ESTABLISHING DOMINANCE AMONG

ALTERNATIVES UNDER AMBIGUITY

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When a decision analyst's goal is to establish a partial ordering of alternatives through dominance tests, then these dominance tests should incorporate ambiguities about outcomes and their likelihood in a manner consistent with the expressions of those providing information. Ambiguity is present, when, for example, one is provided with vague beliefs about how the future is expected to unfold, or imprecise specifications of the consequences of various actions. In general, ambiguity is a condition associated with the non-uniqueness of information caused by the existence of one-to-many, or more generally, many-to-many relations. This condition is prevalent in decision making, since we may have situations which associate one or many probabilities with one or many outcomes.

This research addresses the following challenges: 1) Clarifying and representing beliefs about likelihood and outcomes as they are provided by sources. These representations of ambiguity utilize, but are not limited to, point representations, set representations (c.f. Dempster-Shafer theory), interval representations, and combinations of these.

2) Modifying and extending the stochastic dominance and mean-variance tests so that

they may be useful when outcomes and probabilities are ambiguously described. This involves proposing and proving more general versions of the stochastic dominance theorems, developing foundational results in nonlinear programming, and developing accompanying models and procedures for determining mean-variance bounds. These extensions augment and complement sensitivity analysis, the traditional tool utilized for considering additional uncertainty in decision analysis. It can be shown that typical approaches to sensitivity analysis will not always yield the extreme values of the variance measures.

The usefulness of the research is evident as one notes the widespread use of stochastic dominance and mean-risk tests and yet recognizes that decision-making under ambiguity is a prevalent condition. Decision makers who use such tests could pursue the task of ranking alternatives while explicitly recognizing existing ambiguities. Extending stochastic dominance theorems and mean-risk theorems provide valuable techniques to pare down the number of alternatives considered without burdening the decision maker to provide more information or express utilities or preferences for ambiguities.

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1. Introduction

1.1 Problem Description and Basic Background

When a decision analyst's goal is to establish a partial ordering of alternatives through dominance tests, then these dominance tests should incorporate ambiguities about outcomes and their likelihood in a manner consistent with the expressions of those providing information. This section will establish and describe the setting for such analysis, providing background, foundational material, and further references for the interested reader.

1.1.1 Ambiguity in Decision Analysis

Decision analysis is a field in which the goal is the identification of one or more alternatives which are expected to best satisfy the objectives of the decision maker. Decision analysis is broadly defined as the modeling and resolving of choice problems under risk and uncertainty through the use of preference-based, quantitative procedures. Ambiguity is present, when, for example, one is provided with vague beliefs about how the future is expected to unfold, or imprecise specifications of the consequences of various actions.

In general, ambiguity is a condition associated with the non-uniqueness of information caused by the existence of one-to-many, or more generally, many-to-many relations (Klir and Folger 1988). This condition is prevalent in decision making, since we may have situations which associate one or many probabilities with one or many outcomes.

In decision making, ambiguity is often present in the following senses: ambiguity about probabilities, ambiguity about outcomes, or ambiguity about both. Camerer and Weber (1992) also drew a distinction between these two understandings. However, this bifurcation of ambiguity is not a comprehensive characterization of ambiguity/uncertainty in decision-making. For example, French (1995), Friend and Hickling (1987), and Berkeley and Humphreys (1982), while classifying ambiguity as a form of uncertainty, describe different forms of uncertainty. Among them are: uncertainty about what might happen or what can be done, uncertainty about meaning/ambiguity, uncertainty about related decisions, uncertainty arising from physical randomness or lack of knowledge, uncertainty about the evolution of future beliefs and preferences, uncertainty about judgments (e.g. beliefs, preferences and guiding values), procedural uncertainty, uncertainty about the accuracy of calculations, uncertainty about the appropriateness of a descriptive or a normative model, uncertainty about the extent one possesses agency for inducing changes in the probabilities of subsequent events, and uncertainty about the depth to which to conduct an analysis. Here the discussion is limited to two senses of ambiguity, thus focusing on what Berkeley and Humphreys (1982) call the "core" system within decision theory: a single-level decomposition of immediate acts into consequences.

