

RELATIONSHIP BETWEEN COGNITIVE TYPES OF TEACHER CONTENT
KNOWLEDGE AND KNOWING-TO ACT: A MIXED METHODS STUDY OF
MEXICAN BORDERLAND MIDDLE SCHOOL TEACHERS

MARIA DE LOS ANGELES CRUZ QUINONES

Department of Teacher Education

APPROVED:

Mourat Tchoshanov, Ph.D., Chair

Arturo Olivarez, Ph.D.

Olga Kosheleva, Ph.D.

Ana Huerta-Macias, Ph.D.

Lawrence M. Lesser, Ph.D.

Charles Ambler, Ph.D.
Dean of the Graduate School

Copyright ©

by

Maria de los Angeles Cruz Quinones

2014

Dedication

I dedicate my dissertation to my father who supported me throughout my journey of studies for the PhD and who recently passed away. He taught me to be a hard worker and to never give up.

This dissertation is also lovingly dedicated to my family. A special feeling of gratitude for my loving mother, Maria Dolores, whose words of encouragement and push for tenacity rings in my ears. I dedicate this work and give special thanks to my husband Luis Armando for his understanding and support of this accomplishment. Finally, I am grateful for the unending support from my sisters Reyna, Rosario and Dolores who never left my side and are very special.

PREVIEW

PREVIEW

RELATIONSHIP BETWEEN COGNITIVE TYPES OF TEACHER CONTENT
KNOWLEDGE AND KNOWING-TO ACT: A MIXED METHODS STUDY OF
MEXICAN BORDERLAND MIDDLE SCHOOL TEACHERS

by

MARIA DE LOS ANGELES CRUZ QUINONES, B.S., M.Ed.

DISSERTATION

Presented to the Faculty of the Graduate School of
The University of Texas at El Paso
in Partial Fulfillment
of the Requirements
for the Degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education
THE UNIVERSITY OF TEXAS AT EL PASO

December 2014

UMI Number: 3682455

All rights reserved

INFORMATION TO ALL USERS

The quality of this reproduction is dependent upon the quality of the copy submitted.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if material had to be removed, a note will indicate the deletion.



UMI 3682455

Published by ProQuest LLC (2015). Copyright in the Dissertation held by the Author.

Microform Edition © ProQuest LLC.

All rights reserved. This work is protected against unauthorized copying under Title 17, United States Code



ProQuest LLC.
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106 - 1346

Acknowledgements

I would like to gratefully and sincerely thank my Chair, Dr. Mourat Tchoshanov, for his guidance, understanding, and patience during my graduate studies at the University of Texas at El Paso. I would also like to thank Dr. Ana Macias for her assistance, mentorship, and most importantly, for her friendship. She always was there with the best disposition to help me to accomplish this important milestone in my life. I would like to thank all the individuals who became involved in the process that I went through in order to achieve this accomplishment.

I would like to thank the participants of my study because this study would not have been possible without their participation. I would also like to thank to Ricardo Vizcarra and the rest of the teachers who helped me to verify the translation of one of the instruments used in the study.

Finally, and most importantly, I would like to thank my loving family for their tolerance, support, understanding, and unending encouragement for the completion of my PhD studies.

Abstract

This study analyzed middle school mathematics teachers' content knowledge and its relationship with teachers' "knowing-to act" ability. Understanding what kinds of knowledge has a direct influence on teaching practices and student learning is critical in order to improve teacher education programs and professional development. An Explanatory sequential mixed methods design was used in the study. It involved collecting quantitative data and explaining the quantitative results with in-depth qualitative data. In the quantitative phase of the study, two surveys were administered to N=70 middle school mathematics teachers in the Mexican borderland to assess whether their mathematical content knowledge was related to their "knowing-to act". The correlational analysis of these surveys showed no statistically significant correlation between overall mathematical teacher content knowledge (total score on TCKS) and the "knowing-to act" ability (KtAS). However, a statistically significant correlation between the specific cognitive type of teacher knowledge - models and generalizations - and the "knowing-to act" was reported. The qualitative phase provided a deeper understanding of the quantitative results: the exploration of the "knowing-to act" enacted during mathematics instruction with four middle school mathematics teachers from the quantitative sample was conducted using a specifically designed classroom observation protocol. The analysis of the observation together with the results of the KtAS provided revealing differences among teacher's actions observed and the teacher's responses on the survey. Overall, the analysis of the qualitative data reflected findings from the quantitative phase of the study. Two main findings were reported in the study: (a) the lack of correlation between the mathematical teachers content knowledge and their "knowing-to act" during teaching mathematics, which was reflected by the data collected from the case studies; (b) a statistically significant correlation between knowledge of models and generalizations (T3), which added to the discussion that teachers who performed higher on the cognitive type 3 items of the TCKS were able to know how to act at the moment more frequently than teachers with a limited T3.

