A MEASUREMENT BASED APPROACH TO DESIGNING FAULT-TOLERANT CONTROLLERS FOR MULTIVARIABLE SYSTEMS

A Dissertation
Submitted to the Graduate School
of
Tennessee State University
in
Partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy
in
Computer and Information Systems Engineering

Graduate Research Series No.,

PavanaSirisha Kallakuri
August 2016
A MEASUREMENT BASED APPROACH TO DESIGNING FAULT- TOLERANT CONTROLLERS FOR MULTIVARIABLE SYSTEMS

A Dissertation
Submitted to the Graduate School
of
Tennessee State University
in
Partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy
in
Computer and Information Systems Engineering

PavanaSirisha Kallakuri
August 2016
Copyrighted ©2016

by

PavanaSrisha Kallakuri

All rights reserved
To the Graduate School:

We are submitting a dissertation by Pavanasi Kallakuri entitled "A Measurement Based Approach to Designing Fault-Tolerant Controllers for Multivariable Systems". We recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Computer Information Systems Engineering.

Lee H. Keel  
Chairperson

Sachin Shetty  
Committee Member

Liang Hong  
Committee Member

Sivapragasam Sathananthan  
Committee Member

Accepted for the Graduate School:

Alex Sekwat  
Dean of Graduate School
ACKNOWLEDGEMENTS

It is a pleasure to thank those who made this research possible. I owe deepest
gratitude to my advisor Prof. L.H. Keel for his invaluable guidance. I would like to
thank the Center of Excellence at Tennessee State University for all the support.
Also, I wish to thank all the committee members for their time and help. Above
all, this thesis would never have been completed without the encouragement and
devotion of my family.
ABSTRACT

PAVANASIRISHA KALLAKURI. A Measurement Based Approach to Designing Fault-Tolerant Controllers for Multivariable Systems (under the direction of PROF. LEE HYUN KEEL).

This research introduces two new methodologies to design a set of controllers such that every controller in the set preserves closed-loop stability of a given multivariable plant under prescribed loop failures. The proposed approaches differ from existing techniques in two ways: First, these methods are strictly based on frequency response data of the plant that can be easily measured by experiments. No mathematical models or system identification processes are used. Second, while most control design methods find one controller, the proposed methods design a set of controllers satisfying the control objective. Two approaches are presented with examples illustrating the controller design. Integrity test results of the designed controllers under prespecified loop failures are also presented.
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS ...................................................... iv
ABSTRACT ........................................................................... v
LIST OF FIGURES ............................................................... ix

CHAPTERS

I  INTRODUCTION ................................................................. 1

II  CONCEPTUAL AND DEVELOPMENT DESIGN ................. 4
  2.1  Introduction ............................................................... 4
  2.2  Systems Engineering Management Plan(SEMP) .............. 4
  2.3  Conceptual Design ....................................................... 4
      2.3.1  Need Identification ............................................. 6
  2.4  Preliminary Design ....................................................... 7
      2.4.1  Literature Review ............................................... 7
      2.4.2  System of interest(SOI) .................................... 8
      2.4.3  Research Goal and Objectives .......................... 8
      2.4.4  System Operational Concept ............................ 9
      2.4.5  Alternative approaches for Measurement based Design
              of Set of Fault Tolerant Controllers .................. 9
      2.4.6  System Requirements .................................... 10
      2.4.7  Architecture Definition .................................. 10
  2.5  Subsystem Selection .................................................. 11
      2.5.1  Fault Tolerant Control Approach Selection ........ 11
      2.5.2  Data Based Design Approach Selection ............ 12
      2.5.3  Controller Set Design Approach Selection .......... 13
### III A MEASUREMENT BASED APPROACH TO DESIGN PID CONTROLLERS WITH SYSTEM INTEGRITY

- **3.1 Introduction** .......................................................... 14
- **3.2 Multivariable Controller Integrity Problem** .................. 14
  - **3.2.1 Prespecified Faults:** ........................................... 14
  - **3.2.2 Integrity Controller Design Approach** ..................... 15
- **3.3 Data Based PID Controller Design - Theory** ................... 17
  - **3.3.1 Preliminaries** .................................................. 17
  - **3.3.2 Computation of PID Stabilizing Set** ....................... 18
- **3.4 Implementation of Fault Tolerant PID Controller design Approach** 21
- **3.5 Design Example** ...................................................... 27
  - **3.5.1 Fault Tolerant Controller Design** .......................... 28
  - **3.5.2 Fault Tolerance Test Results** ............................... 32

### IV A MEASUREMENT BASED APPROACH TO DESIGN GENERAL FORM INTERVAL CONTROLLERS WITH SYSTEM INTEGRITY .......................... 35

- **4.1 Introduction** .......................................................... 35
- **4.2 Multilinear Interval Controller Design** ......................... 35
- **4.3 Implementation of Measurement Based approach to Design Interval Controllers** ........................................... 36
- **4.4 3-input 3-output Design Example** ............................... 39
  - **4.4.1 Fault Tolerant Controller Design** ......................... 39
  - **4.4.2 Fault Tolerance Test Results** ............................... 43
- **4.5 Second Order Controller Design Example** ....................... 43
  - **4.5.1 Fault Tolerant Controller Design** ......................... 45
  - **4.5.2 Fault Tolerance Test Results** ............................... 47
- **4.6 Controller Design for Performance Example** .................... 47
  - **4.6.1 Fault Tolerant Controller Design with performance specifications** ........................................... 50
LIST OF FIGURES

1 Systems engineering management plan ........................................ 5
2 System Operational Concept ...................................................... 9
3 Architecture Diagram of SOI ..................................................... 10
4 Block diagram showing feasible Fault Tolerant Control Method alternatives ........................................ 11
5 Block diagram showing Data Based Design Method alternatives ........................................ 12
6 A TITO unity feedback system .................................................... 15
7 A TITO unity feedback System ................................................... 21
8 A TITO feedback system (component-wise view) ............................ 22
9 Equivalent Channel Representation of TITO system ......................... 23
10 SISO closed-loop system with channel $C_1$ ................................... 25
11 SISO channel “equivalent” to Figure 10 ........................................ 25
12 Frequency response data for $G$ (Example 3.5) .............................. 28
13 PID gains for TITO controller (Example 3.5) ................................ 31
14 PID gains for TITO system with closed-loop system integrity (Example 3.5) ........................................ 33
15 Unit step responses in Healthy and Faulty conditions (Example 3.5) 34
16 Open loop frequency response of the $3 \times 3$ system (Example 4.4) .... 40
17 Sets of controllers with system integrity $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2 \times \mathcal{K}_3$ (Example 4.4) ........................................ 44
18 Unit step response of the Healthy and Faulty systems (Example 4.4) 45
19 Frequency response data for $G$ (Example 4.5) .............................. 46
20 Unit step responses in Healthy and Faulty conditions (Example 4.5) 48
21 Frequency response data for $G$ (Example 4.6) .............................. 49
22 Sets of controllers with system integrity $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2$ (Example 4.6) 52
<table>
<thead>
<tr>
<th></th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>Unit step responses With Stabilizing Controller (Example 4.6)</td>
<td>53</td>
</tr>
<tr>
<td>24</td>
<td>Unit step responses With Stabilizing Controller with performance (Ex-</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>ample 4.6)</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Frequency response data for Qball height</td>
<td>55</td>
</tr>
<tr>
<td>26</td>
<td>fault tolerant PID gain region</td>
<td>56</td>
</tr>
<tr>
<td>27</td>
<td>No Failure</td>
<td>57</td>
</tr>
<tr>
<td>28</td>
<td>Actuation Loss during Entire Operation</td>
<td>58</td>
</tr>
<tr>
<td>29</td>
<td>Actuation Loss during Liftoff</td>
<td>59</td>
</tr>
<tr>
<td>30</td>
<td>Actuation Loss in Steady State</td>
<td>59</td>
</tr>
<tr>
<td>31</td>
<td>0-40% Actuation loss during entire operation</td>
<td>60</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

Control design with integrity is a branch of passive fault-tolerant control. Its objective is to maintain stability and acceptable performance of the system despite predetermined component failures, without modifying or reconfiguring controllers. The problem is particularly important for multivariable systems which should be structurally designed if possible to stabilize the closed-loop system even if some loops fail because of faults in sensors or actuators. Most of the passive FTC design techniques are model based, and rely on the mathematical model of the system. Also, these methods will design a single controller.

Mathematical models of the systems are obtained from the laws of physics or the most common way is through system identification process. They require information about the order of the plant. In theory, a System ID procedure should be able to determine the unknown rational transfer function. But, it can fail even if exact (or perfect) data is available. This is especially true when the order of the plant is high. Measurement based controller design approach serves as an attractive complement to model based design. These techniques have important significance in real world control engineering, where models are often unavailable. Controllers are designed directly from measurements made from the plant.

Typical controller design processes will results in a single controller. In this
case, the entire design process has to be repeated to accommodate new design requirements. Also, in complex systems there is a possibility that the controller is not implemented exactly so that small perturbations of the controller coefficients may destabilize the closed-loop control system. These two issues can be resolved by designing a set of controllers instead of a single controller. So that robust controller can be selected from the set, also it will be easier to accommodate new design requirements with out repeating the entire design process. Robustness to plant parameter perturbations can be achieved by adding performance specifications to design. To the best of our knowledge, none of the existing control design techniques are simultaneously providing data based design approach with fault tolerance and robustness.