In the first sense, that is, where there is ambiguity about probabilities, the probability of an uncertain event may not be precisely known, while the outcome associated with this

ambiguous probability may be exactly specified. Most definitions of ambiguity that have appeared in the literature only recognize this sense of the term. Examples include the works of Kahn and Sarin (1988), who defined ambiguity as uncertainty about uncertainty, and Curley and Yates (1985), who characterized ambiguous decision situations as those having probabilities that are uncertain. Savage (1954) didn't use the term ambiguity, but suggested "an aura of vagueness is attached to many judgments of personal probability" (p. 169). Frisch and Baron (1988) established a definition based on concepts mentioned by Fellner (1961) and others: "Ambiguity is uncertainty about probability, created by missing information that is relevant and could be known."[1] Einhorn and Hogarth (1986) suggested that "ambiguity is an intermediate state between ignorance (no distributions are ruled out) and risk (all distributions but one are ruled out). Thus, ambiguity results from the uncertainty associated with specifying which of a set of distributions is appropriate in a given situation. Moreover, the amount of ambiguity is an increasing function of the number of distributions that are not ruled out by one's knowledge of the situation."

In the second sense, that is, where there is ambiguity about outcomes, one recognizes that although the probability of an event may be precisely known, the outcome of the event may not be precisely known. Use of this understanding of ambiguity does not appear to be as prevalent in the literature. Weber (1987) addressed a broader set of uncertainties,

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[1] While not offering a working definition, Ellsberg (1961) described the essence of ambiguity as "the quality depending on the amount, type, reliability, and 'unanimity' of information." His seminal work presented examples and discussions that have, for many, defined the research in this field.

labeling a decision situation *incomplete* if the outcome, utility or probability function is not exactly specified. Concerning the outcome function, Weber noted that the decision maker may not be sure how to evaluate the outcome of an alternative in the attribute space. This exemplifies and could be termed ambiguity over outcomes. Camerer and Weber (1992) specifically contrasted the concept of ambiguity over outcomes with ambiguity over probabilities. They identified aversion to outcome ambiguity when probabilities are known with risk aversion, and aversion to ambiguity about probabilities with ambiguity aversion.

Considering these two senses of ambiguity, one can recognize four classes of decision making, as summarized in Table 1.1. (Classes II and IV are subdivided for later use.) The particular nature of the probability is not an issue. A probability value supplied will usually be considered an epistemic probability, modeling logical or psychological degrees of partial belief, of a person or intentional system. See Walley (1991) for a more complete discussion of distinctions and interpretations within this classification.

Table 1.1 Classes of Decision Making

	No Ambiguity on Outcomes	Ambiguity on Discrete Outcomes	Ambiguity on Continuous Outcomes
No Ambiguity on Probabilities	I point probabilities and single-valued mappings	IIa point probabilities and multivalued mappings to discrete outcome sets	IIb point probabilities and multivalued mappings to outcome intervals
Ambiguity on Probabilities	III probability intervals and single-valued mappings	IVa probability intervals and multivalued mappings to discrete outcome sets	IVb probability intervals and multivalued mappings to outcome intervals

Class I considers cases where it may be meaningful to assign precisely known point probabilities (probability masses) to unambiguous, mutually exclusive, collectively exhaustive states of the world; that is, a probability mass is associated with each outcome. This class is referred to as decision making under risk and is the class that has been studied extensively in the literature. In contrast to Classes III and IV, for a given strategy, the sum of the probabilities over all outcomes is one. As an example of this class, suppose a decision maker views a new product introduction as having a 15% chance of succeeding, with an associated return on investment of 35%, and an 85% chance of failing, with an associated loss of 5%. Although this class of decision making is well understood, one must recognize that theories developed for this class are based on the presumption that a decision alternative has a unique probability distribution. However, uncertainties seldom can be represented with fixed, given probabilities (Winkler 1991).

Class II considers cases where it is meaningful to map a point probability to an ambiguous outcome; that is, a probability distribution maps probabilities to outcomes described by discrete sets or intervals.[2] Probability mass, summing to one, is distributed over the subsets or subintervals of the outcome space. Suppose the outcome space X is countable, i.e., $X = \{x_1, \ldots, x_n\}$. Let 2^X be the power set of X, and $m(A_j)$ be the probability mass attributed to a subset $A_j \in 2^X$. A_j is called a focal element if

