MANAGEMENT OF INVASIVE SPECIES USING OPTIMAL CONTROL THEORY

by

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In my dissertation I will discuss the use of optimal control theory to determine management strategies for an invasive species. I focus on a Diaprepes Root Weevil, which is an invasive species having a substantial negative impact on citrus tree growth in regions such as Florida and California. At the larva stage of the life cycle Diaprepes Root Weevils cause destruction of citrus trees at the root level resulting in loss of citrus crops. This detrimental effect for farmers motivates research into how to minimize the economic loss due to the Diaprepes Root Weevil. For my work, I use optimal control theory to determine levels of pesticide or biological control to apply to the Diaprepes Root Weevil to reduce the economic loss.

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DEDICATION

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Chapter 1

Background

1.1 Mathematical Background

A useful source for the history of control theory is a paper entitled Control Theory: History, Mathematical Achievements and Perspectives [FCZI03]. The article covers highlights from the development of Control Theory, additionally exploring specific topics and examples. Furthermore, the article considers feedback, optimization, controllability, and optimal control. There is also a look at specific examples utilizing control theory, and possible avenues for future study. As mentioned in the article one of the key development of Optimal Control Theory can be traced to Pontryagin. Specifically, there was a book published in 1962, *The Mathematical Theory of Optimal Process*, by L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelize, and E.F. Mishchenko[Pon87]. An important development was the Pontryagin Maximum principle which established necessary conditions to an optimal control problem and relates this to the Hamiltonian, a useful tool for solving optimal control problems. For a more in-depth look we refer the reader to the original book or the book *Optimal Control Applied to Biological Models* by Lenhart, S. and Workman, J.T. [LW07], a

very useful source. The book by Lenhart and Workman covers an introduction to Optimal Control Theory, focusing on a full treatment of continuous time systems, and includes discrete time as well. Additionally, it includes many examples of optimal control applied to biological systems, with both the mathematics and code included.

For specifically Discrete-Time Optimal Control Theory, a good resource is *Optimal Control in Discrete Pest Control Models* by Kathryn Dabbs[Dab10]. The paper gives an overview of how to solve discrete-time optimal control problems and looks at specific models. Another paper on discrete-time optimal control with existence, necessary condition, and uniquness proofs is *Optimal control of gypsy moth populations* by Whittle, Lenhart, and White [WLW08]. In this thesis, we focus on a model which does not fit into this framework, allowing for variations to the mathematical set-up and a full treatment of existence, necessary condition, and uniqueness proofs for the optimal control.

Additionally, there have been many papers linking Optimal Control Theory to biology, a few that we have found useful in our studies: [MS12], [Fil62], [Gra10], [Dab10], [WLW08], [MLW15], [JLPB05], [Ris77], and [Leu93]. Some of these papers also address invasive species as their biological inspiration for implementing Optimal Control. For instance, the Gypsy Moth is a specific invasive species studied in both [WLW08] and [MLW15], which utilize a different model but use Optimal Control Theory to study management, and in [MLW15] include an integrodifference model. In the next section I will explain more about invasive species.

1.2 Biological Background

Our research involves applying control theory techniques to natural resources management, in particular management of invasive species.

Since the beginning of agriculture, people have always had to deal with pests affecting their crops, and developing methods to control the effects. Originally people had to eliminate pests by hand, through picking or mechanical methods, until 2500-1500 B.C. when the Sumerians and Chinese introduced pesticide. Today there is still a great loss of crops to pests. Specifically we consider crops which we use in our daily lives. For instance, there is a loss of approximately 50% of wheat to pests, while cotton loss can exceed 80% [Oer06]. There are various methods applied to combat pests including implementing predators, weeding techniques, biological control agents, and pesticides[Oer06].

Across the world annually there is approximately $40 billion spent on pesticides, while the United States made up a quarter of that cost [PU03, PG97]. Despite attempts to apply pesticide, in the United States there was still a loss 37% of crops, to the ecological pests. Specifically there was 13% lost to insects[Pim05]. Furthermore, even though we have increased pesticide application in the past 50 years by more than a factor or ten, there is still approximately twice as much damage now from insects than then [PMZ+91]. [Pim05]

Another important factor to consider is the human element which affects invasive species. When humans disrupt a territory, the result is a possible response growth in invasive species, with the destruction of the terrain linked to the original species increased chance of eradication [Hob00, Fah02, DL09]. So, humans not only inadvertently encourage the growth of these dangerous invasive species, we also cause the extermination of the preexisting healthy organisms. The resulting inhospitable area

becomes an impediment for both the invasive and native species[Fah02, DL09], shaping the landscape. There has already been research looking into humans affects on the landscape linked to increase in invasions [Hob00, Wit02]. However with the evolution of human society changes are constantly occurring that could influence dangerous invasive species. Additionally, with the increase in the human population of around 5000 million people in the last 65 years, there will be more cases of invasive species and more control required to produce enough crops for the population [CAP16].