In this research, we introduce two measurement based approaches to control design for integrity. First, we propose to design diagonal MIMO PID controllers. This approach is motivated by the result that effectively and completely determines the set of stabilizing PID controllers for a given system described by its frequency response data without mathematical models [18]. Our solution utilizes the classical Individual Channel Design (ICD) approach [9]. In this framework, the multivariable control design problem is reduced to the design of a SISO control for each channel [9, 10]. Then the integrity controllers can be determined by finding the intersection of controller parameters for each failure case considered.

Next, we propose a measurement based method to design diagonal MIMO interval controllers of an arbitrary order. The Mapping Theorem adapted to a measurement based approach is used to determine interval sets of controllers for stabilizing the plant under each failure condition. Controllers with system integrity can be obtained by finding the intersection of controller parameters designed for each
case. The advantage of such a box-like set is that it allows an engineer to independently adjust design parameters to tune controllers. Performance specifications can also be included in the controller design with both methods introduced. Integrity controller design for closed-loop stability and specified performance is illustrated in the examples.

The dissertation is organized as follows, Chapter II gives the details of the conceptual and development design of the controller design method implementation process. Chapter III presents the proposed approach to design Fault Tolerant PID gain region, with example controller design and integrity test results. Chapter IV presents the proposed approach to design Fault Tolerant Interval parameter region for general controller structure, with example controller design and integrity test results. Chapter V details the characterization of fault tolerant PID controllers for quad rotor drone height control. Chapter VI gives the conclusions.
CHAPTER II

CONCEPTUAL AND DEVELOPMENT DESIGN

2.1 Introduction

A systems engineering process is a process for applying systems engineering techniques to the development of systems. This is related to the stages in a system life cycle. The system life cycle is an examination of the system that addresses the phases of its existence to include: system design and development, production and/or construction, system utilization and phase-out and disposal [1]. Some of the relevant aspects of the system engineering design approach are used in our research. The system process that is considered for this research includes three main phases: conceptual design, preliminary system design, detailed system design, system implementation and verification.

2.2 Systems Engineering Management Plan(SEMP)

The Systems engineering management plan(SEMP) provides the activities, milestones, organization and resource requirements necessary for the project completion [1]. Figure.?? details the three year management plan followed for this research.

2.3 Conceptual Design

The first phase in the systems engineering process is the conceptual design stage. It includes the need identification process where the need for the proposed system is identified.
<table>
<thead>
<tr>
<th>Systems Engineering Process Activities</th>
<th>Need Analysis</th>
<th>Literature Review</th>
<th>SOI Identification</th>
<th>System Operational Concept</th>
<th>System Requirements</th>
<th>Architecture Definition</th>
<th>Subsystem Definition</th>
<th>Subsystem Selection</th>
<th>Implementation</th>
<th>Verification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual Stage</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development Stage (Preliminary Design)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development Stage (Detailed Design)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Production Stage</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Systems engineering management palm
2.3.1 Need Identification

In the past few decades, advances in electronics and communications has completely changed the structure of engineering systems. Technical areas like system automation, unmanned operation and control, networked and distributed systems etc are gradually replacing the previous technologies. This revolution helped in realizing more sophisticated operation and control in every field. But, the systems are becoming very complex as the number of devices are increased to adapt new changes. Control systems are effected by the increased complexity in many ways. First, more vulnerability to faults - failure in a single component may lead to catastrophic effects. If not dangerous certain failures might degrade the overall performance. It is essential to design fault tolerant control systems to retain the performance even in the presence of arbitrary failures.

Multivariable/Multi-input multi-output (MIMO) systems can possess the capability of maintaining the closed-loop stability even if one or more feedback loops fail. In other words, carefully designed controllers can be fault-tolerant against loop failures for multivariable systems. In addition to actual loop failures, it is noted that typical sensor or actuator failure conditions (i.e., when the components fail completely) can also be viewed as loop failures. Control systems designed to maintain closed loop stability in the presence of failures are said to have integrity.

Controller design techniques typically rely on a mathematical model of the system. Controller design for closed-loop system integrity is no exception. Because of the increased complexity in multivariable systems finding the exact model of the system becomes very tedious. Controller design methods that does not require mathematical model are an attractive alternative. Recently, measurement based controller
design approaches were introduced as an attractive complement to model based design [18]. In these approaches, controllers are designed directly from measurements made on the plant. These techniques have important significance in real world control engineering, where models are often unavailable or difficult to obtain. Typical controller design processes will result in a single controller. In this case, the entire design process has to be repeated to accommodate new design requirements. Also, in complex systems there is enough possibility that the controller is not implemented exactly so that small perturbations of the controller coefficients may destabilize the closed-loop control system. Designing a set of controllers instead of a single controller is required, so that robust controller can be selected from the set, also it will be easier to accommodate new design requirements without repeating the entire design process. Hence, there is a need to design controllers that provide fault tolerance, requires only measured data for design, and robust for parameter perturbations.

2.4 Preliminary Design

The preliminary design stage starts with need analysis: the steps involved in the need analysis are literature review, defining System Of Interest (SOI), identifying system goals and objectives, defining system operational concept. And then it defines system requirements and system architecture.

2.4.1 Literature Review

In [3], a one plant-two controller configuration was considered. A stable factorization approach was used to design controllers with integrity [4]. This work led to the design of a software package implemented in Mathematica [5]. In [6], a state feedback control law which retains stability against arbitrary actuator failures and parameter perturbations is derived. A method to design a decentralized $H_{\infty}$
output feedback controller to maintain stability and a certain performance of the system under one or more local controller failures was reported in [7].

Generalized nyquist stability criterion gives the common stabilizing gain range for multivariable systems [11]. In [12] procedure to find the independent gain ranges for P and PI controller parameters is reported. A simple method to compute the exact bounds on unidirectional perturbations in uncertain generalized state space systems is provided in [13]. In [14] quasi-LMI approach to compute stabilizing parameter ranges of multi-loop PID controllers is established.

Based on the literature review, there is no measurement based controller design method available which provides fault tolerance and results in a set of controllers instead of a single controller.

2.4.2 System of interest (SOI)

Fault Tolerant Controller design approach for multivariable systems is our system of interest. The approach is measurement based which results in a set of controllers instead of a single controller.

2.4.3 Research Goal and Objectives

This research aims at developing a measurement based controller design methodology to determine sets of controllers that stabilize a linear time-invariant multi variable system under all pre-specified failures. The main objectives are,

- Developing measurement based MIMO controller design method.
- Characterizing a controller set instead of a single controller.
- Identifying characteristics of the system in healthy and faulty situations.
• Developing design methodology for designing diagonal MIMO controller sets for the healthy plant as well as for faulty plants.

2.4.4 System Operational Concept

The controller design approach developed in this research shall provide controller parameter values for MIMO controllers to stabilize any coupled MIMO plant in presence of faults. The input to the design approach is measured data of the MIMO plant. Figure 2 shows the block diagram of the system operational concept.

![System Operational Concept Diagram]

Figure 2. System Operational Concept

2.4.5 Alternative approaches for Measurement based Design of Set of Fault Tolerant Controllers

None of the existing passive fault tolerant controller design methodologies are measurement based and provide a set of controllers at the same time. Our approach
is novel to uniquely fulfill the requirements. So there are no alternative approaches to compare with.

2.4.6 System Requirements

- The design approach shall provide diagonal controllers to stabilize coupled MIMO systems.

- The design approach shall only use measured data as the input.

- The design approach shall result in a set of controllers.

- The resulting controllers shall provide fault tolerance against loop failures until at least one element of the diagonal MIMO controller has non zero output.

2.4.7 Architecture Definition

The controller design method implementation process is broken down into three main subsystems - including Fault Tolerance, designing a set of controllers, making it a data based approach. Figure 3. shows the functional breakdown of the implementation process.

![Architecture Diagram of SOI](image-url)
2.5 Subsystem Selection

This section gives the details of the identified subsystems, and the requirements of each subsystem. Also, it will discuss possible alternative approaches if there are any and the selected approach for each subsystem.

2.5.1 Fault Tolerant Control Approach Selection

The selection criterion for the Fault Tolerant Control (FTC) method is effectiveness of the method in presence of loop failures. Figure 4 shows the available alternatives for the Fault Tolerant approaches.

![Block diagram showing feasible Fault Tolerant Control Method alternatives](image)

**Figure 4.** Block diagram showing feasible Fault Tolerant Control Method alternatives

2.5.1.1 Active FTC Methods:

- The control law changes when fault occurs.

- Requires additional Fault detection, and controller reconfiguration sub systems.

- Examples are Switching between Controllers, On-line reconfiguration of Controllers etc.

2.5.1.2 Passive FTC Methods:

- Same control law is used in healthy as well as faulty situations.
- Maintain stability despite pre-determined faults.

- No additional sub systems are required for fault detection and isolation.

Since possible loop failures and system characteristics under these failures can be pre-determined we do not need additional fault detection and controller re-configuration. Hence, passive FTC Methods are more effective and relevant to the present problem.

### 2.5.2 Data Based Design Approach Selection

The selection criterion for the data based design approach is that, controller set design must be possible. Figure 5 shows the available alternatives for the measurement based design approaches.

![Data Based Design Approach Alternatives]

Figure 5. Block diagram showing Data Based Design Method alternatives

Off all the three available alternatives, frequency data based approach is the only one which is suitable for the controller set design. Hence, frequency data based
approach is selected.

2.5.3 Controller Set Design Approach Selection

The two controller design methodologies that are available right now in the control theory and will result in a set of controllers instead of a single controller are directly used. There are no other alternatives to consider in the selection process.

