowledge of elementary math content. Gwen credits preservice as generally providing her with novel understandings of familiar math ideas, particularly understandings of number quantity and the conceptual connections between mathematics procedures.

It [the methods class] was bringing up things that I knew, but had never specifically heard about, how numbers work together, and their relationship in math—that kind of thing. (Gwen)

Gwen’s revelation about multiplication provides a perfect illustration of a math concept that she knew but had not previously understood in the context of related mathematical ideas.
This is ridiculous to say, I’m pretty sure I hadn’t thought about multiplication as repeated addition until I was in Penn. . . . the minute somebody says it to you, you’re like, “duh, of course that’s what it is,” but I hadn’t thought about it that way before. (Gwen)

Through the preservice program, Gwen also claimed to have been introduced to math strategies and practices that were fundamental to building student understanding of number and quantity, specifically subitizing and counting on.

I don’t think I ever previously thought about the importance of counting on before Charlotte [math methods teacher] . . . said something, and I remember doing it with a few of my kids for . . . the small lesson [while student teaching]. (Gwen)

Throughout my conversations with Gwen, she repeatedly cited the important influence of her math methods course. “In terms of what I do every single day, and how I do it, I think about Charlotte’s class [math methods] a lot (Gwen).”

When compared with the other methods classes she took in preservice, she considers the math methods course to have been the most effective in preparing her to “teach something (Gwen).” In her student teaching experiences, however, Gwen found little that supported her development as a teacher of mathematics.

There were two ways in which Gwen’s time in preservice fell short of her hopes. The first concerned using technologically driven data to improve the teaching and learning of mathematics. The second, and more fundamental, critique offered by Gwen was the inability to see the approaches to math she was being exposed to in her preservice classes realized in an actual classroom. For example, during preservice she fully embraced the idea of project-based learning and while she could see components of it in some of her Penn related school visits, she felt that she never got the complete picture.
So, I would learn all these wonderful, ideal ways to do things in the classroom, and I would be totally on board with it, and then I’d have very mixed success implementing them into the classroom. And even mixed . . . it would be really hard for me to find a classroom that was actually implementing those things. And that is what I feel like I was constantly hounding Grace [Penn GSE Coordinator] for. And she found me a great placement for the second semester, but even then, it’s still Raul [second semester classroom teacher] couldn’t do everything, or wasn’t doing everything the way that I was learning how to do it at Penn for a variety of reasons (Gwen).

**Teaching After Preservice.** “I’ve been a pathetic lone wolf for the most part at my old school (Gwen).

Gwen is entering her third year of teaching and just started a new position as a teacher at Sloane West, a network charter school in Herald City. Herald City is a large, urban district. Before coming to Sloane West, Gwen served for two years as a first grade teacher at Grenich Elementary School, a district-run public school in the same city as Sloane West. Her transition from a district-administered school to the charter system was enabled, in part, by relationships built and strengthened through preservice.

Devon Hunt . . . she did the program [Gwen’s preservice program] a couple of years before I did . . . She is a principal at a different Sloane school, and I’ve been to visit that one a dozen times. I was emailing her about something else, and she was like [sic], “Hey when are you going to come to Sloane? My friend has an opening for second grade.” It was like on the heels of a really horrible week at my public school. I was like, “Okay, I’m ready.” Literally, four days later, I had the job, so it was awesome. (Gwen)

While the process of acquiring the position at Sloan West was very brief, the decision to transition was not haphazard. In fact, Gwen had spent the balance of her tenure at Grenich feeling like a “pathetic lone wolf (Gwen).” Six months before leaving Grenich she had a difficult conversation with her principal in which she asserted her intention to meet her students’ academic needs despite the prescribed curricular
mandates. She argued that her principal’s inability to understand her rationale was the, “ultimate reason” she left Grenich (Gwen).

I was getting reamed out for teaching something that they thought was too basic, and I was like but let me show you the data from three days ago when they [her students] failed to retell a story from beginning, middle, to end. Let me explain to you why I had to take a step back and do it this way—total inability to comprehend the need to do that. I was like, “All right, I’m done here.” (Gwen)

The environment Gwen found at Sloane West differed markedly from the one she had left at Grenich. At Sloane, there was mandatory summer school, a longer school day—7:45am to 4:30pm, and Saturday school every six weeks, but according to Gwen these additional instructional demands were far outweighed by the resources and support Sloane West provided.

It is a long day and the kids come at 7:45….But I mean it’s really structured. It’s night and day difference from my old school. I have so much support, it’s amazing…and free copy paper. Like a copy room full of copy paper. That’s never happened to me for before! (Gwen)

The difference between Grenich and Sloane West is particularly salient in the support Gwen receives for math instruction. At Grenich, Gwen bore the full weight of adapting the curriculum she was given to meet the needs of her students.

Herald City District has a scope and sequence, but at my school there weren’t really the resources to teach it. I tried to follow it my first year, but it was really, really vague . . . Finally another teacher introduced me to Engage New York. From that point on . . . I used that curriculum and used my own curriculum . . . It wasn’t Common Core . . . but it was . . . way better than the one that the school had bought for us. (Gwen)

Conversely, at Sloane West Gwen has a math coach who provides lesson support for all the elementary teachers in the network.
We have this wonderful, wonderful woman for all of the Sloane Schools in Herald City. She makes these little PowerPoints for us and the exit slips . . . . She also uploads to YouTube three or four minutes of her quickly teaching the mini lesson portion of the videos and telling the teachers what the important things are to hit . . . . It shaves hours off my planning time. (Gwen)

Gwen’s admiration for the Sloane math coach goes beyond her appreciation for having her planning time diminished; it is rooted in her perception of the coach’s mathematical expertise. “She’s awesome. [The math coach] knows her stuff, and I trust her (Gwen).” Gwen also identifies a similarity between the Sloane math coach and Charlotte, Gwen’s math methods professor.

I met her [the math coach] and one of the first things she said was one of the exact same things Charlotte said on our first day of math methods, “People would never say, ‘I’m a bad reader’ but a lot of adults have no problem saying, ‘I’m bad at math.’” I was like, “Oh my God, Charlotte said these exact words three years ago.” (Gwen)

Despite Gwen’s faith in her math coach and being generally pleased with her new school placement, she remains troubled by the rigidity of classroom structure and prescribed instructional approach in which she is engaged at Sloane West. Gwen’s frustration is being intensified because of her active participation in progressive education teacher groups which began at Penn, and her strongly held belief that the most effective math instruction is developed in response to student needs. As a result, she is concerned by Sloane West’s dependence upon strategies that, she argues, “would not fly in a middle class school (Gwen).”

Every type of kid deserves the same kind of chance to make meaning and . . . . to understand what’s going on, and it shouldn’t have to be rammed down their throats because of where they live, which is sometimes how I feel. ‘Well this kind of teaching works best for kids from blah, blah, blah background,’ which drives
me up the wall. So, yeah, I think good teaching is good teaching regardless of who your kids are. (Gwen)

Gwen has not shared her concerns with her school leadership, but continues to attempt to balance her deeply held belief in responsive teaching with the instructional and accountability demands placed on her at Sloane West.
Teacher Myles

K-16 Math Background. “It’s funny, I have a lot of memories from elementary school, but none of them involve math” (Myles).

Myles has very few recollections of his math experiences before high school. In fact, his K-16 background is characterized by a lack of emphasis on math. Beginning in middle school Myles decided that his academic inclination, which he called “identity,” was “artistic or literacy-minded” (Myles),

I definitely know that I developed an identity as somebody that fell into that dichotomy of you’re either an artistic or literacy-minded person, or you’re a math and science person . . . [in] middle school and high school, I think I started to get a lot more excited about my language arts classes than my math classes. (Myles)

Myles was on the college track, so he took math class throughout high school, but he did not find any pleasure in the course. He described the classes as “lecturing and you do some problems in your book” (Myles). In fact, Myles had not really considered that his math classes could have been any better.

I do know that I didn’t enjoy those classes [high school math classes], so I think I probably in some way knew it wasn’t good. Although, I don’t know that I knew that math could be good. That sounds terrible but . . . I think I was just kind of like, ‘This is what math classes are. I don’t like math that much, so it makes sense that I don’t enjoy it’ (Myles).

His last two high school math classes were Pre-Calculus/Advanced Math and AP Statistics. Unlike Geometry, the Pre-Calculus class proved to be extremely challenging and the challenge was reflected in Myles’ grades. “I remember definitely getting lower grades on those tests than in any other subject. I was feeling like this just isn’t my strength” (Myles).
Myles’ final high school math class was AP Statistics, a seeming departure for Myles, given his lack of enthusiasm for math. However, Myles’ motivation for taking the course was related primarily to the reputation of the teacher.

I took AP stats. That particular teacher was not a very demanding teacher. AP stats basically seemed like the easiest choice based on who the teacher was; . . . [he was] a little bit lax. He encouraged us not to take the AP stats test. (Myles)

By the time Myles entered college Myles had decided to actively avoid math and ended up passing a math exemption class. When I went to college I tried to avoid math. I took, I think, a calculus course and dropped out during the add/drop period because I just couldn’t do it. I took the exemption test and didn’t have to take another math course in college. (Myles)

Myles majored in English and did not take another math course until he applied to Penn, having been informed that he would be unable to get a Pennsylvania teaching certificate without having a math class on his college transcript. He no longer remembers the name of the online course he took, but he does remember the motivation behind his choice, “It was the easiest one” (Myles).

I definitely think I would be a different math teacher without that experience [preservice program at Penn] for sure . . . I think my math instruction is most influenced by my time there [preservice program at Penn], maybe more than the other subjects. (Myles)

**Learning from Preservice.** Unlike most of the other participants in this study, Myles entered the Penn preservice program with previous teaching experience. When Myles was growing up, his mother worked in an after-school program at a local elementary school and he spent a great deal of time there. He went from a “formal volunteer” to an actual employee and continued to work summers while in college (Myles). After college, Myles became an AmeriCorps literacy tutor and even spent a year as private tutor. Essentially, all his work experience from childhood was connected
to teaching children. Still, Myles maintains that a large part of who he is as a teacher is attributable to Penn.

I feel like, to be honest, to be completely honest . . . I feel like I’m a successful teacher, given where I am in my career. I do owe, not all that, but certainly a big chunk of that to my time at Penn. (Myles)

When considering his overall experience in the Penn preservice program, Myles stated that one of the most impactful components of his time there was the math methods course.

I do think that my math methods course with Professor Abigail stands out for sure. I think what stuck with me . . . maybe more than the other courses . . . I came away feeling like I had a lot more skills that I could put into place the next day. (Myles)

According to Myles, Professor Abigail not only “convinced” him there was a “right way to teach math,” but she helped him understand some of the instructional priorities associated with the “right way (Myles).” Myles summarized these priorities as follows:

[There was] an emphasis on communication . . . on multiple strategies . . . building on other people’s thinking, and building reasoning; rather than, “Here are the five steps you need to follow and memorize.” (Myles)

In math methods, Myles also learned an instructional model that highlighted the priorities identified above. The model involved starting lessons by engaging students in open-ended math problems followed by whole-class discussions that showcased students’ solution attempts. This was a weekly occurrence in the math methods class, and after having seen the impact of the practice on himself and his peers, Myles quickly became convinced of the practices’ potential.
I was just pretty blown away by what some of my classmates had done to get that answer, and what their thinking was. That was eye-opening—just to realize there are so many ways to tackle a seemingly simple problem, a straightforward problem. . . . to realize that there were 20 of us in the class, and each one of us had a different explanation or a different approach. . . . It was really fascinating to see what they had done. It was fun. That was the other thing, it was kind of fun. (Myles)

In Abigail’s instruction and the readings assigned in math methods, Myles also learned about the power of the teachers’ role in questioning students, as well as in rephrasing their work to build understanding and clarify concepts for students who might be struggling. “Abigail, again,” Myles said, “modeled that for us in the beginning parts of those classes too (Myles).” The math methods class also helped Myles to understand that incomplete or incorrect solutions were often worth sharing, that well-handled misconceptions could enhance learning.

The second most impactful experience Myles had in preservice was student teaching. Myles credits student teaching with toppling the illusion of classroom teaching that he had developed from many years of primarily tutoring individuals or small groups of students.

Penn gave me that first realistic, not glamorous, feel of teaching. . . . I worked in schools, but you only have some idea of what it’s like to be a teacher every day. It’s not until you’re there every single day, all day long that you realize what it’s like. (Myles)

In addition to providing Myles with insight into the reality and demands of being an elementary school teacher, his student teaching experience also provided him with a pattern for how to run a classroom.

It’s funny, now I realize so many things I picked up from my mentor teacher [the teacher of the classroom where Myles was a student teacher] once I started
teaching myself. . . . What my classroom mentor did, I realized in my first year in particular . . . just really influenced what I thought a day should look like in a classroom because that was my model. . . . That just really influenced a lot of my thinking still. I’m starting to change what I do as I develop my own routines. (Myles)

Thus, it was Myles’ student teaching experience that most addressed his primary concerns as a new teacher: concerns centered more on the orchestration of managing a classroom than on the finer points of mathematical pedagogy.

I mean, these are the questions I had on my first day. It wasn’t like, “How am I going to teach math?” It was like, “What do I do when they first come in? What do I do when it’s time for recess?”—all those questions. “How do I manage this and keep track of that?” (Myles)

In fact, one of the only regrets Myles identifies in relation to the preservice program is that he did not spend more time visiting other classes to examine other models for the overall management and orchestration of the classroom. He spent the entire year in the same classroom and did not take days off to visit other schools.

I feel like, to be honest, to be completely honest . . . I feel like I’m a successful teacher, given where I am in my career. I do owe, not all that, but certainly a big chunk of that, to my time at Penn (Myles).

**Teaching After Preservice.** Myles has been teaching fifth grade in the same school since he completed the preservice program at Penn. He serves in an unusually diverse school and in a region of the country that has become a haven for individuals seeking asylum. Without question, Myles is pleased with his current placement and sees continuing opportunities for growth. He is a part of a team, participates in professional development and is even making some forays into leadership.
I love my school. I had [sic] a great team, great team. I’m on the leadership committee this year at the school, feeling like it’s a good place for me to be.

Myles’ school uses the enVision math curriculum and he generally follows the lesson structure outlined in those materials. Myles’ rationale for following the curriculum goes beyond the fact that his school has purchased the enVision program. He argues that the curriculum reflects his own beliefs about good math instruction garnered from his teaching experiences, professional development, and primarily his training at Penn. For example, Myles typically begins his lessons with an open-ended question that students are required to solve and discuss before receiving direct instruction. That approach is the model of instruction that Myles learned in his math methods class at Penn.

The first time that I looked at ... enVision, [I was] thinking, “Oh, this is kind of like what ... Abigail had talked about.” That’s kind of how ... lessons with us worked. She would give us, for homework, a math problem. We would always start our class with her going over and sharing our work to solve that problem.10 (Myles)

The way he moderates discussions of student solution strategies reflects what Professor Abigail modeled in preservice. However, he has also been powerfully influenced by the Talk Science discussion strategies he has learned in professional development sessions at his current school.

