

VALIDATION OF COMPUTATIONAL MODEL OF A PIEZOELECTRIC FLOW RATE  
SENSOR

EBEN ROBERTO ESCOBEDO BAEZA  
Master's Program in Mechanical Engineering

APPROVED:

\_\_\_\_\_  
Yirong Lin, Ph.D., Chair

\_\_\_\_\_  
Joel Quintana, Ph.D.

\_\_\_\_\_  
Bill Tseng, Ph.D.

\_\_\_\_\_  
Stephen L. Crites, Jr., Ph.D.  
Dean of the Graduate School

Copyright ©

by

Eben Roberto Escobedo Baeza

2021

## **Dedication**

This work is dedicated to my family, specially to my parents who sacrificed many privileges and time to provide me an education that will help me achieve my professional goals, who taught me the importance of dedication and perseverance, who have walked with me through every step in my life, and who I will always be thankful for raising me in an environment full of love and support.

PREVIEW

VALIDATION OF COMPUTATIONAL MODEL OF A PIEZOELECTRIC FLOW RATE  
SENSOR

by

EBEN ROBERTO ESCOBEDO BAEZA

THESIS

Presented to the Faculty of the Graduate School of  
The University of Texas at El Paso  
in Partial Fulfillment  
of the Requirements  
for the Degree of

MASTER OF SCIENCE

Department of Mechanical Engineering  
THE UNIVERSITY OF TEXAS AT EL PASO

December 2021

## **Abstract**

This study validates a computational simulation of a piezoelectric cantilever beam undergoing different loads by showing a comparison of its results against theoretical calculations. The objective of the computational model is to simulate the behavior of a beam made of a piezoelectric material which can be used as a flow rate sensor due to the ability of the material to convert a mechanical load into an electrical load. After validating the model, different geometries, material properties and loads can be simulated to observe the different behaviors and bring the possibility of selecting the most convenient options according to the needs of the sensor without having to spend money and time in experimental analysis. The results show the similitude between the theoretical and computational analysis of the mechanical and electrical response.

## Table of Contents

Dedication .....	iii
Abstract .....	v
Table of Contents .....	vi
List of Tables .....	viii
List of Figures .....	ix
Chapter 1: Introduction .....	1
Piezoelectrics .....	2
Energy Harvesting Applications .....	4
Flow Sensors .....	6
Piezoelectric Material Properties .....	6
Objective .....	9
Practical Relevance .....	9
Chapter 2: Methodology .....	11
2.1 Theoretical Calculations .....	12
2.1.1 Theoretical Analysis of Rectangular Piezoelectric Cantilever Beam .....	13
2.1.2 Theoretical Analysis Trapezoidal Piezoelectric Cantilever Beam .....	18
2.1.3 Translating Velocity Profile into Drag Force Point Load .....	20
2.2 Computational Analysis .....	22
2.2.1 Grid Independence Study .....	25
2.2.2 Computational Model of Rectangular Piezoelectric Cantilever Beam .....	26
2.2.3 Computational Model of Trapezoidal Piezoelectric Cantilever Beam .....	29
2.2.4 Computational Model of Rectangular Piezoelectric Beam Using Load from Velocity Profile .....	32
Chapter 3: Results .....	34
3.1 Grid Independence study .....	34
3.2 Results for a Rectangular Piezoelectric Cantilever Beam .....	36
3.3 Results for a Trapezoidal Piezoelectric Cantilever Beam .....	40
3.4 Results of Translating Velocity Profile into Drag Force Point Load .....	43

Chapter 4: Conclusions and Future Work.....	44
References.....	46
Vita.....	48

PREVIEW

## List of Tables

Table 1: Properties of Different PZT Types .....	8
Table 2: Properties of PZT Navy Type II .....	11
Table 3: Variations of Design 2.1.1 .....	18
Table 4: Variations of Design 2.1.2 .....	19
Table 5: Compliance matrix of PZT Navy Type II.....	24
Table 6: Stiffness matrix of PZT Navy Type II.....	24
Table 7: Piezoelectric Coupling Matrix of PZT Navy Type II.....	24
Table 8: Permittivity Matrix of PZT Navy Type II .....	25
Table 9: Mesh IDs with corresponding number of divisions.....	25
Table 10: Results of Grid Independence Study Using Different Meshes .....	34
Table 11: Results of Grid Independence Study Using Different Meshes for Different Models..	35
Table 12: Results of Theoretical Calculations of Rectangular Beam .....	36
Table 13: Computational Results of Rectangular Beam.....	37
Table 14: Results of Theoretical Calculations of Trapezoidal Beam .....	40
Table 15: Computational Results of Trapezoidal Beam .....	40
Table 16: Computational Results of Drag Force on Rectangular Beam.....	43