2.5.3.1 Data Based PID Controller Design Method[18]:

This method characterizes entire stabilizing PID gains.

- Requires only, Frequency response data, and Number of unstable poles of the plant.

- Does not require information about order of the plant.

2.5.3.2 Interval Controller design:

The controllers parameter intervals can be designed using the Mapping Theorem, and the Bounded Phase Condition in Multi linear Interval Systems. The interval controller structure, which is described as a axis-parallel box in the controller parameter space, is used here. This design method can be used to any general form controller structure.

The first method i.e., Data Based PID Controller Design Method[18] is already frequency data based controller design method. So it can be directly used for measurement based design. But, the second method i.e., Interval Controller design is a mathematical model based approach. Hence, we have to first make it a frequency data based approach before using this method in the implementation.
CHAPTER III

A MEASUREMENT BASED APPROACH TO DESIGN PID CONTROLLERS WITH SYSTEM INTEGRITY

3.1 Introduction

This chapter presents implementation of proposed measurement based technique to determine a set of stabilizing PID controllers with closed-loop system integrity. A design example with fault tolerant test results is also presented which verifies the design approach.

3.2 Multivariable Controller Integrity Problem

This section gives the details of the prespecified faults considered in this research and presents the proposed approach to integrity controller design.

3.2.1 Prespecified Faults:

The scope of this research is to provide fault tolerance against complete loop failures in coupled multivariable systems. The fault is represented by making output of the controller component corresponding to failed loop with zero value. At any point of time we consider at least one controller in the distributed controller structure is completely functioning.
3.2.2 Integrity Controller Design Approach

Consider a stable, proper TITO plant $G$ and TITO diagonal controller $K$ given by the transfer functions,

$$
G = \begin{bmatrix}
    n_{11} & n_{12} \\
    d_{11} & d_{12}
\end{bmatrix}, \quad K = \begin{bmatrix}
    \frac{N_{k1}}{D_{k1}} & 0 \\
    0 & \frac{N_{k2}}{D_{k2}}
\end{bmatrix}
$$

(1)

![Diagram of TITO unity feedback system](image)

**Figure 6.** A TITO unity feedback system

The problem of determining stabilizing controllers with controller integrity for a given TITO system in Fig. 6 can be formulated as follows. We make the standing assumption that there is no pole/zero cancellation, and thus closed-loop stability is equivalent to roots of the following characteristic polynomial being located in the open left half plane (LHP):

$$
\delta(s) = d_{11}d_{12}d_{21}d_{22}D_{k1}D_{k2} + n_{11}d_{12}d_{21}d_{22}N_{k1}D_{k2}
$$

$$
+ d_{11}d_{12}d_{21}n_{22}N_{k2}D_{k1} + n_{11}d_{12}d_{21}n_{22}N_{k1}N_{k2}
$$

$$
- d_{11}n_{12}n_{21}d_{22}N_{k1}N_{k2}.
$$

(2)

When $K_1$ or $K_2$ fails, the characteristic polynomial will be

$$
n_{22}N_{k2} + d_{22}D_{k2}, \quad n_{11}N_{k1} + d_{11}D_{k1}.
$$

(3)
The problem of designing integrity controllers is now one of determining sets of controllers $K_1, K_2$ such that all roots of the above three polynomials are in the open LHP. The controller integrity problem is equivalent to the requirement of closed loop stability under loop failures when the controller is diagonal. A high-level procedure to solve the controller integrity problem for TITO systems is outlined below:

**Integrity of TITO systems:** A controller set with integrity $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2$ can be constructed as follows:

1. Find sets $\hat{\mathcal{K}}_i$ that stabilize the unity feedback system around $g_{ki}$ for $i = 1, 2$.
   Let $\mathcal{K} := \hat{\mathcal{K}}_1 \times \hat{\mathcal{K}}_2$.

2. Find sets $\hat{\mathcal{K}} = \hat{\mathcal{K}}_1 \times \hat{\mathcal{K}}_2$ that stabilize the TITO system in Fig. 6.

3. Determine $\mathcal{K} = \hat{\mathcal{K}} \cap \hat{\mathcal{K}}$.

**Integrity of 3 × 3 systems:** A controller set with integrity $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2 \times \mathcal{K}_3$ can be constructed as follows:

1. Find a set $\hat{\mathcal{K}}_i$ that stabilizes the unity feedback system around $g_{ii}$ for $i = 1, 2, 3$.

2. Find a set $\hat{\mathcal{K}}_{jk} = \hat{\mathcal{K}}_j \times \hat{\mathcal{K}}_k$ that stabilizes the TITO unity feedback systems with the plants $G_{jk}$ where

   $$G_{jk} = \begin{bmatrix} g_{jj} & g_{jk} \\ g_{kj} & g_{kk} \end{bmatrix}, \quad K_{jk} = \begin{bmatrix} K_j \\ K_k \end{bmatrix}$$

   (4)

   and $(j, k) = \{(1, 2), (2, 3), (1, 3)\}$.

3. Find a set $\hat{\mathcal{K}} = \hat{\mathcal{K}}_1 \times \hat{\mathcal{K}}_2 \times \hat{\mathcal{K}}_3$ which stabilizes the $3 \times 3$ unity feedback system...
with $G$ where

$$G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \quad K = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix}. \quad (5)$$

(4) Determine $\mathcal{K} = \tilde{\mathcal{K}} \cap \hat{\mathcal{K}} \cap \check{\mathcal{K}}$.

Similarly, we can extend the above procedure to $m$-input and $m$-output systems.

For clarity and notational convenience, we focus on two-input two-output (TITO) systems. The results to be presented here however generalize to arbitrary MIMO systems in a straightforward manner.

### 3.3 Data Based PID Controller Design - Theory

This section briefly details the theory involved in the model free approach to PID controller design for SISO systems reported by Keel and Bhattacharya[8]. Our approach rely on this theory to design stabilizing PID gain regions.

#### 3.3.1 Preliminaries

Let $P$ denote a LTI plant and $P(s)$ is its rational transfer function with $z^+, p^+$ ($z^-, p^-$) denoting the numbers of RHP (LHP) zeros and poles, $n(m)$ the denominator (numerator) degrees. Let the relative degree be denoted $r_P$:

$$r_P := n - m$$

We define the signature of $P$ is:

$$\sigma(P) = (z^- - z^+) - (p^- - p^+). \quad (6)$$
Lemma 3.3.1.

\[ r_P = -\frac{1}{20} \frac{dP_{db}(\omega)}{d(\log_{10} \omega)} \bigg|_{\omega \rightarrow \infty} \]  
\[ \sigma(P) = \frac{2}{\pi} \Delta_\infty \angle P(j\omega) \]

where

\[ P_{db}(\omega) := 20 \log_{10} |P(j\omega)|. \]

Assuming that \( P(s) \) has no \( j\omega \) axis poles and zeros, we can also write

\[ \sigma(P) = -r_P - 2 \left( z^+ - p^+ \right). \]

Therefore \( z^+ - p^+ \) can be determined from the Bode plot of \( P \). In particular if \( P(s) \) is stable the Bode plot can often be obtained experimentally by measuring the frequency response of the system. Then the above relations with \( p^+ = 0 \) determine \( z^+ \) from the Bode plot data.

3.3.2 Computation of PID Stabilizing Set

Assume that the only information available to the designer is:

1. Knowledge of the frequency response magnitude and phase, equivalently, \( P(j\omega) \), \( \omega \in [0, \infty) \) if the plant is stable.

2. Knowledge of a known stabilizing controller and the corresponding closed loop frequency response \( G(j\omega) \).

Write

\[ P(j\omega) = |P(j\omega)| e^{j\phi(\omega)} = P_r(\omega) + jP_i(\omega) \]

where \( |P(j\omega)| \) denotes the magnitude and \( \phi(\omega) \) the phase of the plant, at the frequency \( \omega \).
Let the PID controller be of the form
\[ C(s) = \frac{K_i + K_p s + K_d s^2}{s(1 + sT)}, \quad \text{for } T > 0 \] (11)
where \( T \) is assumed to be fixed and small.

**Lemma 3.3.2.** Let
\[ F(s) := s(1 + sT) + (K_i + K_p s + K_d s^2)P(s). \]
and
\[ \bar{F}(s) = F(s)P(-s). \]
Then closed-loop stability is equivalent to
\[ \sigma (\bar{F}(s)) = n - m + 2z^+ + 2. \] (12)

The complete set of stabilizing PID gains can be computed by the following procedure.