It’s been great. The level of discussion, even with the real little kids, the primary, has been really impressive. I’ve been working to incorporate these talk moves and everything into my math because I think it's really a good fit. (Myles)

10 Myles’ school is engaged in partnership with a local university. They are developing a discussion-based science program called Talk Science.
Myles also embraces the direct instruction portion of the enVision lesson. He sees it as particularly vital to students who are struggling. The experiences that convinced him of the critical role of direct instruction preceded his time at Penn and were in fact lessons learned while teaching literacy.

I actually think this one goes back a little bit further than the other stuff . . . Everything I’ve talked about, so far, all started at Penn. This, kind of, goes back to Before, when I was tutoring . . . I was working with kids who were struggling to read. What I’ve seen first-hand with that—certain kids were really benefiting from just direct phonics-based instruction. . . . Kids who were kind of defeated, I could see this working for them. It wasn’t that exciting. It wasn’t that interesting, but the kids are excited because they finally felt successful. I think I carried that with me. (Myles)

Finally, Myles also seems to be developing strategies of his own as he develops as a teacher. He allows students to opt out of guided instruction (a designated enVision math segment) if they don’t need it and go directly to independent practice. He also provides a number of different materials for students to work on once they have completed the assignment for the day.

The idea of giving kids options for when they’ve completed work—I don’t know where that came from. I think I just had a sense that that’s what teachers have to do—give kids options. The trick has been, as I’ve continued to teach, I’ve found better and better resources. I’m able to give better options. (Myles)
Teacher Lacey

K-16 Math Background. “I remember if I didn’t get the right answer the other answer was just wrong . . . there was nothing else with it, you’re just wrong” (Lacey).

Lacey remembers being good at math. It was third grade and the math was, “simple arithmetic.” Lacey continued to perform well in her math classes through the beginning of high school. She took a rigorous sequence of high school math courses that included Algebra I and II, Trigonometry, Calculus I and Calculus II, but found less success with the latter of courses. Overall, she considers her high school math experience to have been “difficult” and “intense.” She had to get a tutor and concluded that her strength in math was limited to computation.

Certain things I was good at . . . computation, like Trigonometry or Algebra I, but then when we got to bigger and in depth concepts, sort of like Algebra I or Calc II or Physics of Calculus, I remember it being a struggle. (Lacey)

Lacey did not take any math classes in college, nor did she feel that all the intense math courses she took in high school had a long-lasting effect on her knowledge of the discipline. “I tested out the summer before I went to college and then apparently forgot everything I learned” (Lacey).

It is noteworthy that Lacey was the only study participant to report feeling fear, dread or anxiety when approaching certain math tasks.

But also math is kinda scary . . . once I got to the second page [of the mathematical knowledge for teaching survey] I was like, okay anxiety is building. I don’t want to do this . . . It [math] still brings up a little anxiety even though I think I’m good at math. It still brings up a little anxiety. (Lacey)

Lacey traced the source of her fear back to her childhood,
I grew up in the Cape Vert so we have, you know, so we’re pretty traditional in terms of learning math, so I remember if I didn’t get the right answer the other answer was just wrong. There was no, there was nothing else with it, “you’re just wrong.” So I remember not wanting to relive that feeling of being wrong at 26, you know. (Lacey)

**Learning From Preservice.** “I remember being in her [Charlotte’s] class and being like whoa, I don’t think I know anything about math . . . and I don’t know the first thing about teaching it (Lacey).”

After graduating from college Lacey served as an AmeriCorps volunteer and “fell in love with teaching kids of color.” She chose to attend Penn after being accepted to numerous other programs because she thought Penn could provide her with the most leverage in transitioning from teaching.

I want to go into teaching, but if I do go into teaching, I want to make sure whatever degree I have will be applicable still to the greater world of education. If I decide to no longer become a teacher, I can pursue different avenues. (Lacey)

She also chose Penn because she thought the program could provide her with the tools to effectively teach “high needs kids.” She had hoped that PENN would give her.

[a] rigorous, top of the line, most-cutting-edge kind of theory . . .  things that could sort of push high needs kids in a very academically-driven kind of focused way—like how to provide targeted instructions for high needs kids, basically, and how to move them efficiently—was my hope. I don’t think that’s happened. (Lacey)

Lacey’s hopes regarding the preservice program, while not completely realized, stood in stark contrast to her specific expectations around preparing to be math teacher,

I remember before coming in [to preservice] thinking that, okay math should be a cinch. I’ll just teach it like . . . I learned it. I remember framing it that way in my head. (Lacey)
The way that Lacey had learned math, however, ran counter to the model presented at Penn. The most critical distinction concerned the roles of teachers and students in the classroom. Lacey’s conception of math instruction posited the teacher as the holder of knowledge and the students as the receivers of knowledge.

I think previously before coming into a teacher prep program that, okay, well, the teacher is kind of always right. You know, the students have to fall into line and the expectation is that the teacher is right, has the right answers and…the students need to…do all of the learning. (Lacey)

In the preservice program at Penn, Lacey was exposed to an instructional model in which all classroom participants were understood to have valuable knowledge to share and all the classroom participants were positioned as learners,

I think Penn did a really good job of sort of setting up a Socratic kind of circle with every one of our classes, and we all realized that we all have something to share. We understand that the professor is the professor, and that they’re disseminating a certain amount of information, but learning is kind of a two-way street when you come to a classroom and that sort of builds the classroom culture. The students feel validated. The teacher feels validated. (Lacey)

This practice of sitting in a circle for instruction, honoring the voices of students and recognizing all classroom participants as learners is one that has been recreated in Lacey’s classrooms.

I sort of take that into even my kindergarten classrooms today. I spend a lot of time sort of listening to them, um building, we build rules together. Um, they share, they share things about themselves every single day. I make sure that I have that component. Even though they’re 5 and I’m 28, I can still learn things from them. (Lacey)

Shifting her thinking in this area was foundational to the learning that Lacey attributes to her time at Penn.
Lacey also learned to focus specifically on the suitability of tasks based on child developmental stages and distinguish between students’ readiness to attend to certain content vs. their readiness to comprehend the mode or context in which the content is presented. She attributes this knowledge to her human development class.

If I didn’t go to Penn . . . I wouldn’t have known if you have a first grader and they’ve mastered certain content in first grade, you don’t just give them second-grade stuff, third-grade stuff, how it’s presented, and then just expect that they can do it—versus making it understandable to the six-year-old mind. They can learn higher concepts, but you still have to present it in a developmentally appropriate way. (Lacey)

Regarding the content that students should learn, Lacey’s perception of math teaching prior to preservice was influenced by a focus on transmitting facts and computing correct answers. In preservice she was encouraged to think about how and why students arrived at answers rather than focusing on getting them to the right answer. She was also introduced to the concept of number sense and the teacher’s role in helping students to develop it. This knowledge transformed her notions of how to teach math in the early grades.

I always thought about it [teaching] as $2 + 2 = 4$. You know, I always thought about it in the concrete way. I never thought about it in the more abstract, “How do you build number sense?” . . . “Why did you get to that answer?” instead of, “This is the answer” . . . We did a number sense activity [in methods class]. . . . I had no idea what that was . . . I remember thinking, if I didn’t have that course or that exposure to what number sense, cardinality, all those different things were . . . I don’t know how I would have taught kindergarten or any elementary grades because I didn’t know those key components. . . . I would think, “Okay we need to practice writing one through ten.” I never thought about: we need to practice showing what one looks like, showing what three looks like in many different ways, and arranging it in different ways. I never thought that a student might see three as only being three things in a row.
Finally, Lacey began to recognize that when given the opportunity, different students will often approach the same problem from a variety of different angles. This was new to Lacey. Providing the space for students to engage in type of problem solving was also novel. “I come from the school of thought where it’s like, okay, this is what the teachers taught you; that’s the way you solve the problem. That’s how you get the answer (Lacey).”

The strategy that Lacey learned in preservice that embodied an emphasis on multiple approaches was the weekly math journal, “I remember that being a big part of our math journal at Penn—that everybody sort of did the same problem, but there were many different ways to get to that (Lacey).” Most of what Lacey learned in preservice about teaching math came from her courses—even courses beyond math methods. Lacey, was, in fact the only teacher to regularly express how her courses in child development and school and society influenced he approach to math instruction. She found that her student teaching experiences offered her very little that could be applied to her subsequent teaching of mathematics.

My first student teaching experience was first grade. I didn’t really have much hands-on experience with the kids . . . . I feel like I wasn’t utilized the best and we did a lot of reading…I didn’t interact with the kids that much at all when it came to math . . . second semester I was at Bledsoe . . . . we didn’t do that much math. There wasn’t anything really that stood out in terms of math instructions from either one of those experiences, which is kind of like a shame looking back (Lacey).
Teaching After Preservice. “But it wasn’t until . . . I got my own class that I realized. . . . I started drawing on all these things that I learned during my time at Penn, but then started making all these different connections (Lacey).”

After completing the Penn preservice program Lacey spent a year as a tutor and is currently in her second year as a kindergarten teacher in a large urban district. She loves the job, but finds it challenging to see her desires to effect change realized in her current position.

I work in a Title I school . . . I love working with students of color and working with that population . . . the longer I teach, I’m starting to find out that it’s a bit difficult to be as effective as I want to be in terms of making that difference in lowering the achievement gap. So I’m finding it very difficult to continue in the current capacity. . . I’m in . . . I like it and then I can see some bigger, bigger issues that—and challenges that I need to face. (Lacey)

Lacey describes her instructional practice as regularly constrained by two major factors, the behavior of her students and the administrators who oversee instruction in her school. She describes her students’ behavior as explosive, impacting her own sense of well-being: “So, okay, working in Bringham . . . it is so stressful, Joy, . . . classroom management is a huge piece because I have a lot of really extreme behaviors—like explosive (Lacey).”

Lacey’s students’ behavior also impacts her practice. In her lesson agenda, she planned a small group demonstration with four students using manipulatives to demonstrate division while the rest of the class observed. She planned to follow the demonstration with a with a whole-class discussion. The small group demonstration, however, was a concession. Lacey would ideally have liked to have all the students use manipulatives.
So [to] have them feel something tangible is sort of different—different ways to interact with the content, especially since I value . . . intelligences just because a lot of kids are visual, or auditory or tactile and I want to make sure that every student has a chance to—has a way to master that content. (Lacey)

She decided, however, that allowing all the students to use manipulatives was not practical.

I would love to just give all of my kindergarteners bears . . . that would be awesome, except that I feel like three of them would throw the bears. . . . so when I think of manipulatives, I try to balance what I know is good teaching and then what I know is actually practical to do in a classroom. (Lacey)

Further complicating her practice are frequent, unannounced evaluations. Lacey fears that if she allows students to use manipulatives and they became unruly, she would be viewed negatively by her administrators.

I feel a little bit ashamed to admit that a lot of my lessons are driven with, “Okay, if somebody walked into the classroom, it still needs to be engaging for the students, absolutely, of course, but it also needs to be a manageable lesson.” Working in Bringham, anybody walks into your classroom at any point, and it can be from the principal, assistant principal to master educators which are outside. It’s like Secret Service almost, sent in from a central office to observe your class. (Lacey)

Overall, regarding implementation, Lacey does not feel that she has much opportunity to implement because of the context of her school. The context to which she refers is not primarily academic, but related primarily to management, discipline and administrative oversight. Her instructional practice is curtailed by these factors, factors she connects to the realities of urban schools.
Teacher Michael

K-16 Math Background. Michael considered himself to be “a big math nerd” throughout his high school and college years. He spent a substantial portion of his K-12 schooling in progressive, alternative environments. In his elementary school he remembers having “vertical classrooms”, “singing”, and “cooking to learn fractions.” Michael attended a progressive independent high school in which applied mathematics, particularly in relation to science, was a central focus. Michael’s high school math teacher, Ellen, also had a distinctive emphasis on writing in mathematics.

We had writing notebooks where we wrote about calculus and how calculus was affecting our daily life, and affecting how we were implementing things in physics . . . documenting and explaining myself, not even to her [Michael’s high school teacher], not to the class, just to myself. Explaining why something is true mathematically and how that relates to my personal understanding of math. That’s the kind of stuff she had us write. (Michael)

He credits his high school math teacher with helping him understand math as a language. He felt so strongly about the importance of writing in math that after he started teaching he purchased a book she had authored on the subject.

I hadn’t owned Ellen’s book myself. I’d only read other peoples’ copies, so I did buy a copy of it just before last school year just to remind myself how to teach math through writing. (Michael)

Michael also had numerous educational and work experiences before preservice that required the effective application of mathematical or general pedagogical knowledge. He is certified to teach English K-12, earned a master’s degree in mental health administration (for which he took several accounting classes), ran a store and worked in
theatrical set design. In addition to requiring the effective application of mathematical knowledge, one of his jobs had connected him directly with math education,

I used to work with a dance company called Tap Team Two. They teach math with tap dancing where they go from school to school to school—or at least they did. This was 15 years ago. They would go from school to school and do an assembly teaching math through tap dance. (Michael)

While he does not credit his work experiences with driving him to pursue math education, he does identify them as powerfully linked to the way he understands and approaches the discipline. He centers his understanding of math around geometry and connects that understanding to his work experience.

The thing that helps me the most is I use geometry as the center of math in my head. . . . I spent a lot of time building stuff . . . TV shows, theater, film and stuff . . . You have to know $A^2 + B^2 = C^2$, or you won’t be able to make a triangle. You need to know different things about proportions . . . I did a lot of that with the kids in first grade just to get them to understand shapes and how they come together. (Michael)

**Learning From Preservice.** “Just because you can speak French doesn’t mean you can teach French” (Michael).

Michael was one of the few teachers who expressed apprehensions about learning to teach math. Michael had lots of positive experiences that impacted both his knowledge of math and his beliefs about good math teaching. Yet, Michael felt that there was a wide chasm between knowing math and teaching math, “Knowing the mathematical ideas, and knowing how to teach them, are two very different things” he said, “Just because I can do the math does not mean I can teach the math (Michael).”

Michael cannot remember exactly when this revelation first became clear to him, but he described it as the time during preservice where he truly felt challenged.
It was when I realized that just knowing the math doesn’t mean I can teach it. I don’t remember, but, boy, did it hit me hard. I don’t know what it was. I can explain how I know it, but I can’t explain how to teach it. (Michael)

In Michael’s case this revelation was met with some guidance. He credits preservice with teaching him how to talk to children about math,

I wouldn’t have known how to say the words that a six-year old needs to hear because . . . I speak 32, I don’t speak six. I didn’t remember what it was like to be a first-grade student, so I needed to be reminded of that. I needed to be reminded of the importance of repetition and the importance of using vocabulary words—defining them, using them properly, asking questions about the vocabulary words. That’s all the stuff that I learned at Penn . . . how to use those words properly. (Michael)

Michael also credits preservice exposing him to educational theory that allowed him to situate his own K-12 math experiences within in the larger framework of math pedagogy.

“Math to self”: that didn’t occur to me on my own. I didn’t realize that . . . this way I had learned division and multiplication on my own. I didn’t realize, “Hey this is a big pedagogy. This is something that lots of people do. This is pretty common, and this is why it works.” Now, because of the schooling, I know why it works, and I learned how best to present this kind of stuff using tangible materials. (Michael)

Michael was also powerfully influenced by mathematics trade books. He had learned to write about math in high school, but he was introduced to math trade books in the preservice methods course. He describes being given the book, An Orange Has Eight Slices, as “one of the best experiences I had at Penn” (Michael).