$m(A_j) \neq 0$. $m(A_j)$ indicates the degree of belief that the actual outcome will be an element of A_j, with no further evidence available for establishing the likelihood of one element in the set over another in the same set. If the outcome space is continuous, X is an interval in $\Re^1$ and $m(A_j)$ denotes the probability mass attributed to a simple or composite subinterval A_j of the outcome space. Here $m(A_j)$ indicates the degree of belief that the actual outcome will be a value in the subinterval A_j, with no further basis for establishing the likelihood of a specific value in that subinterval. The above statements do not imply that the elements of a subset or the points in a subinterval are equilikely; rather they imply that we simply do not have enough information to make a decision about distributing the probability mass more explicity to the members of the subset or to the points in the subinterval. The example presented for Class I above can be extended as follows to make it suitable as a Class II example: the decision maker may feel that 15% of the evidence suggests that the return will be in the interval 30-40%, and 85% of the evidence suggests the loss will be in the interval 0 to 10%. Additional information is not available to distribute the 15% evidence over the 30-40% interval. That is, within the interval 30-40%, the decision maker is in a state of ignorance as to which value will be obtained, and has no basis for further assigning probabilities to the events (see Fishburn 1964, chapter 5). As a second example, suppose that as your disabled car is being towed to a service garage, the driver tells you, based on your brief

[2]Dempster helped formalize these concepts for certain problems in this class in his work on multivalued mappings. Schafer extended the theory, providing a vocabulary within which a person can make judgments about the strength of inconclusive evidence. See Dempster (1967); Shafer, (1976).

description, that in his experience there is an 80% chance the problem is either a broken piston rod, which would cost about $500, or that you need a new timing belt, which would cost around $250. The driver is unwilling to commit the remaining 20% of his belief, but suggests as outside bounds the cost will be in the interval $50-$1500. In the first example, point probabilities are assigned to ambiguous intervals. In the second example, one point probability is assigned to an ambiguous discrete set and the remaining point probability to an ambiguous interval.

In Class III we consider unambiguous outcomes but ambiguous probabilities, where probability intervals or discrete probability sets are mapped to point-valued outcomes. Here the decision maker is facing a situation where the probability p of an outcome lies within the interval $[p_{\min}, p_{\max}]$ (see Curley and Yates 1985). Franke (1978) suggested that many people, when asked for a subjective probability, would be reluctant to specify a unique number and would prefer to specify an interval $\left[p(x)-\underline{\Delta}, p(x)+\overline{\Delta}\right]$ in which the "true" probability, $p(x)$, lies $(\underline{\Delta}, \overline{\Delta} \geq 0)$. These probability intervals accommodate individuals who like to think there is a true probability, even though the theory of subjective probability does not admit such a notion. Clearly we cannot expect these probability intervals to sum to one in the usual sense, but rather, over i possible outcomes, we have $\sum p_{\min,i} \leq 1$ and $\sum p_{\max,i} \geq 1$.[3] As a variation of the product

[3]As discussed later, probability interval assignments cannot be totally arbitrary, but must induce a family of probability distributions which are consistent with the intervals specified and which exhibit additivity and the property of the unit measure distributed over the whole event space.

introduction example of Class I, assume the desired outcome is a yearend bonus that is based solely on management's judgment of the new product's relative success. Suppose the interval 10-20% is said to cover the probability of a very successful product introduction, leading to an unambiguous yearend bonus of $5000. Furthermore, suppose the interval 10-40% is said to cover the probability of a modestly successful product introduction, which would mean a $500 bonus, and the remaining probability is assigned to a weak product introduction, with no bonus as the associated outcome. Thus the probability interval [0.10, 0.20] is mapped to the outcome $5000; the interval [0.10, 0.40] is mapped to the outcome $500; and the interval [0.40, 0.80] is mapped to the outcome $0. As another example, let us suppose we have written a contract with an Olympic-hopeful athlete to endorse our product. If the athlete fails to make the team, an event which one may assess has a probability between 20-40%, the contract terminates, with a net cost to the marketing budget of $10,000. If the athlete makes the team, the contract continues over the next year, and the cost to the marketing budget is $100,000. In both of these examples, the subjective probabilities are given by intervals, and the associated outcomes are given by point values.

In Class IV, the most general, we consider ambiguous probabilities mapped to ambiguous outcomes. To illustrate, suppose in the example of the product introduction that some form of favorable legislation would significantly enhance revenues. Various assessments indicate the probability of legislation being enacted this year is in the interval [0.05, 0.25]. The outcomes associated with passage are the uncertain (ambiguous) positive

impacts on the following year's revenues, estimated increases ranging from 10-20%. Clearly there is a many-to-many relationship here, a multivalued mapping from indeterminate probabilities drawn from probability sets or intervals, to sets or intervals of outcomes.