1. Determine the relative degree of the plant \( r_P = n - m \) from the high frequency slope of the Bode magnitude plot of \( P(j\omega) \).
2. Let \( \Delta^\infty_0[\phi(\omega)] \) denote the net change of phase of \( P(j\omega) \) for \( \omega \in [0, \infty) \).
   Determine \( z^+ \) from
   \[ \Delta^\infty_0[\phi(\omega)] = \left[ (n - m) + 2z^+ \right] \frac{\pi}{2} \]
   which follows from (9) with \( p^+ = 0 \).
3. Fix \( K_p = K_p^* \), solve (13) and let \( \omega_1 < \omega_2 < \cdots < \omega_{l-1} \) denote the distinct frequencies of odd multiplicities which are solutions of (13).
   \[
   K_p^* = - \frac{P_r(\omega) + \omega TP_1(\omega)}{|P(j\omega)|^2} 
   = - \frac{\cos \phi(\omega) + \omega T \sin \phi(\omega)}{|P(j\omega)|} =: g(\omega)
   
   (13)
4. Set $\omega_0 = 0$, $\omega_l = \infty$ and

$$j = \text{sgn} \hat{F}_i (-\infty^-, K_p^*).$$

Determine all strings of integers $i_t \in \{+1, -1\}$ such that:

- For $n - m$ even,

$$[i_0 - 2i_1 + \cdots + (-1)^{i-1}2i_{l-1} + (-1)^{i-1}i_t] \cdot (-1)^{l-l}j = n - m + 2z^+ + 2. \quad (14)$$

- For $n - m$ odd,

$$[i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{i-1}2i_{l-1}] (-1)^{l-l}j$$

$$= n - m + 2z^+ + 2. \quad (15)$$

5. For the fixed $K_p = K_p^*$ chosen in Step 1, solve for the stabilizing $(K_i, K_d)$ values from

$$\left[ \left( K_i - K_d \omega^2 \right) + \frac{\omega_i P_i(\omega_i) - \omega_i^2 TP_i(\omega_i)}{|P(j\omega_i)|^2} \right] i_t > 0,$$

or

$$\left[ \left( K_i - K_d \omega^2 \right) + \frac{\omega_i \sin\phi(\omega_i) - \omega_i^2 T \cos\phi(\omega_i)}{|P(j\omega_i)|} \right] i_t > 0,$$

for $t = 0, 1, \cdots, l$.

6. Repeat the previous three steps by updating $K_p$ over prescribed ranges.

The ranges over which $K_p$ must be swept is determined from the requirements that (14) or (15) is satisfied for at least one string of integers.
3.4 Implementation of Fault Tolerant PID Controller design Approach

Consider a stable, proper TITO plant $G$ and TITO diagonal controller $K$ given by the transfer functions (the argument $s$ is suppressed):

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}, \quad K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$$ \hspace{1cm} (16)

**Assumption 1.** The plant $G$ to be controlled is internally stable.

**Remark 3.4.1.** We assume the plant is stable. Otherwise, under a loop failure (i.e., $K_i = 0$ for some $i$), components in $G$ that do not appear in the closed-loop transfer function will exhibit unstable behavior for nonzero initial conditions. In fact, closed-loop system integrity under loop failures with a unstable multivariable plant is not attainable for this reason.

The unity feedback configuration of plant $G$ and diagonal controller $K$ is shown in Figure 7 and its component-wise view in Figure 8.

![Diagram](image)

**Figure 7.** A TITO unity feedback System

It is known that the TITO system with diagonal controller in Figure 8 is equivalent to the channel representation shown in Figure 9. The design method
utilizing such representation is called *Individual Channel Design* [9]. The channels are given by

\[ C_i = K_i g_{ii} (1 - \gamma h_j), \quad \text{for } i, j = 1, 2. \]  

(17)

where

\[ \gamma = \frac{g_{12} g_{21}}{g_{11} g_{22}} \]  

(18)

\[ h_i = \frac{k_i g_{ii}}{1 + k_i g_{ii}}, \quad \text{for } i = 1, 2. \]  

(19)

The loop interactions are preserved in the individual channels as seen in Fig. 9. The channels \( C_1 \) and \( C_2 \) together are structurally equivalent to the TITO system.

The closed loop system is given by

\[ y_i(s) = R_i(s)r_i(s) + P_i(s)Q_i(s)r_j(s), \quad \text{for } i \neq j \]  

(20)
Figure 9. Equivalent Channel Representation of TITO system

where

\[ R_i(s) = \frac{C_i(s)}{1 + C_i(s)}, \quad (21) \]

\[ P_i(s) = \frac{1}{1 + C_i(s)}, \quad (22) \]

\[ Q_i(s) = \frac{g_{jj} h_j}{g_{jj}}. \quad (23) \]

To proceed, we make the following standing assumption throughout the paper. Let,

\[ G(s) = D_j(s)^{-1} N_g(s) \]

\[ K(s) = N_h(s) D_h(s)^{-1} \]
Assumption 2. det \([D_g(s)D_k(s) + N_g(s)N_k(s)]\) and det \([D_g(s)D_k(s)]\) are coprime.

Under the condition that \(G(s)\) and \(K(s)\) are minimal, the assumption above holds in general when there is no cancellation between plant and controller transfer functions. The following lemma is a direct consequence of the assumption.

Lemma 3.4.1. The closed-loop characteristic polynomial of the system in Figure 7 is the numerator polynomial of \(\det[I + G(s)K(s)]\).

The proof is obvious and omitted here.

The following result shows that stabilization of the TITO system in Figure 7 is equivalent to stabilization of a single input single output system described as “channel \(C_1\)” or “channel \(C_2\)”.

Lemma 3.4.2. Write \(R_i(s), P_i(s), Q_i(s)\) as ratios of two polynomials in \(s\). Then the denominator polynomials of \(R_i(s), P_i(s)Q_i(s), i = 1, 2\), and the numerator polynomial of \(\det(I + GK)\) are identical. Consequently, the given TITO system is stable if and only if \(K\) ensures stability of the transfer functions \(R_i(s), i = 1 \text{ or } 2\).

The proof is a simple calculation and is omitted here.

Remark 3.4.2. Note that the denominators of \(R_1(s)\) and \(R_2(s)\) are identical and also the denominators of each of the closed-loop transfer functions in Figure 9 are the same. Consequently, the stabilization of a TITO system in Figure 7 is effectively reduced to that of a SISO system in Figure 10 with two controllers \(K_1\) and \(K_2\). Note that \(h_2\) is a function of \(K_2\).

Our solution to the TITO integrity problem relies on the result [8] in which for a given frequency response data of a plant, the complete set of stabilizing PID controllers can be determined. The fact that the complete set is obtained is the key
feature which allows us to obtain integrity by intersecting over stabilizing sets. The data based PID controller design method introduced in [8] requires only frequency response measurements and knowledge of the number of unstable poles of the plant.

The problem of determining stabilizing controllers with controller integrity for a given TITO system in Figure 8 can be formulated as discussed in section 3.2.

First, the unity feedback loop shown in Figure 10 is redrawn considering the definition of $h_2$ that is the inner unity feedback loop around $g_{22}$ with controller $K_2$. The block diagram shown in Figure 11 is equivalent to Figure 10.

**Theorem 3.4.1.** Let $\tilde{K_1}$ and $\tilde{K_2}$ be the sets of controllers stabilizing the feedback systems around $g_{11}$ and $g_{22}$, respectively. Let $\mathcal{K}_1(K_2)$ be the set stabilizing the feedback system around the channel $C_1$ (i.e., $g_{11}(1 - \gamma h_2)$) with a specific choice of $K_2 \in \tilde{K_2}$. Then the TITO closed-loop system with a controller $K_1 \in \mathcal{K}_1(K_2) \cap \tilde{K_1}$ and $K_2$ is stable with integrity.

**Proof. Case 1:** Both $K_1$ and $K_2$ are ON (no failure)
The TITO feedback system is stable if the numerator of $\det[I + GK]$ is Hurwitz stable. Here, we have

$$\det[I + GK] = (1 + g_{11}K_1) (1 + g_{22}K_2) - g_{12}g_{21}K_1K_2.$$  \hfill (24)

Note that $K_1$ and $K_2$ are chosen from the set $\mathcal{K}_1(K_1)$ that stabilizes the feedback system $R_1$. From Lemma 3.4.2, the stability of $R_1$ is equivalent to that of the TITO system.

Case 2: $K_1$ fails ($K_1 = 0$)

$$\det[I + GK] = 1 + g_{22}K_2.$$  \hfill (25)

Since $K_2$ is chosen from $\bar{\mathcal{K}}_2$ every element of which stabilizes the feedback system around $g_{22}$, the TITO system remains stable.

Case 3: $K_2$ fails ($K_2 = 0$)

$$\det[I + GK] = 1 + g_{11}K_1.$$  \hfill (26)

Since $K_1$ is chosen from $\bar{\mathcal{K}}_1$ which stabilizes the feedback system around $g_{11}$, the TITO system remains stable. Hence, we conclude that the controller ensures integrity of the TITO feedback system. \hfill $\square$

**Remark 3.4.3.** The procedure can be repeated by selecting $K_1 \in \bar{\mathcal{K}}_1$ first, determining $\mathcal{K}_2(K_1)$, and then selecting $K_2$ from the set $\mathcal{K}_2(K_1) \cap \bar{\mathcal{K}}_2$.

**A Design Procedure:**

1. Determine the stabilizing PID gain set $\bar{\mathcal{K}}_1$ for the SISO system $g_{11}$ in unity feedback loop.
2. Determine the stabilizing PID gain set \( \bar{K}_2 \) for the SISO system \( g_{22} \) in unity feed back loop.

3. Select controllers

\[
\{ K_2^1, K_2^2, \ldots, K_2^n \} \in \bar{K}_2. \tag{27}
\]

4. Determine the stabilizing PID gain sets

\[
\{ K_1(K_2^1), K_1(K_2^2), K_1(K_2^3), \ldots, K_1(K_2^n) \} \tag{28}
\]

as described in Step 1.

5. Find the intersection

\[
K_1^I = \bar{K}_1 \cap K_1(K_2^1) \cap K_1(K_2^2) \cap K_1(K_2^3) \cdots \cap K_1(K_2^n) \tag{29}
\]

and let

\[
K_2^I = \{ K_2^1, K_2^2, K_2^3, \ldots, K_2^n \}. \tag{30}
\]

6. Any controller \( (K_1, K_2) \in K_1^I \times K_2^I \) will ensure closed-loop system integrity of the given TITO system.