Yet, when sharing about the most important thing he had learned in preservice, Michael said that it involved pursuing student understanding. He learned to do this by asking a series of questions that all start with “why?”
The most important thing that I learned about teaching math was asking the kids, “Why is this the case? Why is this true?” That’s something . . . that really resonated with me. That creative high-order thinking . . . created a level of engagement and a level of math-to self-understanding. If you can explain why something is true, that means you understand it, and it’s not just about learning, it’s about understanding. (Michael)

**Teaching After Preservice.** After graduation, Michael served for a year as a substitute teacher. He worked in many different schools, public and private, serving grades K-12. The following year, he taught in an urban public school in a large city—the same city in which he had worked as a substitute. His experience teaching at Bright Elementary School was fraught with challenges, the first of which involved his readiness to teach first grade, “I shouldn’t have done it because I didn’t know anything about first grade (Michael).”

In addition to feeling unready to teach first grade, Michael felt especially unprepared to teach in an “actual” urban school (Michael). Michael did his student teaching in the large urban center of Chatham City, but he was not in a public school. Instead, he was placed at Levering Place, an independent school within the city limits. According to Michael, his student teaching placement was in a “pretend” urban school. He noted that all his students “had two-parent families” and most of them lived in the suburbs (Michael). The ones that lived in the city hailed from the most affluent sections. His school also had limited racial diversity, “I had a half a black kid in my class. . . one. . . girl (Michael).”

That did not prepare me, necessarily, for a Chatham City public school situation. . . so, I didn’t get prepared at all for actually, urban teaching, and I’m really annoyed by that now. . . Yes, so I should have been in an actual urban school, not a pretend urban school. (Michael)
Not only was Bright Elementary a local public school, Michael was teaching in a self-contained, inclusive classroom. One of his students was the initial first-grader to be expelled from the school; another had oppositional defiance disorder. Michael had taken the special education courses in his Penn preservice program, but given the context of his student teaching, felt his training lacked practicality.

I didn’t use the Special-Ed experience [preservice classes] really in my student teaching, but I certainly did in a self-contained inclusive classroom. I spent a specific amount of time without any practical experience, and I prepped a logical experience—sort of trial by fire. (Michael)

Michael has decided not to return to the school, and has no teaching job as his third academic year post graduation begins.

Still Michael holds to his foundational beliefs about what good math teaching looks like. He believes in learner-centered lesson plans in which young students explain a concept or older students write about how math relates to them and makes sense in the world. For Michael, effective math teaching is rooted in enjoyable discovery and participation, so he is looking for other ways to engage students in educational experiences which provide creative challenges and fun activities without the management issues he faced at Bright Elementary.

If you don’t enjoy school you’re not going to show up. . . . If it’s not fun for me, I’m not going to show up as a teacher. I want to enjoy my job . . . whether my job is being a teacher or a student. I would want to have fun if I were the student, so I want to make things fun for the students. It’s much more engaging. (Michael)
Teacher Isa

**K-16 Math Background.** “I think I enjoyed it [math] enough. I don’t think I had a preference or aversion to it” (Isa).

When Isa was a K-12 math student there was an emphasis on encouraging more females to engage in rigorous mathematics courses. For Isa the focus on females in mathematics contributed to enrollment in honors math courses. “I was in . . . accelerated math most years. . . . I’m sure there’s been many years where they’ve tried to identify girls and push girls into math in school (Isa).” Still Isa’s comments about her own math competence in high school reveal a perception that her math ability was not extremely high.

I think I would have said, “I’m okay,”. . . “I’m good in math,” because I wasn’t having trouble with mainstream math class. It was definitely the harder class. But I would have said that. . . . it’s not my favorite subject—or something like that. (Isa)

Isa also found her early forays into advanced math to be challenging, requiring substantial effort. She had to exert substantial effort just to obtain a B.

When I went to the geometry class I think I ended up with a “B” in it, but I worked a lot harder for that “B” than the other classes I was in. And it was a small class, so there were probably 12 kids in it, and I was probably in like the bottom quartile, or half—which at this age, now we look back and I know that seems so silly, but at age 13. (Isa)

Isa also recognized that high school performance impacted college choices and decided it might be wiser to reduce her time investment in math and science for the sake of her overall achievement.
And I think part of it was I felt a sense that you need to perform well in high school to go to college, and math and leader science—it just took so much time. I worked really hard, and so it took so much time. I could balance things better if I took a slightly easier class. (Isa)

From that point on she states, “I took, not the easy classes, but not the hard classes anymore (Isa).”

As a college student, Isa fulfilled her math requirements and subsequently pursued an advanced degree from a prestigious university. After receiving her masters’ degree, she worked in two different industries, with great success, before deciding to transition to education.

I liked what I did very much, but it wasn’t. . . . It’s hard to explain, I liked it, and I learned a lot, and I was working with really smart people, which was something that I wanted to do, but it wasn’t my passion. Even as a hard worker, if it’s not your passion, eventually it kind of drains you. (Isa)

It was not easy to decide to change careers, but after serving as a big sister with the latter company she finally decided to take a leap and transition to education.

**Learning From Preservice.** “I think one of the nicest things about Penn’s program was whether you thought of yourself as a mathematician or not, you totally bought into the importance of math and math instruction by the time you finished it” (Isa). The most transformative aspect of Isa’s preservice math experience was not the content or the methodologies she learned. It was the personal passion of her math methods professor. Her method’s professor, Charlotte, had an enthusiasm for math that made her think that she could also develop a passion for the subject.

I do think that the thing that makes everybody good at what they do is their passion for it. I think Charlotte [math methods professor] had it, so that opened my eyes to it. That made me think I could have it in the future….The enthusiasm
my teachers had for math in my pre-service and their purposeful desire to inspire the same enthusiasm in their students, that stood out in the math methods course in pre-service. . . . That’s one thing I got from it. (Isa)

By the end of the course she had developed, “a greater appreciation for math, a much greater appreciation (Isa).” In retrospect, she postulated that her transition to becoming a math teacher was bolstered by her experience in the math methods course. “I probably wouldn’t have gone ahead and become certified [in math]. . . . if I hadn’t had such a good experience with the class (Isa).”

Isa also learned math education practices in preservice that supported positive instructional outcomes. Noteworthy exercises from the methods class were the weekly math journal questions, eliciting multiple solution strategies from the students in the class. “I enjoyed the problems of the week,” she said “discussion about various strategies to solve a problem, and that deep dive that we took into different problems” (Isa).

The weekly problems were embedded in an instructional strategy that required that students work on problems without direct instruction, then share their strategies with the entire class. Isa found that her facility with using the approach was deficient because her exposure had been limited to the methods class. She found that she lacked substantial opportunity to practice it in her student teaching site.

In our math methods class, opening it with the problem of the day that we discussed . . . surfaced a lot of different solutions and approaches. That’s one I like a lot, but I didn’t have a chance to roll it out as a student teacher as much, so I hope [to be] better at it this year. I think that’s a great way to teach math that I’d like to improve in. (Isa)

Methods class also allowed Isa to learn the language of math often restricted to the educators’ domain.
Then I learned the jargon a little bit. Going from concrete to abstract, part-know and part-unknown . . . all of those . . . I know it sounds so silly. That was important, but when I was in it, I wasn’t thinking I was going to teach math. . . . I wish I could take it [the math methods class] again. (Isa)

Preservice convinced Isa that connecting math to real life experiences was a critical contributor to student engagement, especially in urban schools. “I don’t think you got their interest unless you related it to real life. That was the benefit of the Urban Education Program [Penn preservice program] for me. . . . That was very important to getting engagement around a topic (Isa).”

Finally, she learned about the importance of engaging in standard, daily assessments of student work and using that information to improve instruction:

Walking around the room assessing students, you can think they have it, but they may not have it. I need that [assessment] to look at after class to make certain where they are, to know where they are, so I can intervene. That’s the main reason. . . . I think I learned it in student teaching and at Penn, and I appreciate it a lot the last couple of years. (Isa)

Teaching After Preservice. Isa’s math teaching experience has been very limited. In her first year after completing the Penn preservice program she taught kindergarten and first grade in a turn-around school in a large urban district. Much of Isa’s work at the elementary school was in literacy, and the focus was on controlling things. “You focused on controlling things,” she said, “and . . . I’m going to get out of that habit. I think [that] probably informed my perspective. Maybe the first grade experience did too (Isa).”

In her second year, she transitioned to a new school where she volunteered in the sixth-grade math classes. In that position she was responsible for enrichment classes and
remediation classes. She found that her math knowledge vastly increased by teaching the enrichment course.

It was challenging math, and the content was math you don’t usually get to learn, so I learned a lot teaching it. . . . We studied specialist topics in math, so an example would be a few classes on the Fibonacci sequence for sixth graders. Then diagrams and pictorial problem-solving for the fifth graders. . . . You took a deeper dive on the enrichment topics that you don’t get to spend much time on (Isa).

The lesson format used in the enrichment class was quite similar to the format she learned at Penn which began with engaging students in a challenging problem at the start of the class and building the lesson from that initial experience. Alternately, the lesson plan used for the remediation class was quite different, focusing on direct instruction and homework review.

As she prepares to begin a new full-time position as a sixth-grade math teacher for all-math classes in a high-achieving school district, the issue of how to best design a daily instructional plan for a group of mixed-ability students is heightened. Isa senses a conflict. At this point in her development, she believes that the model used for enrichment may be the more appropriate model for all students, but she is not sure how it would work in a mixed-ability classroom.

I think it [good math instruction] definitely looks pretty much the same across all contexts . . . but you have to tweak it . . . a little bit, to make it work in each room. I don’t want to over stress. . . . I don’t know the answer to that. I would like to think it looks the same, but I’m not sure . . . I think you get the sense of where I am. . . . I like the first model, but I’m trying to fit it to a classroom with students at different levels. Some of them need greater support. . . . I don’t have an answer to your question, but I hope you would see what I would like to do (Isa).
Her current plan is to approach her new position utilizing the structure with which she is more familiar. She suggests giving more opportunity for mucking around to students who grasp the concepts more quickly while offering more support to struggling students:

I’m going to let my students in Pre-Algebra, Honors Pre-Algebra muck around a little bit more because they don’t need as much support and they have more to gain by mucking around, for lack of a better term. Maybe when I know my students better in Math 6, the mainstream math, I would like to get them to a place where they can muck around, but right now I have to make sure they’re feeling supported. I usually let kids who catch on quickly muck around a little more. It’s almost a form of differentiation, but I’m not sure I like that (Isa).

She holds this notion rather loosely and hopes to revisit her interview in the future to see if she still advocates support for the struggling students and mucking around for those with more apparent skill. She learned this from preservice student teaching, not from methods class, and it was affirmed in the models she saw as a professional teacher.

Where I’ve been when the kids need more support, you have to focus on the support. I hate to say it’s one or the other, but sometimes it feels that way. That might be wrong, and I might. . . . I’d love to read this in three years and see. (Isa)
Teacher Daniel

K-16 Math Background. “So I'm not that great when it comes to critical thinking, but I'm good at the procedural part” (Daniel).

As an elementary school student Daniel attended a Catholic school in a densely populated urban area. He considered himself to be, “an amazing math student”, always receiving A’s in math on his report cards, though he quips, “the education [at his elementary school] wasn’t great (Daniel).” Daniel’s math expertise was decidedly one sided as evidenced by his approach to and performance on state tests, “when we took state exams I didn't care about the word problem section, but when there was math computation, I was always in the 99th percentile (Daniel).”

Daniel’s confidence in his mathematical ability was challenged in high school when he was tasked with writing geometric proofs. Daniel found it difficult to produce the required logical explanations: “You have to logically explain how this happened or how it got there. I was really bad at that (Daniel).” It was, however, Daniel’s two college math classes that definitively shattered his positive perception of his mathematical ability.

Then I got to college and I took Calculus and . . . another class where it was mathematical thinking . . . I had no clue what was going on, especially the mathematical thinking class, trying to explain the theory behind the numbers, I was like oh no. I had no concept or understanding of number theories. I was like, “Oh, I'm actually not that great at this.” (Daniel)

Learning From Preservice. “I wasn’t taught to be an expert content-wise in math. I was taught how to teach a lesson (Daniel).”

As seen in the quote above, Daniel is clear that his math pedagogy was influenced by preservice, particularly the math methods course. Yet, when he considers the full
weight of preservice and the experiences that most powerfully influenced his learning, he is drawn to his student teaching experience.

In terms of that whole program, the most endpoint, or the one that I remember, was my student teaching experience. . . . This might sound really wrong . . . I don't want to say that the classes meant nothing, or the theory meant nothing, but I learned most from my mentor teacher [the teacher to whose classroom Daniel was assigned]. And just being in the classroom and doing things. (Daniel)

Daniel seemed to recognize that among his colleagues, his student teaching experience was unique.

I know most people didn't have the same experience I had in my classroom . . . I was pretty early at taking over the classroom . . . I was kind of running the show I want to say in [sic] November. She [Daniel’s mentor teacher] was pretty awesome with that. And I was kind of different than most people. I went in extra. I just wanted to be in the room. We weren't mandated [sic] to go in those other days, but when I had classes in the morning I went. And even if I was only there for two hours, I went. (Daniel)

Even more unique to Daniel is that he taught a math very often during his student teaching experience.

The first half [of the academic year] I did math, only because my mentor teacher still wanted to be in charge of reading. . . . So I did a lot of math, and then, especially second semester, I ran everything. I mean, she was there, but she let me do that. But I did teach a lot of math. (Daniel)

Daniel’s student teaching placement presented him with a dichotomy in terms of approach. On one hand, he states.

Yeah, we can do math talks, yeah, we can do all this stuff, but there's a point when it's just like, “Teach the kids how to do it, so that they know how to do it when they see it on the test,” and I don't like that, but there's just so much pressure on those tests. I could see my mentor teacher having panic attacks when kids don't get order of operations right away and it's like ten dots and that's it. . . . then teaching them straight. I would say that was the biggest conflict because there
wasn't much time in my placement for me to be creative . . . and more about like, “Okay, kids, this is step one, this is step two, this is step three” and then it's like, I do, you do, we do method.  (Daniel)

Daniel does identify specific ways in which the preservice coursework, specifically the math methods class, impacted his knowledge of math and how to teach it. First, math methods course changed Daniel’s personal approach to solving word problems. Daniel had long found word problems unsavory, but in methods class, he was encouraged to embrace new problem solving strategies which he reported using while taking the MKT assessment for this research project.

Before taking Charlotte’s class, I probably wouldn't have answered it [the MKT] as thoroughly as I did . . . because she taught me to be more open and use pictures and things to help you . . . . So I kind of took more time than I normally would. In the past I would just be like, “How do I get there the fastest,” and think about procedures, but she kind of made me think about it more. I have to think. (Daniel)

Daniel’s conversion was prompted in large part by the weekly math problems in which he engaged in methods class. In solving these problems and listening to the solutions of his peers Daniel learned that there were multiple, distinct and yet connected solutions pathways for many math problems. He also had the opportunity to persist in engaging in difficult math problems and be exposed to new ways of thinking about math problems.