This research provided in-service teachers and other participants in the education field with awareness about the active knowledge that is needed to enact the teachers' knowing-to act in teacher preparation programs in Mexico that can be used to support teachers and students in the United States. Further studies are needed in which the association and exploration of other kinds of knowledge for teaching mathematics and students learning can be analyzed. For instance, research on "knowing-to act" in the United States or other countries can also be worthy of a study; how would teachers act in KtA situations during their mathematics instruction in the USA, Canada, or Russia? In addition, this study allows comparisons among Mexico and countries where data is already collected in regards to teacher knowledge in the area of Mathematics, such as Russia, the U.S., Latin American countries, and other countries that participated in the TEDS-M Study 2012.

Table of Contents

Acknowledgements.....	v
Abstract.....	vi
Table of Contents.....	viii
List of Tables	xii
List of Figures.....	xiv
List of Illustrations.....	xv
Chapter 1: Introduction.....	1
1.1 Overview.....	1
1.2 Research Problem	1
1.3 Purpose of the Study.....	4
1.4 Conceptual Framework.....	7
1.5 Philosophical Assumptions.....	8
1.6 Research Questions.....	10
1.7 Significance of the Study.....	10
1.8 The Role of the Researcher.....	11
1.9 Definition of Terms.....	13
1.10 Organization of the Study.....	15
1.11 Summary.....	15
Chapter 2: Literature Review and Conceptual Framework	17
2.1 Overview.....	17
2.2 Literature Review.....	17
2.3 Historical Antecedents of Contemporary Public Education in Mexico.....	17
2.4 Current Situation and New Reform	19
2.5 Educational System in Mexico	21
2.6 Middle Schools in Mexico.....	23
2.7 Mathematics Curriculum in Middle Schools in Mexico.....	26
2.8 Mathematics Teacher Preparation at the Middle School Level	27
2.9 Middle School Mathematics Teacher Preparation Curriculum	29

2.10 Educational Research in Mexico	32
2.11 Conceptual Framework	34
2.12 Shulman Model	34
2.13 Domains of Mathematical Knowledge for Teaching	36
2.14 Content Knowledge	39
2.15 Pedagogical Content Knowledge	42
2.16 Pedagogical knowledge	44
2.17 Knowledge of learners	44
2.18 Relationship between Teacher knowledge and Beliefs	45
2.19 Technological Pedagogical Content Knowledge	47
2.20 Knowing-to and Knowing-about: Knowing-that, Knowing-how, Knowing-why	48
2.21 Conclusions	52
2.22 Summary	53
Chapter 3: Methodology	54
3.1 Overview	54
3.2 Mixed Methods Purpose Statement	54
3.3 Research Questions	55
3.4 Research Design	56
3.5 Context of the Study	59
3.6 Sampling	61
3.7 Quantitative Phase	62
3.8 Interim Phase	69
3.9 Qualitative Phase	72
3.10 Integration Phase	80
3.11 Discussion on Language	80
3.11 Trustworthiness	82
3.12 Ethics	83
3.13 Validity	85
3.14 Summary	85
Chapter 4: Results and Findings	87
4.1 Overview	87

4.2	Research Question 1	88
4.3	Research Question 2	94
4.4	Research Question 3	136
4.5	Research Question 4	170
4.5	Summary	171
Chapter 5: Discussion and Conclusion		174
5.1	Overview	174
5.2	Discussion of Findings for Research Question 1	174
5.3	Discussion of Findings for Research Question 2	178
5.4	Discussion of Findings for Research Question 3	182
5.5	Discussion of Findings for Research Question 4	184
5.6	Conclusion	187
5.7	Implications	193
5.8	Contribution to the field of mathematics education research	194
5.9	Recommendations for Policy and/or practice	195
5.10	Future Research Directions	196
5.11	Limitations	198