3.5 Design Example

Consider a TITO stable plant \( G \) defined as,

\[
G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}
\]

and the controller \( K \) with \( T > 0 \).
\[ K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \frac{K_{i1} + K_{p1} s + K_{d1} s^2}{s(1 + sT)} \begin{bmatrix} K_{i2} + K_{p2} s + K_{d2} s^2 \\ s(1 + sT) \end{bmatrix}. \]

The open-loop frequency response data of the plant \( G \) is shown in Figure 12.

![Bode plots](image)

Figure 12. Frequency response data for \( G \) (Example 3.5)

### 3.5.1 Fault Tolerant Controller Design

**Characterization of stabilizing controller set \( \tilde{K}_2 \):**

The Bode data of \( g_{22} \) is used to determine the PID gains of \( \tilde{K}_2 \) by using the data based design method in [8]. The following is a brief description of the procedure, the
details can be found in [8].

Let

\[ g_{22}(j\omega) = g_{22e}(\omega) + jg_{22i}(\omega) \]  

(31)

From the Bode plot in Figure 12, the estimated high frequency slope is approximately

\(-20 \text{ db/decade} \) and thus \( r_p = n - m = 1 \). From the phase plot in Figure 12, it is

shown that, the total change of phase is \(-\frac{\pi}{2}\). Also \( p^+ = 0 \). Thus, we have

\[ z^+ = \frac{-r_p + p^+ - \frac{2}{\pi} \Delta^\infty_{\omega} \angle g_{22e}(j\omega)}{2} = 0. \]  

(32)

The stability of the closed-loop system consisting of \( g_{22} \) with \( K_2 \) requires the

signature condition

\[ \sigma(\tilde{F}(s)) = (n - m) + 2z^+ + 2 = 3. \]  

(33)

Since \( n - m \) is odd,

\[ [i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{i-1}2i_{i-1}](-1)^{i-1}j = 3. \]  

(34)

where

\[ j := \text{sgn} \left[ \tilde{F}_i(\infty, K_2^*) \right] = -\text{sgn} \left[ \lim_{\omega \to \infty} g_{22}(\omega) \right] = -1. \]  

(35)

It is clear from the signature value that at least one frequency term is required
to satisfy (34). Therefore,

\[ i_0 - 2i_1 = 3 \]  

(36)

The only feasible string of signs satisfying (36) is

\[ \{i_0, i_1\} = \{1, -1\}. \]  

(37)
The next step is to calculate the feasible range of $K_{p2}$. The feasible $K_{p2}$ values are those values at which (38) has at least one positive frequency that satisfies:

$$K_{p2} = -\frac{g_{22r}(\omega) + \omega T g_{22i}(\omega)}{|g_{22}|^2} = : g_2(\omega)$$  \hspace{1cm} (38)

The feasible range of $K_{p2}$ is such that it intersects the graph of $g_2(\omega)$ at least once. Thus, the feasible range is $K_{p2} \geq -2.980$ with $T = 0.001$.

We now fix the value of $K_{p2}$ to any feasible proportional gain and compute the set of $\omega$’s that satisfies (38). Then we solve for the stabilizing $(K_{i2}, K_{d2})$ values using the set of inequalities,

$$\left[ (K_{i2} - K_{d2} \omega_i^2) + \frac{\omega_i g_{22i}(\omega_i) - \omega_i^2 T g_{22r}(\omega_i)}{|g_{22}(j\omega_i)|^2} \right]_i > 0.$$  \hspace{1cm} (39)

The calculated stabilizing controller set $\tilde{K}_2$ is shown in Figure 13(b).

Characterization of stabilizing controller set $\tilde{K}_1$:

Similarly, the Bode data of $g_{11}$ is used to determine the PID gains of $\tilde{K}_1$. The calculated stabilizing controller set $\tilde{K}_1$ is depicted in Figure 13(a).

Characterization of controller integrity regions $\mathcal{K}_1^J$ and $\mathcal{K}_2^J$:

We first select $K_2^1 \in \tilde{K}_2$. Using $K_2^1 \in \mathcal{K}_2$, we compute the frequency response of the "artificial" plant $P_1^i$, where

$$P_1^i = g_{11} \left( 1 - \gamma h_2^1 \right).$$  \hspace{1cm} (40)

The Bode data of $P_1^i$ is used to determine the stabilizing PID gain set $\mathcal{K}_1 (K_2^1)$. This procedure is repeated for a set of controllers

$$\left\{ K_2^2, K_2^3, \ldots, K_2^n \right\} \in \tilde{K}_2$$  \hspace{1cm} (41)
Figure 13. PID gains for TITO controller (Example 3.5)
and the stabilizing PID gain sets

\[ \mathcal{K}_1(K_2^1), \mathcal{K}_1(K_2^2), \mathcal{K}_1(K_2^3), \ldots, \mathcal{K}_1(K_2^n) \]  \hspace{1cm} (42)

are calculated. Then the controller sets that guarantee closed-loop system integrity are given by

\[ \mathcal{K}_1 = \tilde{\mathcal{K}}_1 \cap \mathcal{K}_1(K_2^1) \cap \mathcal{K}_1(K_2^2) \cap \mathcal{K}_1(K_2^3) \cdots \cap \mathcal{K}_1(K_2^n) \]
\[ \mathcal{K}_2 = \{ K_2^1, K_2^2, K_2^3, \ldots, K_2^n \}, \]  \hspace{1cm} (43)

Finally, the controller set \( \mathcal{K}_1 \) is shown in Figure 14(a) and the controller set \( \mathcal{K}_2 \) is shown in Figure 14(b).

3.5.2 Fault Tolerance Test Results

To validate, we select a TITO PID controller from the set and determine unit step responses as follows.

<table>
<thead>
<tr>
<th>Table 3.1. A TITO PID controller selected for test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{p1} )</td>
</tr>
<tr>
<td>0.43</td>
</tr>
</tbody>
</table>

The unit step responses of the closed loop system with the selected PID controller under various failure conditions are shown in Figure 15.

From the unit step responses shown, we can see that, when the controllers are selected from the proposed regions, i.e., \( \mathcal{K}_1 \) and \( \mathcal{K}_2 \), the stability of the system is retained even when any one of the controllers is turned off.

Remark 3.5.1. The procedure above can be extended to 3 or more loops in an obvious way.
Figure 14. PID gains for TITO system with closed-loop system integrity (Example 3.5)
Figure 15. Unit step responses in Healthy and Faulty conditions (Example 3.5)

The controller design constraints that arise with this method are,

- Application to large MIMO systems with higher number of inputs and outputs is computationally extensive. Provided enough computational resources this method can be used to design fault tolerant PID controllers for any MIMO systems with considerable coupling.

- The controller structure is limited to PID.

These constraints are addressed in the proposed Measurement Based Approach to Design General Form Interval Controllers with System Integrity, the method is discussed in detail in the next chapter.
CHAPTER IV

A MEASUREMENT BASED APPROACH TO DESIGN GENERAL FORM INTERVALcontrollers WITH SYSTEM INTEGRITY

4.1 Introduction

In this Chapter, we introduce another measurement based approach to design a set of fault tolerant controllers of an arbitrary structure based on a modified version of the Mapping Theorem. The interval controller structure, which is described as a axis-parallel box in the controller parameter space, is used here. The axis-parallel box-type controller set is clearly desirable because it allows engineers to independently adjust the controller parameters. The entire procedure is free of any mathematical model of a plant and the only information required is the frequency response measurement of the plant.

4.2 Multilinear Interval Controller Design

Let $G$ and $K$ be a plant and a controller, respectively. We also denote by $\mathbf{p}$ a controller parameter vector in $K$. Assume that the vector $\mathbf{p}$ lies in an box-like uncertainty set:

$$
\Pi := \{ \mathbf{p} : p_i^- \leq p_i \leq p_i^+ , \ \text{for all } i \} 
$$

(44)

where $p_i \in [p_i^-, p_i^+]$. Let $V$ be the vertices of the set $\Pi$:

$$
V := \{ \mathbf{p} : p_i = p_i^+ \text{ or } p_i = p_i^- , \ \text{for all } i \} .
$$

(45)
Define
\[
\delta(s, K) := \text{Numerator} \left( \det[I + GK] \right) \quad (46)
\]
\[
\Delta(s) := \{ \delta(s, K(p)), p \in \Pi \} \quad (47)
\]
\[
\Delta_V(s) = \{ \delta(s, K(p)), p \in V \}. \quad (48)
\]

Note that (46) is the characteristic polynomial of the closed-loop system under Assumption 2. Since the coefficients of \( \Delta(s) \) are multi-linear functions of the interval parameters \( p \), by the Mapping Theorem [16, 17], the complex plane image of \( \Delta(s) \) is contained in the convex hull of the image of the vertices. Let \( \text{co}(-) \) denote convex hull of a set (-). Then we have
\[
\Delta(s^*) \subseteq \text{co} \left( \Delta_V(s^*) \right) \quad \text{for each } s^* \in \mathbb{C}. \quad (49)
\]

Let \( \Phi_{(-)} \) denote the angle subtended at the origin by the complex image set (-).

**Theorem 4.2.1.** Let \( K_0 := K(p_0) \in K \) be a stabilizing controller for the given plant \( G \) (i.e., \( \delta(s, K_0) \in \mathcal{H} \), that is, is Hurwitz). Then each member of the interval controller set
\[
K := \{ K(p) : p \in \Pi \} \quad (50)
\]
stabilizes \( G \) (\( \Delta(s) \in \mathcal{H} \)) if
\[
\Phi_{\Delta_V}(j\omega) < \pi \quad \text{for all } \omega \in \mathbb{R}. \quad (51)
\]

The proof is straightforward from the Zero Exclusion Principle found in [17].