And they [the weekly math problems] were conceptual; they were hard. It was very rigorous. I was confused the entire time. But every time we went and she explained process by process, or if . . . other grad students go up to the board and they [showed] how they got the answer, that opened my eyes a lot more than I originally thought. . . . I got to see a method that ties in, or a method that Robin (another math methods student) did that looked completely separate from what I did. (Daniel)
While the methods course succeeded in opening Daniel’s eyes to multiple solution, it did not make him substantially more competent at solving the problems himself: “I wouldn't say I was getting better at it, but I would say that I was learning different ways getting there, but every week when I was doing the problems, I wasn't necessarily getting them wrong (Daniel).”

It is important to note that Daniel despite being able to clearly articulate his learning from both student teaching and coursework, Daniel identified a conflict between the two. While he noticed routines in his classroom that promoted number sense, he was clear that his student teaching context did not allow much space for the kind of instruction he had learned about in his methods class. Even more, Daniel seemed to fully identify with the choices his mentor teacher made.

**Teaching After Preservice.**

But there is something different about knowing the math content versus being able to explain how you were able to get here…It doesn't matter how good you are at math—can you explain it? And can you explain it more than one way? (Daniel)

After completing preservice, Daniel began teaching at a Wide Lake Charter School in a large urban school district and is now starting his third year at the school. Ninety-nine percent of the students at his school are minorities. In his first year at Wide Lake Daniel taught in a fifth-grade self-contained classroom. In his second year, his school departmentalized and he became the English teacher. He continued to teach both fifth and sixth grades giving him an opportunity to teach the same students for multiple years. Now in his third year at Wide Lake, Dennis is again teaching literacy to the same group of students giving him not only a unique perspective on their development but
unique affinity for them. He continues to teach math test prep, but his school day instruction is focused primarily on literacy.

Daniel’s first year of teaching involved a great deal of internal conflict, conflict between a procedural approach to math instruction, to which Daniel was accustomed before preservice, and a more conceptual approach that he experienced in the TEP. Daniel described the TEP approach to teaching math as centering on “critical thinking and have students explore concepts and things not just direct rote memorization.”

In his first year as a 5th grade math teacher, Daniel found himself teaching students primarily to memorize and apply procedures; his focus was not on understanding but primarily on the application of known algorithms. “I felt like I didn’t really teach math the way that I would want it for myself. . . . I was the math teacher that other math teachers were to me. . . . I have felt tremendously guilty about that.”

Daniel provided several reasons for his decision, as first year 5th grade teacher, to teach math as a series of rules to be memorized and calculations to be performed: first, his student’s math ability, second, the pressure associated with standardized tests and third, his own limited ability to teach math conceptually. Daniel started his first year of professional teaching with an attempt to teach math conceptually. His first roadblock was his students’ lack of readiness for grade-level work. “I tried to do conceptual teaching, but it was really difficult because the kids were two, three years behind grade level in math and so for the majority of that year I taught procedurally” (Daniel).

Additional pressure to teach his students primarily to memorize procedures came from the pressure to perform on standardized tests. During Daniel’s first year of
teaching, his students took a baseline math test and failed miserably. He felt pressed to show growth by the end of the year and decided that given the amount of mathematical content they needed to master his only alternative was to teach procedures,

My kids didn’t even know what a fraction was and for me to teach adding…teaching operations involving fractions when they didn’t even understand conceptually what a fraction was or that fraction is less than one. . . it was a nightmare. . . . I resorted to procedural teaching because I had so much to catch up on. (Daniel)

Daniel, however, revealed that it was not only his student’s limitations that constrained his efforts to teach conceptually, it was also his own sense of competence in teaching math with an emphasis on sense-making, critical-thinking and exploration. When faced with the new sixth-grade math teacher’s pre-assessment—dividing and explaining the remainder—he warned her about his students’ lack of conceptual knowledge and clearly identified himself as the cause. “I told her, “I didn’t teach them that [to understand what the remainder represents]. . . I was not good at that. . . . I was not prepared to teach them concepts” (Daniel).

Daniel’s sense of being unprepared is particularly noteworthy because he expressed on several occasions how the preservice program, particularly the math methods course, helped him transition from a procedural perspective on teaching math to a conceptual one. However, Daniel’s preparation, while eye-opening, was insufficient for the work of teaching as he experienced it in his first year.

Can I be honest?. . . . There wasn’t enough taught in grad school [the TEP] in terms of concepts for me to instruct it on my own. Charlotte’s class was great, I learned a lot of concepts in there but there wasn’t enough. It was more like, this is a pedagogical way of teaching it but there wasn’t a like, place value lesson help…it was more like, “you can teach it [place value] with pictures, you can
teach it using manipulatives you can do it by kids thinking on their own.” When I got my own class, I didn’t know how to conceptually do that myself. (Daniel)

Daniel’s experience in his student teaching placement did not help to remediate his lack of confidence or knowledge for teaching conceptually. In fact, his experiences in his placement reinforced the procedural approach to math he had experienced K-16. In his student teaching placement he learned that the way to improve student performance in the short term (i.e. on standardized tests) was to teach procedurally.

Yeah, we can do math talks, yeah, we can do all this stuff, but there's a point when it's just like teach the kids how to do it. So that they know how to do it when they see it on the test. And I don't like that, but there's just so much pressure on those tests. I could see my mentor teacher having panic attacks when kids don't get order of operations right away and it's like ten dots and that's it. And then teaching them straight. I would say that was the biggest conflict because there wasn't much time in my placement for me to be creative…and more about like okay, kids, this is step one, this is step two, this is step three and then it's like I do, you do, we do method. (Daniel)

Daniel enacted this learning in his first years as a professional teacher. He emphasized the acquisition of rote procedures to ensure that his students’ standardized test scores went up.
CHAPTER 5: DISCUSSION OF EMERGING CROSS CASE THEMES

Introduction

In the cases, I highlight the teachers’ interpretations of their experiences, giving attention to their unique perspectives. In this discussion, I look across the cases and highlight commonalities in experience. I analyze the teachers’ development as math educators in the light of the broader body of research on the subject. Looking across the cases, four themes emerged:

1. No problem with math
2. What a class can/cannot teach
3. Persistence
4. Teachers’ interactions with the unique characteristics of urban schools.

Figure 5.1: Organization of Emerging Themes

The themes connect to the experience categories identified in the conceptual framework (See figure 5.1) and to my research questions. My research questions are as follows:
1. How, and in what ways, might preservice education contribute to the development of novice teachers’ knowledge of elementary math content?
2. How, and in what ways, might preservice education contribute to the development of novice teachers’ espoused pedagogical approach for teaching elementary math?
3. How might teachers’ preservice learning be enacted in their early work as elementary math teachers? What, if anything, persists from preservice in novice teachers’ ideology and practice as professional teachers of elementary math.

The first two themes, no problem with math and what a course can/cannot teach, serve as to responses to RQ1 and RQ2. Theme three, persistence, speaks directly to RQ3. Theme four is not a direct response to my initial research questions as I did not initiate any conversations about the context of urban schooling. However, the challenges associated with teaching in urban schools seemed to serve as subtext throughout the many conversations of practice I had with the teachers. The implications of their experiences for the role that preservice plays in the development of novice math teachers could not be ignored.

**Summary of Findings: K-16 Schooling**

While there are differences in the study participants’ k-16 math backgrounds, the overall picture is one of striking similarity. Taken together, the teachers in my study are a well-educated group. All were high-achieving, highly successful k-12 math students. Each took college-track math in high school; four completed calculus or
calculus II during high school and the remaining three completed pre-calculus. Each was admitted into and successfully completed the highly selective, Teacher Education Program at the University of Pennsylvania’s Graduate School of Education (GSE/TEP) and two of the teachers hold a second master’s degree from other universities. It then appears unsurprising that one of the dominant themes in the data collected for this study is no problem with math, and that six of the seven teachers reported no apprehension about learning to teach elementary math prior to beginning preservice. However, as much research has shown, course-taking trajectories and grades do not tell the full story of an individual’s math achievement or their readiness to teach math (Ball, Hill, & Bass, 2005, Ma, 1999). Close inspection of the teachers k-16 math experiences reveals that the study participants’ academic records not only obscured significant gaps in their k-16 math knowledge but also obscured beliefs about learning math that run counter to a constructivist, meaning-making approach to the discipline. In the remainder of this section I detail aspects of the teachers’ k-16 learning experiences and how those experiences led them to perceive that they had no problem with math.

Despite their pattern of high achievement, most of the teachers (6/7) spontaneously reported experiencing a particularly challenging k-16 math course/series of courses one that challenged their perception of themselves as math learners and in some cases completely altered the teachers’ math course taking trajectory. The experience of struggle in mathematics is quite a common phenomenon; the discipline tends to invoke both reverence and fear among US students (Ladson-Billings, 1999). The
study participants’ responses to their experiences of struggle, however, merit investigation.

**Theme One: No Problem With Math**

The seven teachers in this study all claimed to be have been good math students; they felt confident in their abilities to teach elementary math. It was surprising, then, to hear in their k-16 math narratives, all but Michael described an extremely challenging high school or college math course that they claimed made them either drop math or go for “easy” math courses. It seems as if their confidence about teaching elementary math fit with an “easy math” course trajectory. Since most of the teachers’ struggles in mathematics began in high school or college, they were able to preserve the belief that elementary math is easy and that they had the requisite knowledge to teach it well. It is as if these teachers viewed elementary math and high school math as two distinct disciplines: one simplistic and procedural and the other more conceptual and riddled with potential challenges. So, it was a shock to them to discover that elementary math could be difficult to do for them to do. Gwen says it best,

> You kind of worry, ‘Oh my God, maybe I didn't do it right [weekly math problem]. I can't even do elementary math, how the heck am I supposed to teach it if I can't even do it right.’ Yeah, that whole panic.” (Gwen)

As Gwen says, the idea that they would have to teach material that they were suddenly discovering to require substantial effort became a challenge that they embraced. Learning to teach math in preservice seemed to offer the teachers an unanticipated opportunity to revise their beliefs about struggle in math. Instead of seeing struggle as something to be avoided, they found that struggle was an integral part of
learning elementary mathematics—a critical step in the development of conceptual understandings.

This may be the type of struggle that Hiebert & Grouws (2007) describe as productive struggle in mathematics. What it seems to have enabled is the casting off of an underlying belief that struggle is a signal of limited competence (Spiegel, 2012).

The struggle with “hard math” that these teachers described experiencing in high school or college should not be confused with “math anxiety.” Once they embraced the idea that struggle in math could be positive they were able to effectively engage in the difficult math associated with the preservice coursework. Only one teacher, Lacey, persevered throughout program and into her first years of teaching with an overwhelming anxiety about math. Lacey’s fear of being “wrong” was more pronounced than any of the other teachers in the study. Her description of her experience taking the mkt is an example of the way in which this anxiety made it difficult for her to embrace struggling with mathematics as the other teacher in the study did.

But also math is kinda scary. I remember like once I got to the second page [of the mkt] I was like, okay anxiety is building. I don’t want to do this. You know, at this point I’m 28 like why am I afraid. [laughter] I mean there’s an anxiety around math that I think we all need to face, sort of...I grew up in the [Cape Vert]...we’re pretty traditional in terms of learning math so I remember if I didn’t get the right answer the other answer was just wrong. There was no, there was nothing else with it, “you’re just wrong.” So I remember not wanting to relive that feeling of being wrong at 28, you know.

Prior to preservice, she had maintained a perception of herself as a strong math student. This perception rested on her positive experience with math in elementary school. She described struggling in high school, and after passing a college math exemption test, she effectively closed the door on engaging with mathematics. Lacey
intended to “teach math the way that she had learned it.” It was a time when math was simple with easily remembered procedures.

The teachers’ claim as they entered the preservice program that they had no problem with math is typical of most people’s experience of learning math in school over the past fifty years. Despite significant shifts in k-12 educational pedagogy at the research level, these teachers generally experienced elementary math as a procedural, rule-bound enterprise. What seemed to happen as they moved through their preservice program was the opportunity to go through elementary math again — this time gaining a deep understanding of the complexity of the discipline and developing an openness to the benefits associated with getting beyond the procedure and deep into the “fun” of math. This was a very personal moment for them. The power of the math course was the opportunity for the preservice teachers to learn how to learn math.

Theme Two: What a Course Can and Cannot Teach

The Penn TEP is intended to move teachers from a didactic, teacher centered and rule-bound ideology for teaching and learning to an approach that leans toward a constructivist, meaning driven, and discourse rich pedagogy. The math course fits completely within this approach and was identified by teachers as a primary source of learning about math and how to teach it. It is designed to give students the lived experience of learning math in a new way that will enable to implement high leverage instructional practices in math. This theme, then, serves as response to RQ2.

The math methods course was credited by these teachers providing them with practical, concrete learning strategies that they could immediately put into practice in
their student teaching placements. They claimed the “problem of the week” as being THE chief instrument for exploration of the variety of methods that they were being taught.

**Problem of the Week.** Each week at the end of the math methods class the teachers were given a single math problem to solve for homework. The problems were designed to be challenging and focused on content with which teachers were already familiar. They were not given a template or set of steps to follow to solve the problem. They were required to solve the problems in ink, keeping a record of all their solution attempts. All the teachers in the class worked on the same problem. The first activity in each week’s methods class was a public sharing of the teachers’ solutions to the problems.

The problems were intentionally difficult and drew on a wide range of elementary math topics, challenging the teachers’ long-held perceptions of elementary math and calling into question their sense of personal competence in the discipline. According to Abigail [math methods instructor], the weekly math problems were strategically chosen because of their potential to elucidate important math content for students whose content knowledge was limited while simultaneously challenging other students’ perceived mathematical expertise.

The challenge level of the problems did not go unnoticed. Consider the following statement,

> We did these math word problems a day kind of thing. And they were conceptual, they were hard. It was very rigorous. I was confused the entire time. But every time we went and she explained process by process, or if she had other kids ... Not
kids, we’re not kids ... Other grad students go up to the board and they how they got the answer. That opened my eyes a lot more than I originally thought. (Daniel)

Problem-solving became the central activity of this work and speed was dethroned:

teachers were given an entire week to work on the problem. The focus was not on the right answer but on discussing the process used to arrive at a solution.