1.3 Overview

Our plan is to explore management of invasive species using optimal control theory. In part one we will consider a single patch model with no dispersal.

In Chapter 2 we will introduce a basic model which takes into consideration an invasive pest lifecycle and applying a control, for instance a pesticide, a non-persistent short-lived biocontrol agent known as control agent. Furthermore we will prove existence, necessary conditions, and uniqueness for the optimal control. In Chapter 3, we consider what happens when the control persists longer than one time step. Again we will prove existence, necessary conditions, and uniqueness for the optimal control. In Chapter 4, we explore a case study investigating a specific invasive species, Diaprepes abbreviatus, DRW.

1.4 Reference Chart

	Notation	Description
	P_e	Number of eggs
Pest	P_l	Number of larva
Vector	P_p	Number of pupa
	P_a	Number of adult
	γ_1	Egg survival
	γ_2	Transition rate egg to larva
	θ_1	Fecundity rate of female adults
Pest	θ_2	Adult survival
Matrix	ζ_1	Larva survival
	ζ_2	Transition rate larva to pupa
	ν_1	Pupa survival
	ν_2	Transition rate pupa to adult
Initial	ϕ_e	Initial Proportion eggs
Pest	ϕ_l	Initial Proportion larva
Vector	ϕ_p	Initial Proportionpupa
	ϕ_a	Initial Proportionadults
Control	N	Number of control agents
	α	search efficiency/encounter rate of control
Cost	β_1	loss of harvest per square meter per time steo
Function	β_2	cost of control per square meter per time step
Nematodes Persist	μ	mortality/degradation of control agent

Chapter 2

Basic Model

2.1 Parameters

We denote pests by the vector P and control by the vector N. We consider a system where it is possible to apply control every time step, hence we establish a discrete-time model with constant time steps. The pest life cycles and dynamics, we used a 4×4 matrix, A, taking into account the pest eggs (P_e), larva (P_l), pupa (P_p), and adults (P_a). Note this can be generalized and applied to pests with a larger or smaller number of stages; additionally the matrix can characterize different pest stages. Let:

$$A = \begin{bmatrix} \gamma_1 & 0 & 0 & \theta_1 \\ \gamma_2 & \zeta_1 & 0 & 0 \\ 0 & \zeta_2 & \nu_1 & 0 \\ 0 & 0 & \nu_2 & \theta_2 \end{bmatrix}.$$

The control is applied only to the larva stage P_l, or the second stage of the pest stages. The control search/application efficiency is denoted by α, and accounts for how likely a control agent is to encounter a pest larva. Below is the formulation of the pest dynamics with the control included in the larva stage, where t is a time step.

$$\mathbf{P}(t+1) = A_l\mathbf{P}(t) \tag{2.1}$$

$$\begin{bmatrix} P_e(t+1) \\ P_l(t+1) \\ P_p(t+1) \\ P_a(t+1) \end{bmatrix} = \begin{bmatrix} \gamma_1 & 0 \cdot e^{-\alpha N(t)} & 0 & \theta_1 \\ \gamma_2 & \zeta_1 \cdot e^{-\alpha N(t)} & 0 & 0 \\ 0 & \zeta_2 \cdot e^{-\alpha N(t)} & \nu_1 & 0 \\ 0 & 0 \cdot e^{-\alpha N(t)} & \nu_2 & \theta_2 \end{bmatrix} \begin{bmatrix} P_e(t) \\ P_l(t) \\ P_p(t) \\ P_a(t) \end{bmatrix}$$

We denote initial values by

$$\begin{bmatrix} P_e(0) \\ P_l(0) \\ P_p(0) \\ P_a(0) \end{bmatrix} = \begin{bmatrix} \phi_e \\ \phi_l \\ \phi_p \\ \phi_a \end{bmatrix}.$$

2.2 Cost Function

We constructed the cost function by breaking it down into the control and pest components. Specifically, if we look at the cost incurred to an environment by an invasive pest there will be the loss of profit from the pest existing in the environment and the cost to purchase control to apply to the environment to deal with the pest.