### 4.3 Implementation of Measurement Based approach to Design Interval Controllers

We now show how to evaluate the condition given in Theorem 4.2.1 for a set of controllers from knowledge only of \( G(j\omega) \) for \( 0 \leq \omega < \infty \).
Let \( K(s, p) = N_k(s, p)D_k(s, p)^{-1} \) be a model of the controller and the matrix \( G(j\omega) \) the measured frequency response of the plant. Define
\[
\Delta_G(j\omega, p) := \{ \delta_G(j\omega, p) = \det [D_k(j\omega, p) + G(j\omega, p)N_k(j\omega, p)], p \in \Pi \} \tag{52}
\]
and the image of the vertices
\[
\Delta^Y_G(j\omega) = \{ \delta_G(j\omega, p), p \in V \}. \tag{53}
\]

**Lemma 4.3.1.**
\[
\Phi_{\Delta^Y_G}(j\omega) = \Phi_{\Delta^Y_V}(j\omega) \text{ for all } \omega \in \mathbb{R}. \tag{54}
\]

**Proof.** Let \( G(s) = D_g(s)^{-1}N_g(s) \), then
\[
\delta(s, p) = \text{Numerator} (\det [I + GK])
= \text{Numerator} \left( \det \left[ \frac{D_g(s)^{-1} (D_g(s)D_h(s, p) + N_g(s)N_k(s, p)) D_k(s, p)^{-1}}{\det [D_g(s)] \det [D_h(s,p)]} \right] \right).
\tag{55}
\]

From Assumption 2, we have
\[
\delta(s, p) = \det [D_g(s)D_h(s, p) + N_g(s)N_k(s, p)]. \tag{56}
\]

Recall
\[
\Delta_V(s) = \{ \delta(s, p), p \in V \}. \tag{57}
\]

Multiplying by \( D_g(s)^{-1} \), we have
\[
\frac{\Delta_V(s)}{\det [D_g(s)]} = \left\{ \frac{\delta(s, p)}{\det [D_g(s)]} = \det [D_h(s, p) + G(s)N_k(s, p)], p \in V \right\} = \Delta^Y_G(s). \tag{58}
\]
Since $\det [D_g(j\omega)]$ is a constant for every $\omega \in \mathbb{R}$, it is easy to see that the angle subtended by the set is equal to that subtended by the vertices:

$$\Phi_{\Delta_Y}(j\omega) = \Phi_{\Delta_Y}(j\omega) \quad \text{for all} \quad \omega \in \mathbb{R} \quad (59)$$

Thus, from

$$\Delta_G(s^*) \subset \text{co} (\Delta^Y_G(s^*)) \quad \text{for each} \quad s^* \in \mathbb{C}, \quad (60)$$

we conclude the following.

**Corollary 4.3.1.** Let $K_0 := K(p_0) \in K$ be a stabilizing controller for the given plant $G$. Then each member of the interval controller set

$$K := \{ K(p) : p \in \Pi \} \quad (61)$$

stabilizes $G$ if

$$\Phi_{\Delta_Y}(j\omega) < \pi \quad \text{for all} \quad \omega \in \mathbb{R}. \quad (62)$$

**Remark 4.3.1.** The significance of Corollary 4.3.1 is that condition (62) can be checked from the knowledge of only plant frequency response $G(j\omega)$ and the "vertex controller".

The fault tolerant interval controllers are designed using the following steps,

1. Identify the prespecified fault scenarios.

2. Design Stabilizing Interval Parameters for Healthy and each of the failure cases separately by following procedure.

   a. We first start with nominal controller parameter values $p_0$.
b. Then expand the intervals of all parameters, $p_i^- \leq p_i \leq p_i^+$.

c. Identify the vertices $V$.

d. Characterize,

$$\Delta^V_G(j\omega) = \{ \det [D_k(j\omega,p) + G(j\omega)N_k(j\omega,p)], p \in V \}$$

e. Check the phase condition in (9).

f. Increase the interval until the phase condition is not satisfied.

3. Take the intersection of all stabilizing interval regions calculated in the previous step. The result is the desired fault tolerant parameter intervals.

4.4 3-input 3-output Design Example

Consider the open-loop frequency data of a 3-input 3-output plant $G$ shown in Figure 16.

The desired controller is of PID type, denoted by $K$ with $T > 0$.

$$K = \begin{bmatrix}
\frac{K_{D1}s^2 + K_{P1}s + K_{I1}}{s(1 + sT)} & \frac{K_{D2}s^2 + K_{P2}s + K_{I2}}{s(1 + sT)} & \frac{K_{D3}s^2 + K_{P3}s + K_{I3}}{s(1 + sT)}
\end{bmatrix}$$  \hspace{1cm} (63)

4.4.1 Fault Tolerant Controller Design

In this design, we need to consider a three stage simultaneous stabilization problem. First, determine the controller sets $\tilde{K}_i$ stabilizing $g_i$ for $i = 1, 2, 3$. This represents the case of two loop failures. The second stage is to determine the controller sets $\tilde{K}_{jk} = \tilde{K}_j \times \tilde{K}_k$ stabilizing the plants $G_{ij}$ where $(j, k) = \{(1, 2), (2, 3), (1, 3)\}$. 
Figure 16. Open loop frequency response of the 3×3 system (Example 4.4)
This represents single loop failure cases. The final and third stage is to determine the controller set stabilizing the $3 \times 3$ plant. This represents the case with no loop failures.

The stabilizing controller intervals are designed for the following 3 cases,

2-loop Failures: $\mathcal{K}_i$ stabilizing $g_{ii}$, $i = 1, 2, 3$.

1-loop Failure: $\mathcal{K}_{jk} = \mathcal{K}_j \times \mathcal{K}_k$ stabilizing $G_{jk} = \begin{bmatrix} g_{jj} & g_{jk} \\ g_{kj} & g_{kk} \end{bmatrix}$, where $(j,k) = \{(1,2), (2,3), (1,3)\}$.

$$K_{jk} = \begin{cases} K_j \in \mathcal{K}_j & 0 \\ 0 & K_k \in \mathcal{K}_k \end{cases}$$

No Failure: $\mathcal{K}$ stabilizing $G$.

For the two loop failure cases, the parameter space box corresponding to each controller set with $T = 0.001$ is as follows.

For $\mathcal{K}_1$,

$$4.7402 \leq K_{D1} \leq 7.2598, \quad -0.7598 \leq K_{P1} \leq 1.7598, \quad 3.7402 \leq K_{I1} \leq 6.2598.$$  \hspace{1cm} (64)

For $\mathcal{K}_2$,

$$3.5449 \leq K_{D2} \leq 6.4551, \quad -0.4551 \leq K_{P2} \leq 2.4551, \quad 4.5449 \leq K_{I2} \leq 7.4551.$$  \hspace{1cm} (65)

For $\mathcal{K}_3$,

$$0.3105 \leq K_{D3} \leq 3.6895, \quad 0.3105 \leq K_{P3} \leq 3.6895, \quad 3.3105 \leq K_{I3} \leq 6.6895.$$  \hspace{1cm} (66)
For the single loop failure cases, the parameter space box corresponding to each controller set is as follows.

For $\hat{K}_{23}$,

\[
3.8848 \leq K_{D2} \leq 5.1152, \quad 0.3848 \leq K_{P2} \leq 1.6152, \quad 4.8848 \leq K_{I2} \leq 6.1152, \\
1.8848 \leq K_{D3} \leq 3.1152, \quad 1.1848 \leq K_{P3} \leq 2.4152, \quad 4.8848 \leq K_{I3} \leq 6.1152.
\]

(67)

For $\hat{K}_{13}$,

\[
4.9043 \leq K_{D1} \leq 6.0957, \quad 0.1043 \leq K_{P1} \leq 1.2957, \quad 3.9043 \leq K_{I1} \leq 5.0957, \\
1.9043 \leq K_{D3} \leq 3.0957, \quad 1.4043 \leq K_{P3} \leq 2.5957, \quad 4.4043 \leq K_{I3} \leq 5.5957.
\]

(68)

For $\hat{K}_{12}$,

\[
4.4238 \leq K_{D1} \leq 5.5762, \quad 0.2238 \leq K_{P1} \leq 1.3762, \quad 3.4238 \leq K_{I1} \leq 2.5762, \\
3.4238 \leq K_{D2} \leq 4.5762, \quad 0.9238 \leq K_{P2} \leq 2.0762, \quad 4.4238 \leq K_{I2} \leq 5.5762.
\]

(69)

The set of stabilizing PID controllers for the given 3-input 3-output system is:

\[
4.4629 \leq K_{D1} \leq 5.5371, \quad 0.2629 \leq K_{P1} \leq 1.3371, \quad 3.4629 \leq K_{I1} \leq 4.5371, \\
3.4629 \leq K_{D2} \leq 4.5371, \quad 0.6629 \leq K_{P2} \leq 1.7371, \quad 4.4629 \leq K_{I2} \leq 5.5371, \\
1.4629 \leq K_{D3} \leq 2.5371, \quad 1.6629 \leq K_{P3} \leq 2.7371, \quad 4.4629 \leq K_{I3} \leq 5.5371.
\]

(70)
We project these sets on to their respective coordinate subspaces and find the intersections. The PID controller set with the property of system integrity is found (see Figure 17).

$$4.9043 \leq K_{D1} \leq 5.5371, \quad 0.2629 \leq K_{P1} \leq 1.2957, \quad 3.9043 \leq K_{I1} \leq 4.5371,$$

$$3.8848 \leq K_{D2} \leq 4.5371, \quad 0.9238 \leq K_{P2} \leq 1.6152, \quad 4.8848 \leq K_{I2} \leq 5.5371, \quad (71)$$

$$1.9043 \leq K_{D3} \leq 2.5371, \quad 1.6629 \leq K_{P3} \leq 2.4152, \quad 4.8848 \leq K_{I3} \leq 5.5371.$$

### 4.4.2 Fault Tolerance Test Results

For verification, we selected a controller within the family $K$:

$$K = \begin{bmatrix}
\frac{5.1s^2 + 0.8s + 4.2}{s(1 + sT)} & \frac{4.1s^2 + 1.2s + 5.1}{s(1 + sT)} & \frac{2.1s^2 + 2s + 5.2}{s(1 + sT)} \\
\end{bmatrix}. \quad (72)$$

The unit step responses of each output of the healthy closed-loop system as well as the closed-loop systems under all possible single and two loop failures are shown in Figure 18.