In these discussions, multiple solution pathways were prized and attention to and awareness of these pathways was claimed by six of these teachers as central to the shift in their perceptions about math. Myles described the shift thusly,

I think what I realized though is just how many different ways there are to tackle a problem. That was eye-opening to me when we would do some math as a class… in my math methods course. It's just amazing. It was eye-opening to realize that there were 20 of us in the class, and each one of us had a different explanation or a different approach....I was just pretty blown away by what some of my classmates had done to get that answer and what their thinking was. (Myles)

**Student Teaching.** Six of the seven continued to embrace this approach in their thinking about teaching but only two were able to take a problem-solving approach (termed an “inquiry” approach in their science class) into their first years of teaching; however, none reported being able to fully implement the approach in their student teaching. What they could do was follow the course requirements such as implementing their lesson plans in their two-week takeovers. They included in their plans what they had begun to think of as a “right way” to teach math. This meant an emphasis on a core set of instructional priorities: making room for student discourse, prioritizing understanding and meaning-making, engaging in problem solving and highlighting multiple solution pathways.
While their intentions for student teaching are still evident in the CRLA’s that they developed for our conversation (see Table 5.1) about their current practice, their implementation during student teaching was quite limited. None worked with mentor teachers who were fully implementing the strategies and approaches that the student teachers were learning. On the contrary, some, like Daniel, taught in classrooms where the teacher was under significant pressure to have her students perform well on standardized tests. As a result, her lessons would often devolve to simply teaching students to memorize or apply an algorithm,

I could see my mentor teacher having panic attacks when kids don't get order of operations right away and it's like ten dots and that's it. And then teaching them straight. I would say that was the biggest conflict because there wasn't much time in my placement for me to be creative…and more about like okay, kids, this is step one, this is step two, this is step three and then it's like I do, you do, we do method. (Daniel)

Anderson & Stillman (2011) have noted the commonness of the disparity between the pedagogical imperatives of urban student teaching placements and those emphasized in reform oriented preservice programs. The size and orientation of this study limit drawing conclusions but the findings do tentatively suggest that teachers may learn that meeting the performance demands of urban classroom (high stakes tests, etc.) sometimes requires going against your beliefs.

This is not to say that student teaching wasn’t useful in shaping their general approach to classroom management, student teaching interaction, and essential student-centered approach. As Gwen said,

My other teacher, Jamie, was amazing...From her I learned a lot about how to help kids who are in crisis and in your classroom. Make sure you're addressing
their needs as humans and not just 3rd grade students. Generally, how to treat children. Which I already, as a human I already kind of knew, but I hadn't really seen it done before. That's what I feel like I took away from her classroom the most (Gwen).

What emerges here, however, is the centrality of the math methods course in the development of preservice teachers’ knowledge of math. Importantly, this claim by these student teachers stands in contrast to the prevailing notion that preservice teachers learn the most from their student teaching experience.

**Theme Three: Persistence**

Having documented the substantial shifts that these teachers made in their understanding of math and how to teach it during their preservice year, I now turn to the ways in which the teachers’ preservice experiences are reflected in their early work as teachers of mathematics. This addresses my third research question,

RQ3. How might teachers’ preservice learning be enacted in their early work as elementary math teachers? In other words, what, if anything, persists from preservice in novice teachers’ professional practice?

My findings indicate that core shifts in math knowledge and pedagogy that the teachers attributed to the preservice course persisted into their early years of teaching. Two distinct types of persistence emerged from the data. The first was *ideological persistence* defined here as changes in teachers’ espoused pedagogical approach or beliefs about how to teach math that show up in their talk about teaching math. The second type is *persistence in implementation* or enacted pedagogical approach that is consistent with the pedagogy of the preservice course and shows in their classroom practice.
**Ideological Persistence.** I used the design of an individual lesson as a primary tool for pursuing evidence of ideological persistence, a Constructed Response Lesson Agenda or CRLA (see Table 5.1) along with an interview that focused on the CRLA. The Constructed Response Lesson related to how they would teach to a standard. The standard provided to teacher was from the Common Core State Standards for Math, Grade 3 CCSSM, CCSS.Math.Content.3OA.A.2,

Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

There appeared to be a remarkable similarity in how the teachers would approach shaping an introductory math lesson on the proposed standard. Additionally, the instructional approaches taken by the teachers to crafting to an ideal lesson were strongly aligned with the format of weekly math problem.
Table 5.1 Results of Constructed Response Lesson Agenda (CRLA)

<table>
<thead>
<tr>
<th>Name</th>
<th>T. Gwen</th>
<th>T. Myles</th>
<th>T. Lacey</th>
<th>T. Michael</th>
<th>T. Isa</th>
<th>T. Daniel</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Order of Lesson</strong></td>
<td>Kids work first- Initial problem-in small groups</td>
<td>Kids work first- Initial problem-in small groups</td>
<td><strong>KWL</strong></td>
<td>Kids work first- Initial problem/regroup their own bodies</td>
<td><strong>Model writing a division problem</strong></td>
<td>Kids work first- Initial problem, w/manipulatives— 2 problems in small groups</td>
</tr>
<tr>
<td></td>
<td>Public sharing</td>
<td>Public sharing</td>
<td><strong>Fishbowl-4 students</strong></td>
<td>Multiple configurations with sharing and instruction in between</td>
<td>Guided problem— gradual release</td>
<td>Public sharing</td>
</tr>
<tr>
<td></td>
<td>Direct instruction, Guided practice</td>
<td>Direct instruction, Guided practice</td>
<td><strong>Discussion</strong></td>
<td>Small Group project-beans poster</td>
<td><strong>IP</strong></td>
<td>Direct Instruction with guided notes</td>
</tr>
<tr>
<td><strong>Independent Practice (IP)</strong></td>
<td>IP</td>
<td>IP</td>
<td><strong>Read Aloud</strong></td>
<td>Poster Session/Diary Walk</td>
<td><strong>Assessment</strong></td>
<td>Small Group Practice</td>
</tr>
<tr>
<td></td>
<td>Small group work</td>
<td>Small group work</td>
<td><strong>Assessment</strong></td>
<td>Public Discussion</td>
<td>Assessment</td>
<td>Assessment/Exit ticket</td>
</tr>
<tr>
<td><strong>Review of IP</strong></td>
<td>Review of IP</td>
<td>Review of IP</td>
<td>Assessment</td>
<td>Assessment</td>
<td>Assessment</td>
<td>Assessment</td>
</tr>
</tbody>
</table>

As shown in the chart above, five of the six teachers planned lessons in which the initial and central lesson activity engaged students in solving a single problem or a problem solving set (up to two problems). Students were expected to work in small groups (3 of 4) to solve the problem(s) and in each case the students were to be engaged in the problem before receiving instruction on how the problem should be solved. The problem-solving period was to be followed by a whole class discussion focusing on student solutions to the problem. In three of these four cases, teachers would have followed the public discussion with direct instruction, practice problems and an assessment. When questioned about the rationales for their choices, the teachers
regularly emphasized multiple solution pathways, conceptual understanding, sense-making, teaching all students, the importance of learning from peers, valuing student thinking and the need for teachers to build on student thinking in their formal instruction—all essential aspects of the lesson of the week, hence of the framework of the math course.

**Persistence in Implementation.** The data around persistence in implementation was less uniform. Six of the seven teachers in my study demonstrated ideological persistence but only, one, Myles, reported consistently reflecting his espoused pedagogical approach in his daily math instruction. The rest of the teachers demonstrated a considerable range in their ability to translate their espoused pedagogical approach into their teaching practice and tended towards implementing an individual practice/set of practices or mathematical ideas. Crystal, Gwen, Lacey and Isa indicated partial implementation and Daniel and Michael indicated little to no implementation.
Table 5.2 Summary of Implementation

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Reported Implementation Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Myles</td>
<td>Daily lessons include an initial problem solved in small groups, a public sharing of work that highlights multiple approaches, direct instruction and independent practice.</td>
</tr>
<tr>
<td>Crystal</td>
<td>Uses models to build understanding. This was learned in the methods class. In her current context, she is “exploring how to model ratios.” Also uses “[number talks and student discourse.” Students learn from peers.</td>
</tr>
<tr>
<td>Gwen</td>
<td>Utilizes a version of the problem described in Table 5.1. Student work on an initial problem is followed by public sharing of solutions with an emphasis on invented algorithms. Occurs daily (for a maximum of 30 minutes), separate from core math lesson.</td>
</tr>
<tr>
<td>Lacey</td>
<td>Socratic circle- regularly engages students in sharing their unique perspectives; values student voice. It is unclear how frequently this happens in math.</td>
</tr>
<tr>
<td>Daniel</td>
<td>Number talks- engaged students briefly in the practice at the end of his first year. It went well but Daniel did not continue the practice.</td>
</tr>
<tr>
<td>Michael</td>
<td>Does not report any evidence of implementation. Has left the profession.</td>
</tr>
<tr>
<td>Isa</td>
<td>Uses an initial problem in her current math lessons but only for advanced students. Her use of the model is limited to high achieving students.</td>
</tr>
</tbody>
</table>

**Alignment Matters.** The data for persistence in implementation is limited but points strongly and consistently towards the importance of alignment between the teacher’s espoused pedagogical approach instructional imperatives and norms of the school. I am defining alignment as agreement or consistency in approach (Webb, 1997). In analyzing the relationship between ideological persistence and persistence in implementation, a primary factor appears to be the degree of alignment between the instructional context of the school and the novice teacher’s espoused pedagogical approach. The instructional context of the school includes the school’s instructional imperatives (e.g. what is monitored or supported), the school curriculum and the instructional norms evidenced in the practice of other teachers. Where the teacher’s
espoused pedagogical approach and the instructional context of school were aligned, instances of persistence emerged.

Figure 5.2: Novice Teacher’s Persistence in Implementation

Crystal epitomizes the power of this alignment. Between her first and second year of teaching her school shifted their prescribed instructional approach to one that was more aligned with the approach Crystal learned in preservice, but she does not report using the approaches until her school context shifted to one that supported her espoused pedagogy.

I would say that the shift in our organization has had a tremendous impact on my teaching. The way that they [her school] want us to approach teaching and learning has had the biggest impact, but because of [Abigail’s] class, I felt more comfortable doing it than I would have had I not been exposed to that
class…They [her school] have really gotten on board with Penn's philosophy to teaching and learning (Crystal).

The most consistent implementation occurred when multiple points of alignment existed between the school context and the teachers espoused pedagogical approach. In this study, Myles had the greatest alignment between his espoused pedagogical approach and his enacted pedagogical approach, and, he also identified the greatest alignment between his espoused pedagogical approach and the approach to instruction norms of his school. Myles had the benefit of a complementary curriculum, school support in the form of professional development and a team of teachers embracing a similar approach (i.e. the benefit of instructional norms).

It is important at this juncture to re-establish that teachers’ espoused pedagogical approach is central even to the way they operate in aligned school contexts. For example, and to his credit, Myles recognized that the enVision curriculum could be engaged without attending to core aspects of the pedagogical approach he learned in preservice. He surmised that without the preservice experience his approach to instruction using the enVision curriculum would have been different,

I think I probably would be more tied to the book [enVision], which like I said, enVision does a decent job of incorporating some of that stuff into the book, but I think I'd be much more focused on, here are the steps, and here's the procedure and probably not doing a lot of discussion. I'd probably have a lot less patience for wrong answers, to be honest, because I might not have seen the value in that. I might have seen it as, that's a misconception, I better clear that up really fast whereas usually a student after enough talking and thinking will clear it up themselves, which is a lot more powerful. (Myles)

Myles interpreted the enVision text through the lens of his espoused pedagogical approach. As Remillard (2003) notes, “teachers bring their own beliefs and experiences
to their encounters with curriculum to create their own meanings. Through using them, teachers interpret the intentions of the authors” (Remillard, 2003, p.12). This reaffirms that persistence in the implementation of constructivist oriented math pedagogies requires alignment of both teacher’s espoused approach and the school’s instructional context: for novice teachers, a supportive instructional context is necessary but insufficient.

**Struggling Students.** Even in cases where the teacher’s espoused approach and the school’s instructional context are aligned, struggling students still present novice teachers with an extraordinary challenge. I am defining struggling students as those who seem unable to grasp grade level work because they do not possess the requisite skills or knowledge expected to be mastered at previous grades. Myles, contended that he gives students who need it, a direct method for solving a problem.

A lot of kids don't really feel comfortable until they have some concrete stuff to follow (Myles)

Sometimes, you've just got to tell them what to do. Some kids just need that life line, I think, of "What do I do?"...Like I said, for some kids in reading, as in math, it's just, they can feel real lost. They just need a life line. They need to be told like, "Is there something I can do here. I can't come up with my own strategy right now. I don't even know what you mean. I need something." (Myles)

Myles clearly attempted to first engage struggling students in pedagogies that highlighted meaning making, but there was a tendency among the teachers to rely on or revert to a more didactic, procedural approach to instruction when faced with struggling students. In fact, Isa’s suburban school [professional placement] instantiated a system that provided advanced students with a problem based lesson format and remedial students with practice and homework review. When Isa began teaching in that district
she continued the pattern and argued that she wasn’t sure if the problem approach would work with struggling students.

**Theme Four: Teacher Interactions with the Unique Contexts of Urban Schools**

For Myles and Isa, the number of struggling students in their school context was limited. For Daniel, Gwen, Lacey and Michael, who taught in a high needs urban school, teaching struggling students represented the bulk of their instructional practice. Urban schools, at the most fundamental level, are schools located in urban or heavily populated areas. However, urban schools often have an additional set of attributes (or increased frequency of certain attributes) potentially unrelated to population density. These attributes include highly impoverished communities with high concentrations of non-white students, low rates of student achievement and the least qualified teachers (Lankford, Loeb & Wyckoff, 2002). Challenging student behaviors are also often more frequent and more severe in urban schools (Lassen, Steele & Sailor, 2006). Four teachers from the study, Michael, Daniel, Lacey and Gwen, taught in high-needs, urban schools after completing the Penn TEP. They felt enormously challenged with teaching in these schools and all but Gwen argued vehemently that their preservice experience had not prepared for them particularities of urban schooling.

**My Students are Behind.**

Caught between his students’ lack of readiness and mounting accountability pressure from his school (in the form of standardized tests) Daniel quickly reverted to a didactic, procedural approach to math instruction, limiting his persistence in implementation.
I tried to do conceptual teaching but it was really difficult because the kids were two, three years behind grade level in math and so for the majority of that year I taught procedurally (Daniel).

In addition, Daniel’s attempts to teach in a manner aligned with his espoused pedagogies revealed limitations in his own understanding of the content. He had embraced a pedagogy in preservice methods that required a level of content mastery that he did not possess. Daniel’s struggle was as much about his own knowledge deficit as it was about his students’ knowledge deficit.

Gwen did push back against the tendency to teach struggling students in ways that differed from her espoused pedagogy. When her attempt to address the administration fell on deaf ears, she left her first professional teaching assignment. Despite her second school having increased math support and a 30 minute math period dedicated to inquiry based problem-solving, she argued the school still maintained an approach to instruction that disenfranchised certain students.

‘Well this kind of teaching works best for kids from blah, blah, blah background,’ which drives me up the wall. So, yeah, I think good [math] teaching is good [math] teaching regardless of who your kids are. (Gwen)

I recently learned that Gwen left that school as well, challenged by her growing sensitivity to the discrepancy between how to teach struggling students vs. high achieving students.

**I Cannot Get to Instruction.** A second set of teachers found themselves limited in their ability fully focus on instruction. A safe and well managed classroom is often seen as a prerequisite to effective instruction (Bloom, 1995). Yet, “managing student behavior can be one of the greatest concerns and a laborious exercise for prospective
teachers in urban schools” (McKinney, Campbell-Whately & Kea, p.16). Both Michael and Lacey epitomized this phenomenon. The challenges they experienced with classroom management prevented them from engaging fully the challenges of instruction. After two years, Michael, who had a more constructivist k-12 experience than any of the other teachers in the study, left the profession. He traced his inability to manage the behaviors in his professional classroom to his lack of exposure to urban classrooms during preservice (see Michael’s case for more detail).