Since destruction of the environment is catastrophic we expect a nonlinear term for the cost of pest damage. Specifically, when there is a low density of the pests, we expect the affected specie will not suffer large losses, but at a high density of pests the mortality rate becomes exponentially large. Furthermore, since we don't have a functional term we use the square which ensures mathematical uniqueness. For mathematical convenience we choose to model the cost of pest damage as $\beta_1 P_l(t)^2$.

The exponential increase of damage ensures that control will be applied at some point, which is a desirable feature in the cost function because it prevents plant death as a result of too high pest density.

In addition to the cost related to pest damage, we need to consider the cost of purchasing the control agent which is $\beta_2 N(t)$. So β_2 is the price of a single control unit. So the total cost is cost due to pest damage, $\beta_1 P_l(t)^2$, plus the cost of using control, $\beta_2 N(t)$,

$$Cost = \beta_1 P_l(t)^2 + \beta_2 N(t)$$

where β_1 and β_2 will be determined by the specific invasive species.

2.3 Optimal Control Problem

Realistically, there is going to be a maximum amount of control we can purchase and apply. We denote N_{max} as the maximum amount of control at any time step we can apply to the environment.

The set-up over our Optimal Control Problem is to minimize the objective functional for T time steps

$$J(N) = \sum_{t=0}^{T-1} \beta_1 P_l(t)^2 + \beta_2 N(t)$$

subject to

$$
\begin{aligned}
P_e(t+1) &= \gamma_1 P_e(t) + \theta_1 P_a(t) \qquad & P_e(0) = \phi_e \\
P_l(t+1) &= \gamma_2 P_e(t) + \zeta_1 e^{-\alpha N(t)} P_l(t) \qquad & P_l(0) = \phi_l \\
P_p(t+1) &= \zeta_2 e^{-\alpha N(t)} P_l(t) + \nu_1 P_p(t) \qquad & P_p(0) = \phi_p \\
P_a(t+1) &= \nu_2 P_p(t) + \theta_2 P_a(t) \qquad & P_a(0) = \phi_a
\end{aligned}
\tag{2.2}
$$

where $N(t) \geq 0$ for all t and $N \in \mathbf{N} = \{N : \{1, ..., T\} \rightarrow \{x \in \mathbb{R} | 0 \leq x(t) \leq N_{max}, t = 1, 2, ..., T\}\}$.

We will prove the existence and uniqueness of the optimal control, which we denote by $\mathcal{N}$. We will also prove necessary conditions for the optimal control $\mathcal{N}$. The proofs roughly follow the proofs in *Optimal Control of Gypsy Moth Populations* by Whittle, Lenhart, and White [WLW08]. The existence proof roughly follows from *Optimal Control in Discrete Pest Control Models* by Kathryn Dabbs[Dab10]. A useful source for proofs in Optimal Control theory is *Optimal Control Applied to Biological Models* by Lenhart, S. and Workman, J.T. [LW07].

Note in the following proofs each $\mathcal{P}_e, \mathcal{P}_l, \mathcal{P}_p, \mathcal{P}_a$ is a function of $\mathcal{N}$. Similarly each $\mathcal{P}_e^\varepsilon, \mathcal{P}_l^\varepsilon, \mathcal{P}_p^\varepsilon, \mathcal{P}_a^\varepsilon$ is a function of $\mathcal{N} + \eta\varepsilon$.

2.3.1 Existence

Theorem 2.3.1. *There exists $\mathcal{N} \in \boldsymbol{N}$ which minimizes $J(N)$.*