## List of Figures

Figure 1: Diagram Showing Coordinate Axis System.....	13
Figure 2. Schematic of Beam.....	16
Figure 3: Front view of Trapezoidal Beam.....	19
Figure 4: Wind Tunnel Velocity Profiles.....	21
Figure 5: Mesh ID 12 on Design 2.1.1.6.....	26
Figure 6: Geometry of Rectangular Beam (Dimensions of Design 2.1.1.6).....	27
Figure 7: Boundary Conditions on Rectangular Beam.....	28
Figure 8: Geometry of Rectangular Beam (Dimensions of Design 2.1.2.1).....	30
Figure 9: Boundary Conditions on Trapezoidal Beam.....	31
Figure 10: Boundary Conditions Using Drag Force from Velocity Profiles.....	33
Figure 11: Comparison of theoretical and computational normal stress results of rectangular beam.....	38
Figure 12: Comparison of theoretical and computational normal strain results of rectangular beam.....	38
Figure 13: Comparison of theoretical and computational deflection results of rectangular beam.....	39
Figure 14: Comparison of theoretical and computational voltage ratio results of rectangular beam.....	39
Figure 15: Comparison of theoretical and computational normal stress results of trapezoidal beam.....	41
Figure 16: Comparison of theoretical and computational normal strain results of trapezoidal beam.....	42
Figure 17: Comparison of theoretical and computational deflection results of trapezoidal beam.....	42

Figure 18: Comparison of theoretical and computational voltage ratio results of trapezoidal beam

..... 43

PREVIEW

## Chapter 1: Introduction

Being able to monitor the flow and predict possible instabilities in systems that involve compressors can prevent adverse events. Rotating stall and surge are flow instabilities that can result in a decrease of efficiency, overtemperatures in the burner and turbine, large inlet overpressures, and decrease the blade life due to increased temperatures and stress produced by the unsteady flow. Surge is a large axisymmetric flow oscillation of the total annulus averaged flow through the entire compression system. When rotating stall occurs, cells of stalled flow rotate around the circumference of the compressor but the annulus averaged flow remains steady. These instabilities appear when the compressor operating point reaches the surge limit in the compressor map where the rotational speed, the pressure rise, and the mass flow are related [7, 19]. The implementation of a rapid response flow rate sensor can provide useful information for the characterization of the flow, and eventually prevent instabilities. The piezoelectric effect can be used to design a sensor that allows the required real-time monitoring to avoid reaching an operating point that leads to negative effects.

The word “piezo” is derived from the Greek word “piezein”, which can mean pressure, to be pressed or squeezed and thus the word “piezoelectricity” is interpreted as “electricity from pressure”. This phenomenon is present in some materials that have the ability to generate an electrical charge when they are subjected to a mechanical load. When a mechanical stress is present in a solid material, there is also a mechanical strain present which is translated as deformation. The direct piezoelectric effect is when a material is under these mechanical quantities and it produces an electrical charge of a proportional magnitude. When the inverse happens, an electric field causes the piezoelectric material to undergo strain, it is called the converse piezoelectric effect [2, 6, 8]. For the piezoelectric effect to happen, the material must possess polarity. In some cases, the

materials already have polarity because of the symmetry of the crystal class but it is not present in other crystal classes nor in isotropic bodies. Poling is the process where a strong electric field is applied to an originally isotropic polycrystalline ceramic to give it the polarity that yields the piezoelectric properties. Poling is similar to the process of magnetizing a magnet [8].

The first applications of piezoelectric materials appeared in the late 1910s. Throughout the years, the direct and converse piezoelectric effect has been used in devices for many different purposes. In some cases, the piezoelectric materials are the main factor of the device but sometimes just a component. These materials have been used in resonators, watches, crystal and ceramic filters, SAW filters, delay lines, ultrasonic transducers, underwater acoustic devices, underwater microphones and speakers, fish-finders and diagnostic acoustic devices [6].

## PIEZOELECTRICS

The piezoelectric effect involves mechanical and electrical loads which magnitudes are dependent of each other. The constitutive equations of piezoelectric materials that relates mechanical and electrical quantities are given by Eq. (1)

$$\begin{bmatrix} \epsilon \\ D \end{bmatrix} = \begin{bmatrix} s & d \\ d & k \end{bmatrix} \begin{bmatrix} \sigma \\ E \end{bmatrix} \quad \text{Eq. (1)}$$

Where  $\epsilon$  is the mechanical strain,  $D$  is the electric displacement,  $\sigma$  is mechanical stress,  $E$  is the electric field,  $s$  is the compliance matrix,  $k$  is the matrix of dielectric constants, and  $d$  is the piezoelectric constants. In this equation, the mechanical strain and the electric displacement are the outputs obtained when a mechanical stress and/or electric field are present (these are the inputs). All the variables in Eq. (1) are represented by matrices because their material properties can vary on the three directions as well as the inputs can be applied in any direction resulting in different magnitudes for the output in each direction. A more detailed explanation will be presented in