Michael’s story is not unusual. In fact, recent studies of attrition demonstrate that 17% of teachers stop teaching after five years (Gray & Taie, 2015) and that high-poverty, urban schools are disproportionately impacted by teacher turnover and attrition, losing up to 20% of their staff each year (Ingersoll, 2004). As Ingersoll has argued, the reasons for staff departure are varied but often include student-discipline and lack of administrative support.

Significant numbers of those who depart from their jobs in these schools report that they are hampered by inadequate support from the school administration, too many intrusions on classroom teaching time, student discipline problems and limited faculty input into school decision-making. (Ingersoll, 2004, p.2).

Lacey, who taught in a large, poor, urban metropolis also struggled to create a classroom environment that was conducive to learning. Unlike Michael, Lacey was no stranger to urban school environments. Still she did not possess the skills to create a learning community in her classroom nor to engage parents in the support of their students’ education. She attributes her lack of preparation to preservice.

Once I got into the classroom I was like oh, wow, I met some things that they [the Penn TEP] did not teach…For example, classroom management, that was a huge
thing, especially dealing with just urban schools in general… it is so stressful Joy. I teach kindergarten and classroom management is a huge piece because I have a lot of really extreme behaviors like explosive, like OCD, ADHD (Lacey).

Michael complained of lack of exposure, but Lacey seems to demonstrate that exposure to urban environments does not ensure that one will obtain the skills necessary to successfully navigate those environs.

As previously mentioned low achievement and lack of grade level knowledge is often a common characteristic of high-need urban schools. These issues exist in all schools but at greatly increased magnitudes in high needs, urban schools. I argue that addressing urban related context issues in preservice is critical because those issues either obscure discussions about teaching math or require novice teachers to make significant adaptations in the enactment of their espoused approach to teaching math.
CHAPTER 6: CONCLUSION

Summary of Findings

The research on teacher knowledge of mathematics has long shown that preservice teachers’ disciplinary or content knowledge tends to be thin and rule bound (Ball, 1988; Ball 1990). Kennedy (1999) argued that the university-based preservice experience is ideally situated for the task of preparing teachers to transition from a procedural approach to math to a more constructivist approach, but questioned the ability of preservice to succeed at the task. The findings from my research strongly suggest that there is great potential for preservice to substantially alter the initial frames of reference that teachers bring to preservice (Kennedy, 1999), especially for teachers whose k-16 math experiences have been dominated by procedural, top-down approach math. My research also affirmed that teachers continue to arrive at their preservice programs with only a procedural grasp of elementary mathematics. Not only had the majority of teachers in this study experienced a singularly computational approach to elementary math as elementary school students, they were also blithely unaware of the significant limitations of those experiences. Consequently, they maintained a sense of confidence concerning their mastery of the elementary forms of the discipline, the “simple arithmetic.” The teachers’ procedural understanding of math was compounded by a perception that struggle served as a marker of limited ability.

Notwithstanding, in less than four months, the Penn TEP seemed to upend more than twelve years of teachers’ passive indoctrination about the nature of math and how to teach it. The math methods course served as the primary instigator of this change. The
course instructors were intentional in their efforts to challenge teachers’ personal history-based beliefs (Holt-Reynolds, 1992). The challenges to the teacher’s history-based beliefs were not merely expounded as theoretical propositions; they were elucidated by engaging teachers in the novel experience of solving complex elementary math problems. The experiences, embedded discourse rich instructional methods, elucidated new approaches to problem solving, and, possibly of greatest import, allowed the teachers to see familiar elementary math content in a new way (Ebby, Faculty Interview, 2017).

Their efforts had substantial impact on the teachers who reported that they would be different teachers today had they not had the preservice experience, typically more rule bound and concerned about guiding students through each step of a problem. They attributed concern for sense making through the discovery of multiple solution pathways directly to their math methods course. The evidence for this second finding is quite strong replicated in the background survey, interview data and the CRLA.

A third cluster of findings is related to the way that preservice learnings were evident in the teachers’ professional practice. There was a striking similarity in the espoused pedagogical approach for teaching elementary math post-graduation across the six teachers in the study. They exemplified both ideological persistence and persistence in implementation. Ideological persistence was evident two to three years post-graduation. Persistence in implementation varied more substantially, being powerfully influenced by the alignment between school context and the teacher’s espoused approach, and quite profoundly by the particularities of the urban school context. Regardless of
context or level or preparation, novice teachers tended to demonstrate a tendency to revert to didactic, procedural approaches to math instruction when faced with struggling students. The strength of these findings is moderate as I did not observe these teachers in their schools teaching lessons but have based all evidence of persistence on self-report.

**Implications for Future Study**

Having demonstrated the potential impact of preservice education on novice teachers’ math development the implications for future study center on how to deepen and sustain that impact. The most vulnerable teachers in this study served in contexts where classroom management, especially of explosive student behaviors was inadequate. Bloom (1995) contends that,

> Children are only able to learn within a context of safety and security. But for many children today, neither their homes nor their schools are safe places for living, much less learning. This situation shows no indication of reversing itself in the foreseeable future. Educators then, must face what becomes an intolerable burden - how to educate children who are disturbed, distracted, hyperaroused and whose behavior often interferes with their own learning and the learning of others (Bloom, 1995, p.1).

The sanctuary classrooms literature connects the oft challenging nature of student behavior in schools to trauma and providing teachers with a model for addressing trauma that shifts the root cause of many challenging behaviors from deviance to injury (Bloom, 1995). Further research is warranted into the ways that sanctuary classroom models and the accompanying skillsets can be provided to preservice teachers, especially those preparing for urban contexts.

Secondly, future research must consider the most effective models for continuing university support of novice teachers throughout their professional careers (Rust,
personal communication, 2017). The model of preservice that ends with graduation is becoming obsolete. The challenge for implementation identified in my research was not the absence of transformation in teachers’ pedagogical approach but the inability to implement an espoused approach in an inhospitable or insufficiently aligned school contexts. In no case, was the novice teacher positioned to impact the instructional imperatives of the school. The options, for the teachers, devolved to succumb (both to positive or negative pressures) or leave.

Finally, further research is needed in identifying the skills and understandings novice teachers need to teach through the gap. In using the term teach through the gap I am referring to the distance between a student’s expected level of content mastery and inquiry skill and their actual level of content mastery and inquiry skill. One of the major challenges faced by teachers in this study, both within and outside of urban schools involved teaching struggling students [those who were not progressing in time/step with grade level standards]. Research that aims at providing novice teachers with concrete strategies for teaching through the gap, especially in the context of high stakes testing is much needed.

The purpose of this study was to tell the story of preservice education through the lens of successful preservice experiences, particularly as they relate to the teaching and learning of elementary mathematics. In elucidating the experiences of teachers whose preservice experiences have been successful, I have challenged the stereotype that preservice is ineffective. I have illuminated the understanding that teachers continue to present at preservice with limited mathematical understandings. I have also called
attention to the ways in which high need urban school contexts raise concerns for
preservice instructors and novice teachers. The results of the research may serve to
support teacher educators and those designing teacher preparation programs in thinking
more deeply about the different ways that preservice teachers develop the math subject
matter knowledge requisite for teaching and the convergence of factors that contribute to
shifts in pedagogical approach.
## APPENDIX I: PLAN FOR DATA COLLECTION, ANALYSIS AND DEFENSE

<table>
<thead>
<tr>
<th>Research Timeline</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 24, 2014</td>
<td>Proposal Hearing</td>
</tr>
<tr>
<td>April - September 2015</td>
<td>Data Collection</td>
</tr>
<tr>
<td>October 2015</td>
<td>Transcription (rev.com)</td>
</tr>
<tr>
<td>November 2015 - March 2016</td>
<td>Data Analysis</td>
</tr>
<tr>
<td>April - December 2016</td>
<td>Writing</td>
</tr>
<tr>
<td>December 2016</td>
<td>Turn in Final Draft</td>
</tr>
<tr>
<td>December 2016</td>
<td>Final Defense</td>
</tr>
</tbody>
</table>
Hello! My name is Joy Anderson Davis. Many of you may remember me from your preservice program at the University of Pennsylvania. I co-taught the preservice field seminar during your tenure at Penn. You may also remember that I am a doctoral student at Penn. I am currently in the process of collecting data for my dissertation and am emailing in the hopes that you will consider sharing with me about your preservice experiences. I am hoping to learn more about the ways in which your preservice experience impacted what you know and understand about teaching elementary math.

Please consider completing the following online survey. The survey has three parts. Part one is a consent form that includes information about my study and requests your consent to participate. The second part consists of questions about your math background and your perceptions of your experiences learning to teach elementary math at Penn. Part three is a questionnaire about teaching elementary math. There is a request at the end of part two for your contact information if you are open to being contacted for two follow up interviews.

Please remember that completing the survey does not obligate you in any way to participate in the follow up interviews and only those individuals who are open to participating beyond this initial survey need to provide information about their identity.

You can complete the survey by clicking the link below or pasting in into your browser.

Thank you in advance for your time. I truly, truly appreciate it. If you have any questions or concerns do not hesitate to contact me at pearson.joydavis@gmail.com or 267 597 4381.

Joy Anderson Davis
APPENDIX III: BACKGROUND SURVEY

There are several questions in this survey that ask you to rank the significance and positivity of your preservice learning experiences. Please be aware that these questions are referring only to the impact of those experiences on your knowledge of elementary math and your understanding of the best ways to teach it. These questions ARE NOT an indicator of your overall satisfaction or dissatisfaction with your teacher preparation program or experience.

By clicking the following button you certify that you have read the consent form and give your consent for participation in the study.

Consent

1. Gender

2. Age Band (Choose one)
   21-25        26-30        30-39        40+

3. Racial/Ethnic Affiliation
   □ Asian/Pacific Islander
   □ Black/African American
   □ Hispanic
   □ White

4. Reflect on your own mathematical learning. Which sentence(s) best describes the message you received during your years as a mathematical learner (K-16)\textsuperscript{11}
   □ Arriving at the correct answer to a problem is the most important thing.
   □ The methods used to solve a problem are just as important as arriving at the correct solution.
   □ There is only one way to solve a problem and it is the teacher’s way. The teacher is responsible for telling students that one way.

\textsuperscript{11} envision MATH® Common Core © 2012 Problem-Based Interactive Learning © 2012 Pearson, Inc.
There are many ways a problem can be solved that will ultimately lead to the correct solution. It is the responsibility of students to pursue a variety of ways and justify their reasoning.

5. Did you have teaching experience before starting your teacher preparation program at Penn? Check all boxes that describe your previous teaching experience.

- full-time paid
- student teaching
- volunteer
- summer program
- foundational (ELA, math, science, etc.)
- extra-curricular (arts, music, etc.)
- other ______________

6. Did you teach elementary math in the 2013-2014 school year?

What grade level(s) did you teach? (check all that apply)

<table>
<thead>
<tr>
<th>Grade</th>
<th>2013-2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

How often did you teach math? (check one)

- Daily
- Bi-weekly
- Weekly
- Other________

7. How long do you anticipate you will work as a classroom teacher?

- <5 years
- 5-10 years
- 11-20 years
- Career teacher

For questions 8-15 please identify your level of agreement with each statement. Please remember that references to “preservice” includes your entire preservice experience from the time you began at GSE until commencement (e.g. fieldwork, coursework, conversations with your supervisor, etc.)
8. I learned a lot of elementary math content during preservice.
   
   Strongly Agree  Agree  Neutral  Disagree  Strongly Disagree
   
   Comment:

9. I learned a lot of elementary math content in my methods class.
   
   Strongly Agree  Agree  Neutral  Disagree  Strongly Disagree
   
   Comment:

10. I learned a lot of elementary math content during my fieldwork experience.
    
    Strongly Agree  Agree  Neutral  Disagree  Strongly Disagree
    
    Comment:

11. I learned a lot about how to teach elementary math in preservice.
    
    Strongly Agree  Agree  Neutral  Disagree  Strongly Disagree
    
    Comment

12. I learned a lot about how to teach elementary math in my methods class.
    
    Strongly Agree  Agree  Neutral  Disagree  Strongly Disagree
    
    Comment

13. I learned a lot about how to teach elementary math during my fieldwork experience.
    
    Strongly Agree  Agree  Neutral  Disagree  Strongly Disagree
    
    Comment

14. I use a lot of the math content I learned in preservice in my current teaching assignment.
    
    Strongly Agree  Agree  Neutral  Disagree  Strongly Disagree
    
    Comment

15. The way that I currently teach math is reflective of the pedagogy I learned in preservice.
    
    Strongly Agree  Agree  Neutral  Disagree  Strongly Disagree
    
    Comment
16. I learned most of the math content that I teach in/at ____________________________

17. The person or institution that most influenced my teaching style is ____________________________

18. If you identified GSE as significantly influencing your pedagogical approach please identify at least one way that your experience at GSE influenced your pedagogical approach.

___________________________________________________________________________

___________________________________________________________________________

If you are open to being contacted regarding further participation in this study please type your name and email address in space below.

When you click done you will automatically be redirected to the remainder of the survey questions.
APPENDIX IV: SAMPLE MKT SURVEY QUESTIONS

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

   Which statement(s) should the sisters select as being true? (Mark YES, NO, or I’M NOT SURE for each item below.)

   a) 0 is an even number.
      Yes
      No
      I’m not sure

   b) 0 is not really a number. It is a placeholder in writing big numbers.
      Yes
      No
      I’m not sure

   c) The number 8 can be written as 008.
      Yes
      No
      I’m not sure

2. Ms. Chambreaux’s students are working on the following problem: Is 371 a prime number? As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

   a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
   b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
   c) Check to see whether 371 is divisible by any prime number less than 20.
   d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

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12 “We have publicly released a small set of items from our projects’ efforts to write and pilot survey measures…. However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge…We ask users to keep in mind that these items represent steps in the process of developing measures. In many cases, we released items that failed, statistically speaking, in our piloting; in these cases, items may contain small mathematical ambiguities or other imperfections.” (Learning Mathematics for Teaching, 2008, p.2). For the complete set of released items see http://lmt.mspnet.org/index.cfm/17924.
APPENDIX V: INTERVIEW 1 [SUBJECT MATTER KNOWLEDGE]

Thank you for agreeing to participate in my study. Please tell me a little about what you have been doing since you completed your program. Where do you teach? What grade levels? How is it going?

Follow up on MKT & Subject matter knowledge background

3. Briefly describe your experience taking the MKT. Did it seem easy? Difficult?

4. Do you think your performance on the MKT is an accurate measure of your current understanding of elementary math content

5. When and how do you think you learned the knowledge required to answer those questions?

6. Do you think you could have successfully completed that survey before your preservice program?

7. Can you tell me about your math experiences before preservice?

8. Elementary school, high school, college, work?

9. Did you consider yourself to be a good or competent math student?

10. When you started preservice did you anticipate that you would have any difficulty learning to teach math or with your math methods class? Why? Why not?

11. There were several questions at the end of the MKT about your perception of your mathematical competence. You answered X. Do you feel confident about your answer? What influenced your responses?

12. Math experiences in preservice

13. Okay I want to take you back to your teacher preparation experience. Do you have your portfolio available? Have you had a chance to preview it? If not take a few moments now to look it over, especially the sections on math.