From Figure 18 it is clear that the designed controller is stabilizing all three outputs in case of healthy, single loop failure and two loop failure cases.

### 4.5 Second Order Controller Design Example

We consider the Wood and Berry distillation process found in [?] for model based design. For our design, we generated the open-loop frequency data $G(j\omega)$ of the process shown in Figure 19.
Figure 17. Sets of controllers with system integrity \( \mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2 \times \mathcal{K}_3 \) (Example 4.4)
Figure 18. Unit step response of the Healthy and Faulty systems (Example 4.4)

In this problem, we design a second order controller of the form:

\[ K = \begin{bmatrix} \frac{a_1 s + b_1}{s^2 + c_1 s + d_1} \\ a_2 s + b_2 \\ \frac{a_3 s + b_3}{s^2 + c_2 s + d_2} \end{bmatrix} \]  \hspace{1cm} (73)

4.5.1 Fault Tolerant Controller Design

The objective is to determine bounds on the controller parameters so that system integrity is obtained.

\[ a_1^- \leq a_1 \leq a_1^+, \quad b_1^- \leq b_1 \leq b_1^+, \quad c_1^- \leq c_1 \leq c_1^+, \quad d_1^- \leq d_1 \leq d_1^+, \]
\[ a_2^- \leq a_2 \leq a_2^+, \quad b_2^- \leq b_2 \leq b_2^+, \quad c_2^- \leq c_2 \leq c_2^+, \quad d_2^- \leq d_2 \leq d_2^+. \]  \hspace{1cm} (74)
In this design, we need to consider a two stage simultaneous stabilization problem. First, determine the controller sets $\tilde{K}_i$ stabilizing $g_{ii}$ for $i = 1, 2$. This represents single loop failure cases. The final stage is to determine the controller set stabilizing the $2 \times 2$ plant. This represents the case with no loop failures.

The stabilizing controller intervals are designed for the following 2 cases,

**1-loop Failure**: $\tilde{K}_i$ stabilizing $g_{ii}$, $i = 1, 2$.

**No Failure**: $\tilde{K}$ stabilizing $G$.

By taking the intersection of the intervals obtained for the cases described above we characterize the integrity region, the fault tolerant interval parameters are given
below.

\[ a_1 \in [94.68, 98.43], \quad b_1 \in [12.25, 14.47], \quad c_1 \in [69.25, 72.88], \quad d_1 \in [96.02, 99.77]. \]
\[ a_2 \in [6.92, 9.25], \quad b_2 \in [0.75, 2.66], \quad c_2 \in [62.75, 64.73], \quad d_2 \in [46.75, 49.04]. \]

\[ (75) \]

4.5.2 Fault Tolerance Test Results

To verify, we select the following controller parameter values for testing.

<table>
<thead>
<tr>
<th>( a_1 )</th>
<th>( b_1 )</th>
<th>( c_1 )</th>
<th>( d_1 )</th>
<th>( a_2 )</th>
<th>( b_2 )</th>
<th>( c_2 )</th>
<th>( d_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.29</td>
<td>13.04</td>
<td>71.38</td>
<td>96.86</td>
<td>8.67</td>
<td>1.23</td>
<td>63.75</td>
<td>48.36</td>
</tr>
</tbody>
</table>

The unit step responses of the closed loop system with these gains are shown in Figure 20.

These techniques can be extended to include performance of the closed loop system. In the next section, we show how to include performance specified by gain margin, phase margin, and \( H_{\infty} \) norm.

4.6 Controller Design for Performance Example

The performance attainment problem can be reduced to robust stabilization of modified families of real and complex plants. This is due to the fact that the vertex property continues to hold and the Mapping Theorem is applicable [17, 18] for solving robust stability problem of these families. The procedure for performance attainment can be described as follows.

1. The problem of achieving a guaranteed gain margin \([M_{\min}, M_{\max}]\) is equivalent to simultaneously stabilizing the plant \( P(s) \) and the family of real plants

\[ P^\varepsilon(s) = \{ MP(s) : M \in [M_{\min}, M_{\max}] \}. \]
Figure 20. Unit step responses in Healthy and Faulty conditions (Example 4.5)

2. The problem of achieving a guaranteed phase margin $\theta_m$ is equivalent to simultaneously stabilizing the plant $P(s)$ and the family of complex plants

$$P^c(s) = \left\{ e^{-j\theta} P(s) : \theta \in [0, \theta_m] \right\}.$$  \hspace{1cm} (77)

3. The problem of achieving a $H_\infty$ norm for complement sensitivity function $T(s)$, that is, $\|W(s)T(s)\|_\infty < \gamma$, is equivalent to simultaneously stabilizing the plant $P(s)$ and the family of complex plants

$$P^c(s) = \left\{ P(s) \left[ 1 + \frac{1}{\gamma} e^{-j\theta} W(s) \right] : \theta \in [0, 2\pi] \right\}.$$  \hspace{1cm} (78)

The solutions of the above problems are facilitated by the fact that the multilinear structure of the characteristic equation continues to hold, and thus Corollary 4.3.1 applies.
In this example, we design a set of fault-tolerant PI controllers that guarantee the prespecified gain and phase margin. The TITO process considered is found in [14] for a model based design. For our design, we generated the open-loop frequency data of plant $G$ as shown in Figure 21.

![Figure 21. Frequency response data for $G$ (Example 4.6)](image)

Both loops need guaranteed gain margin $GM = 2$ and phase margin $PM = 20^\circ$. Let the PI controller to be designed be, denoted by $K$ with $T > 0$.

$$K = \begin{bmatrix} \frac{K_{P1}s + K_{I1}}{s(1 + sT)} & 0 \\ 0 & \frac{K_{P2}s + K_{I2}}{s(1 + sT)} \end{bmatrix}.$$  \hspace{1cm} (79)

where $T = 0.001$. 
4.6.1 Fault Tolerant Controller Design with performance specifications

The objective is to determine bounds on the PI parameters so that system integrity is obtained.

\[ K_{P_1}^- \leq K_{P_1} \leq K_{P_1}^+, \quad K_{I_1}^- \leq K_{I_1} \leq K_{I_1}^+, \quad K_{P_2}^- \leq K_{P_2} \leq K_{P_2}^+, \quad K_{I_2}^- \leq K_{I_2} \leq K_{I_2}^+. \]  

(80)

The stabilizing controller intervals with performance are designed for the following 2 cases,

**1-loop Failure:** $\tilde{K}_i$ stabilizing $g_{ii}$, $i = 1, 2$.

**No Failure:** $\tilde{K}$ stabilizing $G$.

Using the interval controller design procedure described, we determine the axis-parallel parameter space box $\tilde{K}_1$ in the $(K_{P_1}, K_{I_1})$ space that stabilizes the SISO plant $g_{11}$ with required gain and phase margins. Similarly, we also determine the axis-parallel parameter space box $\tilde{K}_2$ for the plant $g_{22}$.

\[ (\tilde{K}_{P_1} \in [-7.5, 10], \tilde{K}_{I_1} \in [-6.5, 12]), \quad (\tilde{K}_{P_2} \in [-0.5, 62.5], \tilde{K}_{I_2} \in [-0.5, 71.5]) \]

(81)

Let

\[ p^0 := [\hat{K}_{P_1}^0, \hat{K}_{I_1}^0, \hat{K}_{P_2}^0, \hat{K}_{I_2}^0]. \]  

(82)

The nominal stabilizing controller gains required to obtain interval controller for TITO system are chosen from $\tilde{K}_1$ and $\tilde{K}_2$:

\[ \hat{K}_{P_1}^0 = -1, \quad \hat{K}_{I_1}^0 = -2, \quad \hat{K}_{P_2}^0 = 20, \quad \hat{K}_{I_2}^0 = 19. \]

(83)

Then we obtain $\hat{K}$:

\[ \hat{K}_{P_1} \in [-3.2, -0.2], \quad \hat{K}_{I_1} \in [-2.5, -0.1], \quad \hat{K}_{P_2} \in [0.5, 95.5], \quad \hat{K}_{I_2} \in [0.5, 90]. \]  

(84)
To complete the design, we find the intersection of the controller intervals characterized for healthy, and two faulty cases by projection. The set of TITO controllers that guarantee the given gain and phase margin requirements with the property of closed-loop system integrity is found as

\[ K_{P1} \in [-3.2, -0.2], \quad K_{I1} \in [-2.5, -0.1], \quad K_{P2} \in [0.5, 62.5], \quad K_{I2} \in [0.5, 71.5]. \]

The controller gain intervals for performance are shown within the stabilizing gain intervals in Figure 22.