Proof. Each P_e, P_l, P_p, P_a is continuous as a function of N at every time step by Equation 2.2. Define $B^+ = \{(N(1), ...N(T))|N \in \mathbf{N}\}$. We note that there is a natural isomorphism between $\mathbf{N}$ and B^+. Considering $J : \mathbf{N} \to B^+ \to \mathbb{R}$, we see that J is continuous as a function of N. We have that B^+ is a compact subset of $\mathbb{R}^T$ in the standard Euclidean topology. Thus, $\inf\limits_{N \in \mathbf{N}} J(N)$ exists. Hence, we have a sequence $N_k \in \mathbf{N}$ such that $\lim\limits_{k\to\infty} J(N_k) = \inf\limits_{N\in\mathbf{N}} J(N)$, with corresponding $P_{e_k}, P_{l_k}, P_{p_k}, P_{a_k}$ sequences. Thus we can find subsequences $N_{k_j}, P_{e_{k_j}}, P_{l_{k_j}}, P_{p_{k_j}}, P_{a_{k_j}}$, such that $\lim\limits_{j\to\infty} J(N_{k_j}) = \inf\limits_{N\in\mathbf{N}} J(N)$, $N_{k_j} \to \mathcal{N}, P_{e_{k_j}} \to \mathcal{P}_e, P_{l_{k_j}} \to \mathcal{P}_l, P_{p_{k_j}} \to \mathcal{P}_p, P_{a_{k_j}} \to \mathcal{P}_a$. Therefore, there exists $\mathcal{N} \in \mathbf{N}$ which minimizes $J(N)$.

□

2.3.2 Necessary Conditions

Adjoint System: Define the following terminal value system, called an adjoint system:

$$\lambda_e(t) = \lambda_e(t+1)\gamma_1 + \lambda_l(t+1)\gamma_2$$

$$\lambda_l(t) = 2\beta_1\mathcal{P}_l(t) + \lambda_l(t+1)\zeta_1 e^{-\alpha\mathcal{N}(t)} + \lambda_p(t+1)\zeta_2 e^{-\alpha\mathcal{N}(t)}$$

$$\lambda_p(t) = \lambda_p(t+1)\nu_1 + \lambda_a(t+1)\nu_2$$

$$\lambda_a(t) = \lambda_e(t+1)\theta_1 + \lambda_a(t+1)\theta_2$$

$$\lambda_e(T) = 0,\ \lambda_l(T) = 0,\ \lambda_p(T) = 0,\ \lambda_a(T) = 0.$$

These adjoints, λ, are useful in establishing the formulas and necessary conditions for the optimal control. Additionally adjoints are effective for computational purposes, specifically the forward backward sweep discussed later. Note the adjoints are constructed by

$$\lambda_e(t) = [\beta_1 \mathcal{P}_l(t)^2 + \beta_2 \mathcal{N}(t)]_{P_e} + \mathcal{P}_e(t)_{P_e}\lambda_e(t+1) + \mathcal{P}_l(t)_{P_e}\lambda_l(t+1) + \mathcal{P}_p(t)_{P_e}\lambda_p(t+1)$$

$$+\mathcal{P}_a(t)_{P_e}\lambda_a(t+1),$$

similar construction follows for the other adjoints. The adjoints were formulated by Pontryagin and colleagues, the adjoints variables preform a function similar to that of Lagrange multipliers.[LW07]

Theorem 2.3.2. *If there exists an optimal control $\mathcal{N}$, then there exists an adjoint system 2.3.2 and*

$$\mathcal{N}(t) = \begin{cases} 0 & if\ \frac{\beta 2}{\alpha} > \xi(t) \\ \frac{1}{\alpha}\ln[\frac{\alpha}{\beta_2}\xi(t)] & if\ \frac{\beta 2}{\alpha} \leq \xi(t) \end{cases}$$

where $\xi(t) = \zeta_1 \lambda_l(t+1)\mathcal{P}_l(t) + \zeta_2 \lambda_p(t+1)\mathcal{P}_l(t)$.

Proof. Since we have that $\mathcal{N}$ minimizes $J(N)$; for all sufficiently small $\varepsilon > 0$ and for all $\eta \in \{\eta = (\eta(1), ..., \eta(T)) | \eta(t) \leq 1, t = 1, ..., T\}$ we have that $J(\mathcal{N} + \eta\varepsilon) \geq J(\mathcal{N})$. To determine the structure of the control consider directional derivatives of the cost J, we will take a directional derivative of functional J; for the directional derivative in the direction of η with $\varepsilon > 0$ sufficiently small and $0 \leq \mathcal{N} + \eta\varepsilon = \mathcal{N}^\varepsilon \in \mathbf{N}$ we have that:

$$0 \leq \lim_{\varepsilon \to 0^+} \frac{1}{\varepsilon}[J(\mathcal{N} + \eta\varepsilon) - J(\mathcal{N})]$$