14. Thinking about your preservice experiences and what you learned from that experience, what stands out to you? Don’t just think about math, think in general?

   a. What experiences are most memorable?
   b. How, if at all, did those experiences impact you?
1. Now I want you to think specifically about teaching elementary math and what you need to know about math to do it well. What preservice experiences come to mind—experiences that helped you learn the math you need to know to teach elementary math well?

   a. What made that experience noteworthy?
   b. The context?
   c. The people with you?
   d. A previous experience?

How, specifically, did your mathematical knowledge change in and thru that experiences?

e. Probes: Can you tell me about your (math methods course, student teaching experience, interactions with your supervisor, specific projects or assignments)? Did you consider that experience significant? Why or why not? How, if at all, do you think that experience contributed to your knowledge of mathematics?

Was there ever a time during your preservice experience when you felt like your mathematical knowledge or what you thought you knew about math was being challenged?

f. Can you tell me about that experience?

How would you describe the knowledge that is required to teach math? What do you think you need to know about math to teach it well?

g. What experiences have convinced you of that?

Is there anything you think you wouldn’t know about elementary math, that you think is important to know or that you currently rely on knowing in your current work, had you not done preservice?

h. Can you tell me about the experiences during which you acquired (or began acquiring) that knowledge?

Where you think about the place and time when you acquired most of the math knowledge you need to teach elementary math what place and time come to mind?

i. Can you tell me about the experiences you had there?

j. How did those experiences contribute to what you know about elementary math?
Is there anything else you want to share with me about your experiences learning the content commonly associated with learning and teaching elementary math?

Thank you for your time. Have a wonderful rest of the day!
The Common Core Standards Initiative. Common Core State Standards for Grade 3 are divided into five knowledge domains: Operations & Algebraic Thinking, Number & Operations in Base Ten¹, Number & Operations—Fractions¹, Measurement & Data, and Geometry. The second standard for operations and algebraic thinking at grade 3 is as follows:

Represent and solve problems involving multiplication and division

- 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret \(56 \div 8\) as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as \(56 \div 8\).

Q1. Can you put the standard into your own words?

Q2. What would be the goal or objective of your introductory lesson for this standard?

Q3. Describe the components of your lesson in sequential order. For each component (or step) identify 2 things: teacher/instructor actions and student actions. Also, please specify any materials you would use.
APPENDIX VII: INTERVIEW 2 [PEDAGOGICAL APPROACH]

Thank you for participating in this second interview. I received your completed lesson agenda. Before we discuss it, I would love for you to tell me a little bit about any teaching experiences you had before preservice.

Background

Where did you teach? For how long?
What did you think good math teaching looked like before you started preservice?

Lesson agenda talk

Okay, I would like to have you talk thru the thinking behind the choices/steps in your lesson agenda. As you talk through each section of your lesson agenda try to identify how your choices reflect your beliefs about good math instruction. Feel free to elaborate on important aspects of the activities/steps you have written.

As the interviewee is reflecting on their lesson agenda, I will compile a list of their expressed conceptions, ideas or beliefs about good teaching. When they are done, I will go through the list idea by idea to allow them to expound on it.

I will read the list of ideas you have expressed about good math instruction. After each one I would love for you to talk more about that idea. You do not have to add anything to idea but if there is more you want to say, feel free.

After they have had a chance to further expound on their expressed beliefs about good teaching I will again reread the list one by one and ask them to briefly describe experiences they have had (anywhere) that were critical to their adoption of the particular belief.

I am going to read through the list of ideas you have expressed again giving you a chance after each one to think of the experience (s) that were critical to you adopting this understanding.
Preservice experiences

I then reviewed one by one the experiences they identify that are from preservice and asked:

Can you describe the experiences more fully?
   Probe—Who was there? What as the context?

What made that experience noteworthy?
   Probe—The context? The people with you? A previous experience?

How, specifically, was your picture of what it means to teach math well impacted or changed by that experience?

I would like for you to think only of your time in preservice. As you reflect on it and the ways in which your program was designed to help you develop a picture of what good teaching looks like what experiences stand out as personally meaningful?
   Probe: Were there lectures, particular readings, assignments, interactions with particular people, interactions with students?

Can you describe the experience(s)?
   Probe: Who was there? What was the context?

What made that experience noteworthy?
   Probe: The context? The people with you? A previous experience?

How, specifically, was your picture of what it means to teach math well impacted or changed by that experience?

Was there ever a time during your preservice experience when you felt like your understanding of what it means to teach well was being challenged?
   Can you tell me about that experience?

General beliefs about good math instruction:

How would you describe what it looks like to teach math well?

What experiences have convinced you of that? Do you think your description is true in all contexts?

Is there anything you think you wouldn’t know about how to teach math, that you think is important to know or that you currently rely on knowing in your current work, had you not done preservice?

Can you tell me about the experiences during which you acquired (or began acquiring) those understandings?
When you think about the experiences through which you were most powerfully exposed to the beliefs/understandings you now hold about good math instruction, what place comes to mind?

Can you tell me about the experiences you had there? (if we have not discussed them already).

How did those experiences contribute to what you know about elementary math?

Is there anything else you want to share with me about the experiences you have had that influenced your picture of what constitutes good math?

Thank you for your time. Have a wonderful rest of the day!
APPENDIX VIII: FACULTY/staff INTERVIEWS

Thank you so much for agreeing to speak with me. This interview semi-structured so you can feel free to add to or elaborate on questions I ask as you feel inclined! (Smile) My goal here is to get a good sense of the goals of your program and how those goals are enacted in program structures and assignments. Let’s start off with some basic demographic info.

1. What is your primary role in the Penn TEP program?
2. How long have you served in that role
3. Are there are roles you serve in the program?

Course Instructors

1. How would you describe the overall goals of your program?
2. What are the goals of your course?
3. How does your course support the overall program goals (instructors only)
4. What aspects of your students’ overall program best compliment or support your course? (instructors only)
5. How much time do you have for your course (how many sessions)? (instructors only)
6. How much time would you have in a perfect world?
7. What do you think is most important for your students to get from your course?
8. How is course designed to ensure this learning?
9. Assignments? Projects?
10. If you had no limits what would you include in your course that is not currently a part of your course?
11. Do you think your students will be prepared to teach elementary math when they finish your course?
12. If yes, why
13. If no, what else will they need
14. In the context of teaching mathematics, how would you complete the statement, “A novice math teacher should be know and be able to. . . .
15. Imagine you were given a few teacher belief statements (statements that focused on what good math instruction looks like) and told that one of them belonged to a TEP alum.
16. What clues in the statements would help you pick out the TEP alum?

Program Directors
17. How would you describe the overall goals of the program?
18. What kind of teachers are you hoping to create?
19. Imagine you were given a few teacher belief statements (statements focused on what
good instruction looks like) and told that one of them belonged to a TEP alum.
   a. What clues in the statements would help you pick out the TEP alum?
20. How are the program goals enacted in and thru the structures and experiences built
    into the program?
21. What is the trajectory of experiences in which TEP teachers are engaged from the
    start of their program till its completion?
22. What is the role of the supervised/supported teaching placement in your program?
23. What is the role of coursework?
24. Are there any other components of your program besides coursework and
    supervised/supported teaching?
    a. Can you describe them?
    b. Can you tell me about the ones that are specifically designed to enhance teachers’
       ability to teach elementary math? How are they designed to accomplish this goal?
25. Are there components of your program that are associated but not administered
    thru/by the formal program? Can you tell me about these?
    a. Are there quintessential TEP experiences? Can you describe them?
       Probe—These do not have to be courses—they could be anything, even internal
       experiences
26. Aside from instructors, are there other individuals within your program or associated
    with your program whose role is to support teachers’ acquisition of subject matter and
    pedagogical knowledge?
27. Does your program have a stance on the role of subject matter knowledge in
    teaching? What is that stance? How do you ensure that students gain the requisite
    knowledge?
28. How do you define a successful preservice teacher? A successful novice teacher?

Is there anything else about your program that you think it would be important for me to
know?

Thank you again for your time. It has been a pleasure and I wish a continued success in
this important work.
APPENDIX IX: PARTICIPANT CONSENT FORM

Preparing Successful Teachers of Mathematics

Joy Davis
Pearson.joydavis@gmail.com

You were selected as a possible participant in this study because you completed the Teacher Education Program, Graduate School of Education University of Pennsylvania or the Teach for America Graduate School of Education University of Pennsylvania Partnership in May of 2013 OR you serve as faculty/staff for one of the aforementioned programs. Your participation in this research study is voluntary.

Why is this study being done?

The purpose of this study is to explore the ways in which preservice education influences teachers’ mathematical knowledge and approach to teaching math.

What will happen if I take part in this research study?

If you volunteer to participate in this study, the researcher will ask you to do the following:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Timeframe</th>
<th>Eligibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete a brief survey about math background and preservice experience and a brief mathematical knowledge survey.</td>
<td>October 15 – November 5, 2014</td>
<td>All members of the TEP or TFA cohorts that completed their programs in 2013 are asked to participate in the background surveys.</td>
</tr>
<tr>
<td>Interview 1</td>
<td>December 1- December 19, 2014</td>
<td>Only those participating in the full study</td>
</tr>
<tr>
<td>Complete a lesson plan outline and participate in a follow up interview (Interview 2)</td>
<td>December 1- December 19, 2014</td>
<td>Only those participating in the full study</td>
</tr>
<tr>
<td>Participate in a single interview about the goals of TEP</td>
<td>October 15-November 5, 2014</td>
<td>Faculty/Staff of TEP</td>
</tr>
</tbody>
</table>
How long will I be in the research study?

Participation can begin as early as October 15, 2014 and will end December 30, 2014. Participation in the full study will require approximately 5-6 hours over the 2.5 months of the study. Some study participants will be asked to review notes from their personal interviews to check for accuracy of the interviewer’s codes.

Are there any potential risks or discomforts that I can expect from this study?

There are no anticipated risks or discomforts.

Are there any potential benefits if I participate?

You will have the opportunity to tell your story and may benefit from reflecting on your preservice experiences.

Will I be paid for participating?

Each participant who completes the full study will receive a $100 gift card.

Will information about me and my participation be kept confidential?

Any information that is obtained in connection with this study and that can identify you will remain confidential. It will be disclosed only with your permission or as required by law. All paper and pencil assessments will be kept in a locked drawer in my office and will be accessible only to me. All interview transcripts and recordings will be kept on a laptop secured by a password known only to me. All email surveys will be sent to an email address created for the purposes of this study. Again, only I will have access to these surveys. I do reserve the right to disclose the data to my dissertation committee as necessary and only for the purposes of this research project. When discussing the any data with my supervising committee all names and identifiers will be removed.

What are my rights if I take part in this study?

- You can choose whether or not you want to be in this study, and you may withdraw your consent and discontinue participation at any time.
- Whatever decision you make, there will be no penalty to you, and no loss of benefits to which you were otherwise entitled.
- You may refuse to answer any questions that you do not want to answer and still remain in the study.

Who can I contact if I have questions about this study?
If you have any questions, comments or concerns about the research, you can talk to the one of the researchers. Please contact:

Howard Stevenson  
Joy Davis – pearson.joydavis@gmail.com

UPenn Office of Regulatory Affairs Institutional Review Board

If you have questions about your rights while taking part in this study, or you have concerns or suggestions and you want to talk to someone other than the researchers about the study, please call the UPenn IRB at (215) 573-2540 or write to:

UPenn Office of Regulatory Affairs  
3624 Market Street, Suite 301S  
Philadelphia, PA 19104

SIGNATURE OF STUDY PARTICIPANT

__________________________________________  
Name of Participant \textbf{AND} email address

______________________________  
Signature of Participant \hspace{2cm} Date

SIGNATURE OF PERSON OBTAINING CONSENT

__________________________________________  
Name of Person Obtaining Consent \hspace{2cm} Contact Number

______________________________  
Signature of Person Obtaining Consent Form \hspace{2cm} Date
Teacher Gwen
The Common Core Standards Initiative, Common Core State Standards for Grade 3 are divided into five knowledge domains: Operations & Algebraic Thinking, Number & Operations in Base Ten¹, Number & Operations—Fractions¹, Measurement & Data, and Geometry. The second standard for operations and algebraic thinking at grade 3 is as follows:

Represent and solve problems involving multiplication and division

- 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

--------------------------------------------------------------------

Q1. Can you put the standard into your own words?

Students will be able to contextualize division problems. They will understand that the dividend is the total amount of things you have. The divisor is EITHER the amount of groups you can make (in which case the quotient is the amount in each group) OR the amount in each group (in which case the quotient is how many groups you have). Students need to develop a story problem, given an equation of this sort.

Q2. What would be the goal or objective of your introductory lesson for this standard?

SWBAT create a story problem given a division equation.

13 Teacher Crystal only participated in the first interview. Teacher Daniel did not complete a written version of his CRLA. He described his lesson format orally during Interview 2.
Q3. Describe the components of your lesson in sequential order. For each component (or step) identify 2 things: teacher/instructor actions and student actions. Also, please specify any materials you would use.

1) Presentation of Problem: \( 56 \div 8 = 7 \). What question would have this equation as its solution?
   - Teacher and Students: Read problem.
   - Teacher: Call on students to retell the problem (without looking at it). 3 students will be called on: first, one who almost definitely understands, second, one who exemplifies most of the class and third, a student who struggles to attend to math class.
   - Students: Retell the problem.

2) Inquiry
   - Students: In groups, students develop a context for the problem. They have access to manipulatives.
   - Teacher:
     - Tracks students’ work in assessment tool
     - Asks guiding questions (i.e. “What are those cubes?” “What does it mean to divide by 8?”)

3) Sharing
   - Teacher: Pre-selects 2 groups to share out (groups who contextualized it in different ways).
   - Students: Share work or listen to peers

4) Direct Instruction
   - Teacher explicitly demonstrates how to contextualize a different problem (i.e. \( 45 \div 9 \)): There are 45 cookies. I have 9 friends with whom I want to share. How many cookies would they each get? AND There are 45 cookies. I put 9 into each bag. How many bags did I make?
   - Students: Eyes on teacher.

5) Independent Practice: Worksheet on this skill.
   - Teacher: Floats around class offering help/pausing whole class for brief re-teach, if necessary
   - Students: Complete worksheet
   - At conclusion, teacher selects 2-3 problems to review whole class

6) Exit Slip
   - Students do independently
   - Teacher does nothing, but uses data from this to design next day’s lesson/reteach block.
**Teacher Michael**
The Common Core Standards Initiative, Common Core State Standards for Grade 3 are divided into five knowledge domains: Operations & Algebraic Thinking, Number & Operations in Base Ten¹, Number & Operations—Fractions¹, Measurement & Data, and Geometry. The second standard for operations and algebraic thinking at grade 3 is as follows:

Represent and solve problems involving multiplication and division

- 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

---

**Q1. Can you put the standard into your own words?**

Use the act of separating a quantity of objects or symbols into equal shares to show the act of division.

**Q2. What would be the goal or objective of your introductory lesson for this standard?**

Students will be able to separate objects into equal shares in order to show that a quantity can be made up of a number of equal parts.