### 4.6.2 Fault Tolerance Test Results

To verify the result, we select two PI controllers within the designed parameter space boxes one from stabilizing region, and one from stabilizing region with performance for testing.

<table>
<thead>
<tr>
<th>Table 4.3. PI gains considered for test</th>
<th>( K_{P1} )</th>
<th>( K_{I1} )</th>
<th>( K_{P2} )</th>
<th>( K_{I2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stabilizing controller</td>
<td>-1.5</td>
<td>-2.6</td>
<td>1.4</td>
<td>95</td>
</tr>
<tr>
<td>Stabilizing controller with performance</td>
<td>-0.72</td>
<td>-1.5</td>
<td>0.65</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The unit step responses of the closed loop system with stabilizing PI gains are shown in Figure 23.

The unit step responses of the closed loop system with stabilizing PI gains with performance are shown in Figure 24.

From Figure 23 and Figure 24 it is clear that the pi gains designed for performance are giving better transient and steady state response when compared with stabilizing PI gains.
Figure 22. Sets of controllers with system integrity $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2$ (Example 4.6)
Figure 23. Unit step responses With Stabilizing Controller (Example 4.6)

Figure 24. Unit step responses With Stabilizing Controller with performance (Example 4.6)
CHAPTER V

DESIGN OF FAULT TOLERANT HEIGHT CONTROLLERS FOR QUADCOPTER DRONE

5.1 Introduction

Initial step in any drone operation is, lifting the drone to a minimum height before doing any required position maneuvering. So stable operation of the drone in reaching initial height is a must requirement. Actuator effectiveness loss is the most common failure effecting required performance and operation of the quadcopter. This loss can occur due to external disturbances caused by environment and hardware, or due to the network communication errors which results in changed control effort values. Hence, we aim at designing Fault Tolerant Height controllers against the actuation loss, so that stable height control is achieved even in the presence of pre specified percentage loss in actuation. In this chapter new procedure to design Fault Tolerant Height controllers for Quad rotor drone Qball-X4 will be introduced. Fault Tolerant PID Controller gain region will be designed using the proposed method and designed controllers are to be tested for fault tolerance. This design example is different from the previous ones in two ways.

- The type of failure considered here is percentage loss in actuation or loss of actuator effectiveness (not complete loop failure).

- The system we are controlling is SISO, i.e., from height input to the measured height output.
5.2 Stabilizing PID Controller Design

The open-loop frequency data (magnitude and phase) of the drone height control system $G_{ht}(j\omega)$, is shown in Figure 5.2. The 3D simulation model provided by Quensr is used to obtain this data, where we can give required height input and measure the actual height output.

![Bode Diagram](image)

Figure 25. Frequency response data for Qball height

First, PID controller set for stabilizing the system $G_{ht}$ in unity feedback loop is designed using Data Based PID set design method and is shown as $PID_{healthy}$ in Figure 5.3.

5.3 Designing FTC region

Let, $X$ be the pre specified percentage loss in actuation. The FTC design procedure starts with designing stabilizing gain regions for different percentage values of actuation loss. Let, $PID_X$ be the stabilizing PID gain region which stabilizes the
plant \((1 - \frac{1}{X})G_{j\omega}\) in unity feed back loop. For example, \(PID_{10\%\text{loss}}\) is the stabilizing PID gain region which stabilizes the plant \(0.9G_{j\omega}\) in unity feed back loop.

In this design, the maximum actuation loss is considered is 40\%. The stabilizing PID gain regions \(PID_{10\%\text{loss}}, PID_{20\%\text{loss}}, PID_{30\%\text{loss}},\) and \(PID_{40\%\text{loss}}\) are designed to stabilize the plants \(0.9G_{j\omega}, 0.8G_{j\omega}, 0.7G_{j\omega},\) and \(0.6G_{j\omega}\) respectively. The regions are shown in Figure 5.3. The stabilizing fault tolerant PID gain region \(PID_{FTC}\) is the intersection of all these regions with \(PID_{\text{healthy}},\) i.e.,

\[
PID_{FTC} = PID_{\text{healthy}} \cap PID_{10\%\text{loss}} \cap PID_{20\%\text{loss}} \cap PID_{30\%\text{loss}} \cap PID_{40\%\text{loss}} \quad (85)
\]

![Figure 26. fault tolerant PID gain region](image-url)
5.3.1 Controller Integrity Test Results

The controller $C_1$ is selected from healthy i.e., no failure region, and the controller $C_2$ is selected from designed fault tolerant region. The closed loop step response with $C_1$, and $C_2$ in healthy as well as faulty situations is tested for stability. The closed loop step response with $C_1$, and $C_2$ in healthy situation is as shown in Figure 5.3.1.

![Graphs showing No Actuation Loss, Output with Normal Controller, and Output with FTC controller](image)

Figure 27. No Failure

The closed loop step response with $C_1$, and $C_2$ when there is an 20% actuation loss during entire operation is shown in Figure 5.3.1.

The closed loop step response with $C_1$, and $C_2$ when there is an actuation loss during the lift off is shown in Figure 5.3.1. And the closed loop step response with $C_1$, and $C_2$ when there is an actuation loss in the steady state is shown in Figure 5.3.1.

From the Integrity Test Results it is clear that the designed controller is ensuring stability when there is an actuation loss in between 0 – 40% in healthy and
Figure 28. Actuation Loss during Entire Operation

as well as all three failure situations. But the normal controller is only stabilizing
the system in healthy case and the system is unstable in all the faulty situations.
Figure 29. Actuation Loss during Liftoff

Figure 30. Actuation Loss in Steady State
Figure 31. 0-40% Actuation loss during entire operation
CHAPTER VI

CONCLUSIONS

In this research, the problem of designing fixed order controllers that preserve closed-loop stability under various loop failures is considered. It is noted that sensor or actuator (complete) failures lead to loop failures when a diagonal controller is used. Two passive FTC design methodologies for multivariable systems are implemented. Both approaches are strictly based on frequency response data of the plant and no mathematical models of the plant are used, and results in a set of controllers.

The first approach reduces the stabilization problem of a multivariable system to that of a SISO system, and takes advantage of available techniques in model free PID design. We rely on a recent result that determines the entire set of PID controllers for a SISO plant. The controller design constraints that arise with this method are, application to large MIMO systems with higher number of inputs and outputs is computationally extensive. The controller structure is limited to PID.

The design constraints of the first method are addressed in the proposed Measurement Based Approach to Design General Form Interval Controllers with System Integrity. This method, requires the knowledge of nominal stabilizing controller and results in box-type of controller sets in controller parameter space. Such controller sets are useful because it allows engineers to tune each controller parameter independently. While the approach can be used with arbitrary order controllers (the first technique uses PID type), it requires a nominal stabilizing controller for each failure
case considered.

The proposed approaches can be used to design controllers of arbitrary structure, e.g., PID, FOC, SOC, for any MIMO system with sufficient coupling. Four design examples for different MIMO systems with different stability and performance requirements are presented to illustrate the versatility of the proposed controller design methodologies. The designed controllers are tested for Fault Tolerance, and it is observed that the stability of the closed loop system is retained in presence of pre-specified loop failures.
REFERENCES


BIO-DATA

PAVANASIRISHA KALLAKURI
1744 Haley’s hope Ct, Nashville TN. (508) 873-2563
pskallakuri@gmail.com

EDUCATION

• PhD in Computers and Information Systems Engineering from Tennessee State University, USA (G.P.A. 3.9/4.0).

• MS in Computers and Information Systems Engineering from Tennessee State University, USA (G.P.A. 3.9/4.0).

• BE in Electrical and Electronics Engineering from Andhra University, INDIA (G.P.A. 3.9/4.0).

WORK EXPERIENCE

• Graduate Research Assistant, COE-ISEM at Tennessee State University, USA. Sept 2009-Till date.

• Adjunct Faculty, Dept. of ECE at Tennessee State University, USA. Aug 2014-May 2015.

• Scientist/Engineer SC, Satish Dhawan Space Centre, ISRO, India. Dec 2006-April 2009.

TECHNICAL SKILLS

Control Design: PID controller design, State Feed Back and Observer Design, Robust Control design ($H_{\infty}$, PM, GM), Multi Agent Formation and Trajectory control, Fault Tolerant Control design, Individual Channel Design.


PROJECTS

- Design of Robust Height Controllers for Unmanned Ariel Vehicle Qball-X4.
  - Quarc (Matlab Simulink based software).

- Trajectory Controller design for quad copter ARdrone-2.0.
  - Matlab Simulink desktop real-time module.

  - Robust Control Design Tool- Matlab.

- Undergrad ECE Control Systems Lab Setup.
  - Embedded C (ECPMV executive software).

- Data Based PID controller Design for a magnetic Levitation System (M.S Thesis).
  - Embedded C (ECPMV executive software).

- Embedded micro controller for launch vehicle attitude control.
- VHDL (Libero IDE).

- **PLC based Traction Control system.**

  - Ladder Programming (Allen Bradley PLC).

**PUBLICATIONS**


- P.Kallakuri, Navid Mohsenizadeh, L.H.Keel and S. P. Bhattacharyya, A Collaborative Research on Multivariable Fixed Order Controller Synthesis and

- P.Kallakuri, L.H.Keel and S. P. Bhattacharyya, Data Based Design of PID Controllers for a Magnetic Levitation Experiment, in Proc. of 18th IFAC World Congress, Milan, Italy, 2011