1.1.2 Stochastic Dominance and Expected Value-Variance Tests

Stochastic dominance tests and expected value-variance (E-V) tests are frequently used to attempt to establish dominance among potential alternatives (Yetzhaki 1982).

1.1.2.1 Stochastic Dominance Tests

Overview

Stochastic dominance tests can be employed to assist decision makers in ordering uncertain alternatives. These tests require specification of the alternatives' probability distributions over outcomes and the assumption that the decision maker is maximizing his or her expected utility in the spirit of von Neumann-Morgenstern expected utility theory. With these assumptions, decision alternatives can be partitioned into dominated and non-dominated sets on the basis of the alternatives' probability distributions and the decision maker's utility function class (e.g. Bawa 1982; Weeks 1985; Levy 1992).

The current use of stochastic dominance tests assumes probability distributions are known and unique. As argued above, however, specifying unique probability distributions may be unjustifiable, primarily due to the presence of ambiguity.

Furthermore, ignoring the existence of ambiguity may invite skepticism about the stochastic dominance results.

Review of utility theory concepts underlying stochastic dominance tests

Stochastic dominance theorems assume the decision maker subscribes to four expected utility axioms. The basic premise of utility theory is that the set of outcomes of an alternative can be mapped to real numbers in such a way that the decision maker will attempt to maximize expected utility. Let a particular distribution $\omega_r \in \Omega$, $r = 1,2,\ldots$ be called a prospect. That is, ω_r represents the set of ordered pairs $(x_i, m(x_i))$ describing a particular mapping of point probabilities to single-valued outcomes. The utility value of an alternative represents the relative desirability of the decision alternative. The four axioms, following Whitmore and Findlay (1978), are listed below:

1) Complete Ordering Axiom: The preferences of the decision maker among the prospects of Ω satisfy the following two conditions:
    a. For all pairs ω_1, $\omega_2 \in \Omega$ the decision maker either prefers ω_1 to ω_2, ω_2 to ω_1, or is indifferent between them. Only one of these three possibilities is true for any pair.
    b. If for any ω_1, ω_2, $\omega_3 \in \Omega$, the decision maker prefers ω_1 to ω_2 and prefers ω_2 to ω_3, then the decision maker also prefers ω_1 to ω_3.
2) Continuity Axiom: If the decision maker prefers prospect ω_1 to ω_2 and prefers ω_2 to ω_3, then a prospect $\omega_{1,3}$ can be constructed as a convex combination of ω_1 and

ω_3 such that the decision maker is indifferent between $\omega_{1,3}$ and ω_2, where

$\omega_{1,3} = \{(\omega_1, p), (\omega_3, 1-p)\}$, $0 \le p \le 1$.

3) Independence Axiom: If a decision maker is indifferent between prospects ω_1 and ω_2, and between ω_3 and ω_4, then the decision maker is indifferent between $\omega_{1,3}$, a prospect constructed as a convex combination of ω_1 and ω_3, and $\omega_{2,4}$, a prospect constructed as a convex combination of ω_2 and ω_4, provided the probabilities p_1 and p_2 associated with ω_1 and ω_2, respectively, are equal.

4) Unequal Probability Axiom: Assume that a decision maker prefers prospect ω_1 to ω_2. If prospects $\omega_{1,2}$ and $\omega'_{1,2}$ are constructed such that:

   $\omega_{1,2} = \{(\omega_1, p_1), (\omega_2, 1-p_1)\}$ and $\omega'_{1,2} = \{(\omega_1, p_2), (\omega_2, 1-p_2)\}$, with $0 \le p_2 < p_1 \le 1$, then $\omega_{1,2}$ is preferred over $\omega'_{1,2}$.

Stochastic dominance theorems also presume that a decision maker's preference structure can be described by a given class of utility functions. Following Fishburn and Vickson (1978), assume that $u(\cdot)$, the decision maker's utility function, is well-defined and finite over all possible outcomes. That is, if I is the smallest interval such that all possible values of a measure of interest X take values in I, then $u(X)$ is defined and finite on I. Also assume that X is bounded from below.

Let U_1 be the class of utility functions whose member functions are strictly increasing. That is, $U_1 = \{u : u, u'$ are continuous and bounded on I, and $u' > 0$ on $I^0\}$. Here u' denotes the first derivative of the function u with respect to x and I^0 denotes the interior