**Q3. Describe the components of your lesson in sequential order. For each component (or step) identify 2 things: teacher/instructor actions and student actions. Also, please specify any materials you would use.**

*Hook: (10 min) Have students stand. If group is a prime number, remove one student to be the "assistant". Have students divide themselves into equal groups. If there is confusion about the instruction, let the students know the appropriate number of groups into which they need to divide. Ask students: How many groups are there? How many people are in each group? How many students are there total? Write equation on the board. Example: 3 groups of 8 students makes 24 students. 3x8=24*

Have students regroup into a different number of groups. Write equation on the board. Discuss the fact that the number of students hasn't changed, but the other numbers have changed.

In mixed groups of 3 or 4 students, have students create a poster representing an equation.
Materials: Quantities of dried beans, white glue, poster board, markers

Distribute non-prime number of beans to each group. Tell students to create a poster dividing the beans into equal shares. Share posters with the group.
Teacher Isa

The Common Core Standards Initiative, Common Core State Standards for Grade 3 are divided into five knowledge domains: Operations & Algebraic Thinking, Number & Operations in Base Ten¹, Number & Operations—Fractions¹, Measurement & Data, and Geometry. The second standard for operations and algebraic thinking at grade 3 is as follows:

Represent and solve problems involving multiplication and division

- 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Q1. Can you put the standard into your own words?

Students must be able to describe a real life situation where 56 objects are divided into 8 groups of 7 when given the expression 56 divided by 8.

Q2. What would be the goal or objective of your introductory lesson for this standard?

SWBAT write or draw a division expression and its quotient as a set of objects.

Q3. Describe the components of your lesson in sequential order. For each component (or step) identify 2 things: teacher/instructor actions and student actions. Also, please specify any materials you would use.

I’m a little burnt out right now, Joy, and I have five minutes before our call. You are definitely not going to get my best work here, and I apologize.

1. **INTRO:**
TEACHER: today, we are going to turn division problems into stories about things we can put in groups. the problem will be the title of the story. then, we will share them with our math partners. it’s fun because you can make a math problem into anything that you can put in groups. the only rule is that you MUST make the story fit the numbers in the division problem that I give you. i will show you how i might make a story, and then we can do one together.

2. **MODELING**
drawing a problem out of a hat: 15 divided by 3

TEACHER: ah, i got 15 divided by 3. 15 is my dividend, so that is what I’m putting into groups. 3 is my divisor, so that will be the number of groups. I need to write about 15 things that go in 3 groups... I know 15 divided by 3 is ...(pointing to class) (class: “5”), yes, 5!, so I will have 5 in each of my 3 groups.

hmmm... what do i like a lot?? I love reading! Ok, I’ll write about 15 books. I can divide them into groups of five on three shelves.

I’ll draw a quick photo for my story first. (teacher draws a book shelf with 3 shelves, and 5 books on each shelf, narrating as she draws.)

Now, I’ll write my story... (on smart board, teacher writes story under photo)

*Ms. Wells had 15 books to put on 3 shelves. Each shelf has 5 books!*

When I finish, I’ll share my story with my partner.

Let’s do one together now. Please take out your white boards and markers, and molly will draw our division problem out of the bowl.

3. WE DO

ah, we got 10 divided by 2. we know that our dividend is 10 and our divisor is 2. which one tells us the total number of objects? (yes, 10 the dividend) and which tells us the groups? (yes, 2 the divisor)... Let’s close our eyes and think, ‘what could we write about? what does our class love?’

Ok, open your eyes, and raise your hand if you want to make a suggestion...(surface suggestions and tease out which one’s are good examples to divide into groups. land on dividing our pet fish into bowls)

Fish is a great example because we can put them in two different bowls. I will draw our bowls on the board while you draw on the top half of your white boards.

how many fish should we draw in each bowl if we are dividing the 10 fish into 2 groups? 5, yes, let’s draw five fish into the two bowls.

Ok, now, we need to write the story of our drawing. Turn to your partner and share a sentence about the picture we drew.

Great, raise your hand if you want to share your story. (Surface answers and give feedback. Land on a good example of a sentence story for the 10 divided by 2 story.)
Let’s all write our sentence story on our boards, “We divided our ten fish into two bowls of five fish.”

Now, I’ll give you your division problem. When you get it, take a minute to think about what you will write about.

1 minute later. When you are ready, you can go to your seat to draw your picture and write the story about your problem. Stay here if you need help thinking of a story, and I can help you.

When we are all working, I will walk around the room to see how you are doing. When you finish tell your math partner about your story. If you finish early, you can take more problems and make a few stories.

INDEPENDENT WORK

Teacher circulates, comments and highlights strong work. Differentiate with the difficulty of the division problem each student receives and allow ESL students to draw the story, rather than write a sentence if necessary.
Teacher Lacey

The Common Core Standards Initiative, Common Core State Standards for Grade 3 are divided into five knowledge domains: Operations & Algebraic Thinking, Number & Operations in Base Ten¹, Number & Operations—Fractions¹, Measurement & Data, and Geometry. The second standard for operations and algebraic thinking at grade 3 is as follows:

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- 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Q1. Can you put the standard into your own words?

Student should be able to dictate or show how to divide a number into equal shares/parts; for example, if a student is given a number of objects they should be able to share those number of items equally among friends or put them into piles where each puke has the same number of object dispersed.

Q2. What would be the goal or objective of your introductory lesson for this standard?

SWBAT show/demonstrate their understanding of division
SWBAT define division in their own words

Q3. Describe the components of your lesson in sequential order. For each component (or step) identify 2 things: teacher/instructor actions and student actions. Also, please specify any materials you would use.

Materials: manipulatives, pencil, paper

Lesson 1:
KWL (K: What students know, W: what do they want to know and)
Meaning of Division and Quotient
Read aloud
(“L” of KWL chart what they learned)

Lesson 2: Equal Share (share apples among 3 people)
# of objects ÷ # of groups = # of objects

Lesson 3: Equal Groups (Share 3 apples among each group)
# of objects ÷ # of groups = # of objects
Represent division as equal groups

Lesson 4: Word Problems of equal share and equal groups
Apply the knowledge of equal share and equal groups in the word problems.

Lesson 5: Compose their own word problems
The Common Core Standards Initiative, Common Core State Standards for Grade 3 are divided into five knowledge domains: Operations & Algebraic Thinking, Number & Operations in Base Ten¹, Number & Operations—Fractions¹, Measurement & Data, and Geometry. The second standard for operations and algebraic thinking at grade 3 is as follows:

Represent and solve problems involving multiplication and division

- 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Q1: Students will be able to interpret division with whole numbers as: a process of breaking a total amount (dividend) into a certain number (divisor) of equal groups, and counting the amount within each equal group; a process of breaking a total amount into equal groups of a certain number (divisor) and counting the number of equal groups.

Q2: My goal for an introductory lesson targeting this standard would be that students could model the breaking up of total amount into equal groups using manipulatives. Through modeling, students should also be able to write the division equation that their manipulatives represent.

Q3: Provocation: Students, in groups of 3, 4 or 5, would be given a number of manipulatives that can be divided evenly among the number of people in that group. Teacher would present a word problem that could be acted out by sharing the manipulatives equally among group members. Teacher would float from group to group, observing/listening.

Direct instruction: Students will be called on to explain their process and answers. Teacher would explain that, whether they knew it or not, the students have just done division. With use of a graphic organizer or worksheet, teacher would guide students through writing a division expression based on what they had done with manipulatives. Teacher will then pose another problem for groups to solve, this one asking students to use the same number of manipulatives to see how many groups of a certain number they can make. Teacher will then guide them through writing this new division expression, asking if they see any connections between the two expressions. Space would also be provided for students to draw this division.
**Guided practice:** Teacher will model one or two more problems, using the manipulatives to show the division – in both ways – and then writing the division expression.

**Independent practice:** Working from a book or the board, students will work independently to model, and then express, some division expressions. Teacher will walk around to support struggling students. Students looking for a challenge could write their own word problem requiring division.
APPENDIX XI: SAMPLE INTERVIEW

Teacher Michael

Sample Interview [snapshot]; Interview 2;

Joy: I'm going to go with that and just do a little bit of an explanation about the lesson agenda talk that we're going to have through this interview because it's going to be a little bit different than the previous format. I'm going to ask you to talk through the thinking, which you sort of already started doing, behind the choices and steps in your lesson plan agenda. As you talk through each section of your lesson agenda, I would love for you to try to identify how your choices reflect your belief about good math instruction. You can feel free to elaborate on important aspects of the activities that you've written down. Something may only show up as [inaudible 00:04:09] in your actual agenda but you might actually say this is really important, I want to elaborate on it, that's completely fine.

As you're reflecting, I'm going to compile a list of the ideas and conceptions about the teaching that seems to be coming up as you're talking. I'm going to try to use your words as much as possible. Then after you're done, I'm going to go through the list idea by idea to give you another chance to expand on the things that you say. I'll read that list of ideas back to you and you can talk more about any one of those or you can say I've said all I need to say about that particular idea. Cool?

Teacher Michael: Okay.

Joy: Okay I'm going to just go back to your agenda and you can talk me through step by step your choices and the reasons behind your choices, why you wanted to do a certain thing first or second or third.

Teacher Michael: You just want me to go from the first step?

Joy: Mm-hmm (affirmative) yep, from first step that would be great.

Teacher Michael: The first thing I want to do when I'm teaching this particular standard is get the kids active and on the direct to self connection, a math self connection. I always like to start a lesson like this with the kids being very personally involved in it. Before they look at symbols and numbers, they can understand quantities and groups. I think the best way that kids can see quantities and groups is to use themselves and their craft as the total quantities and themselves as the counter, it makes it much more
personal for them. If they can relate to it then they're going to remember it better. As long as the children are involved they can learn, Ben Franklin said very famously, "Tell me and I forget, teach me and I remember, involve me and I learn." That's been a big philosophy of mine always.

To get the kids up and give them a lot of freedom, I don't want to tell them the number of groups they have to get into. If there are 24 kids there are a whole lot of different ways they can group, but I want them to figure out what equal groups really means and that there are different ways of making the same quantity out of groups, out of equal groups. The only reason it would be a problem would be if a number is prime but I could use the same kind of thing to show kids what a prime number is. They can't divide into equal groups. It's using the same strategy to teach other things. You can use the same strategy for teaching multiplication and division, since they're commutative.

At any rate, they get up, they move into groups. I try to do as little guidance as possible and have them figure out the number of groups that they have to get into and how many people are in each group. If it doesn't work, you've got to change and figure out what to do. In that plan I said 10 minutes but actually it might even take longer than that. To do 2 groups like that it could take 20 or 25 minutes, it depends on the intelligence's and the leadership of the classroom, who's taking charge of the setting on the groups.

We have the kids divide into their groups and then when they're all divided into a group I write the number of groups on the board, the number of kids in each group without putting the operators in, just so they see the numbers and know what those numerals represent. Then they can reorganize into another number of groups, another different sized groups and I write the same things on the board again, showing them that the total quantity doesn't change, just the number of groups and the number of people in the groups change. Then I can put in some operators and put in multiplication operators or division operators depending on how I write it on the board. Since this particular lesson was supposed to be about division I would put the quantity, the total on the last and say, "24 kids were divided into 3 groups of 6," and then write the equation on the board.

Joy: Now tell me a little bit about the dry beans portion, is that a second portion?

Teacher Michael: If kids are working in smaller groups I can have them decide how many total they want, I can give them a cup full of beans they can use all of them or some of them but I like to give kids a lot of freedom so that
they can come up with their own equation and if they all have a different number, nobody's copying off of anybody. Then gluing stuff is fun, kids love it. I like gluing stuff down [crosstalk 00:11:02]

Joy: I don't know any kid who doesn't like glue. I have not met a kid yet who doesn't like glue.

Explanation of List of Pedagogies Employed

Joy: I have heard a lot of very interesting things as you were talking...I would love for you to elaborate on each one, if you feel like you said all that you want to say about that particular idea, that's fine. As you elaborate on it, just highlighting again what it is about that particular idea...What it is about that particular idea that makes you think it's really important or apart of good math instruction. Then I would love for you to also add where you think you picked up that idea. As I read back through them, give me a, if you want to elaborate more on the idea, how it's connected to the math instruction and then where you think you learned it, what convinced you, what experience or whatever happened that made you think this is important, this matters.

The first thing you mentioned was this idea of math to self connection, maybe not the first but one of the first that I heard several times, this math to self connection.

Teacher Michael: Yeah it's being involved in your own learning. Just like in upper grades you can write about what you think about math stuff. In lower grades you can discuss it and kids have a much easier time discussing things that they can personally relate to. If there is a math to self connection, kids are more likely to stay engaged and feel personally involved in math so they won't say, "Well I don't even have to know this because it has nothing to do with my life." If there is math to self connection then they know that yeah, it does have to do with their life.

Joy: You said a second, this is related, but that kids do best by using their own bodies, their own person. I would tell you that one is a related one because then you can tell me where [inaudible 00:17:39] related ideas came from for you.

…

Teacher Michael: Some of it I knew from my own experience in elementary school but I didn't know why it was important until I was at graduate school. I
certainly didn't understand any of the words. Math to self, that didn't occur to me on my own. I didn't realize that this way that I had learned division and multiplication on my own, I didn't realize hey this is a big pedagogy, this is something that lots of people do. This is pretty common and this is why it works. Now, because of the schooling I know why it works and I learned how best to present this kind of stuff using tangible materials.

Joy: I'm sorry, I didn't want to interrupt go ahead.

Teacher Michael: No I'm done.

Joy: Can you tell me a little bit about the elementary school experience that you had? You were saying this is something you experienced in elementary school. Can you tell me a little bit more about the actual experience?

Teacher Michael: My own learning?

Joy: Mm-hmm (affirmative) your own learning in this way, you were saying, "I sort of experienced this but didn't have a formal name for it, didn't realize it was a common pedagogy but I knew this, experienced it in elementary school."

Teacher Michael: As a student I made posters, I painted math and I didn't know that I was painting math until later. I know I used to work with a dance company called Tap Team Two. They teach math with tap dancing where they go from school to school to school, or at least they did, this was 15 years ago. They would go from school to school and do an assembly teaching math through tap dance. It's a really cool-

Joy: It is.

Teacher Michael: It's very mathematical. It just reinforced in my mind, yes there are so many different ways you can teach math.

Joy: You were at Germantown all the way through? When you say your elementary school experience you're still referring to Germantown or [crosstalk 00:21:14]

Teacher Michael: No actually I was in Columbus, Ohio until whatever grade it was. I was at a public alternative weird school. It was the 70's, these things happen. It was vertical classrooms for second grade. We used cooking to learn fractions. Gosh, I forgot about that.
Joy: I'm glad you're remembering it for this interview, I really am.

Teacher Michael: Yeah we cooked food to learn fractions, oh my gosh. That was so cool. Now I want to ... I made potato [inaudible 00:22:16] and I made a certain number of them and I remember measuring it out and I remember writing a cookbook and it was a [inaudible 00:22:22]. Gosh now I've got, I may have to write a lesson plan on cooking to learn fractions.

Joy: Cooking with fractions. Wow.

Teacher Michael: Yeah it was all the things that I was personally involved with that I remember the best. I don't remember any worksheets that I did, but I do remember cooking to learn fractions.
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