References	202
Appendix A: Omar's case study	209
Appendix B: Rogelio's case study	222
Appendix C: Maria's Case Study	241
Appendix D: IRB Proposal	260
Appendix E: Knowing-to Act Survey (KtAS)	266
Appendix F: Teacher Knowledge Survey	270
Appendix G: Teacher Content Knowledge Survey	272
Appendix H: Interviews' Transcription	289
Appendix I: Field notes Expanded Observations.....	327
Appendix: J Observation Protocol	369
Vita	372

List of Tables

Table 2.1: Middle School Enrollment since 1925.	26
Table 2.2: Middle School Mathematics Curriculum in Mexico and USA: Comparison Chart.	27
Table 2.3: Normal Superior Institution Degree's Plan (DGESPE, 2010).	30
Table 3.1: Demographic Information of Participant Middle Schools (SNIEE, 2010).	60
Table 3.2: Criteria for Case Selection Based on the Quantitative Phase.	70
Table 3.3: Qualitative Data Sources.	79
Table 4.1: Review of Research Questions, Methods and Data Sources.	87
Table 4.2: Raw Scores of the Outlier Participants Removed.	90
Table 4.2: The Quartiles by Survey.	95
Table 4.3: Observations Table.	100
Table 4.4: Knowing-to act Situation Observed in Rosa's Classes.	104
Table 4.5: Frequency Table of the Situations in the Category <i>Student Misconceptions</i>	105
Table 4.6: Frequency Table of the Situations in the Category <i>Student Difficulties</i>	106
Table 4.7: <i>Emerging Situations</i> During Rosa's Classes.	106
Table 4.8: Frequency Table of the KtA Situations in the Category of <i>Emerging Situations</i>	107
Table 4.9: Teacher's Action Codes Item 1 KtAS.	109
Table 4.10: The frequency Table of Rosa's Actions in Item 1 on KtAS.	110
Table 4.11: Correspondence of the Triangulation Between the KtAS Survey and Classroom Observations in Item 1.	112
Table 4.12: Teacher's Action Codes Item 3 KtAS.	113
Table 4.13: Correspondence of the Triangulation Between the KtAS Surve and Classroom Observations for Item 3.	114
Table 4.14: Teacher's Actions Codes Item 9 KtAS.	116
Table 4.15: The Frequency of Rosa's Actions in the Item 9 KtAS.	117
Table 4.16: Correspondence of the Triangulation Between the KtAS Surveyand Classroom Observations for Item 9.	117
Table 4.17: Correspondence of the Triangulation Between the KtAS Survey and Classroom Observations in Item 2.	120
Table 4.18: Teacher's Action Codes under Item 10 of the KtAS.	122
Table 4.19: Frequency Table of Teacher's Actions in Item 10 of the KtAS.	124
Table 4.20: Correspondence of the Triangulation Between the KtAS Survey and Classroom Observations in Item 10.	124
Table 4.21: Teacher's action Codes in <i>Emerging Situations</i>	127
Table 4.22: Knowing-to Act Situations Observed in Omar's Classes.	128
Table 4.23: Frequency Table of the KtA Situations in Omar's Classes.	130
Table 4.24: Knowing-to act Situations Observed in Rogelio's Classes.	131
Table 4.25: Frequency Table of the KtA Situations in Rogelio Classes.	132
Table 4.26: Knowing-to Act Situations Observed in Maria's Classes.	134
Table 4.27: Frequency Table of the KtA Situations Identified in Maria's Classes.	135
Table 4.28: Reason Codes for the Differences in Rosa's Responses.	144
Table 4.29: Themes and Codes Assigned to Rosa's Reasons.	147
Table 4.30: Reason Codes for the differences in Omar's responses.	152
Table 4.31: Themes and Codes for Omar's Reasons.	154
Table 4.32: Reason Codes for the Differences in Rogelio's Responses.	158

Table 4.33: Themes and Codes of Rogelio's Reasons.....	160
Table 4.34: Reason Codes for the Differences in Maria's Responses.....	163
Table 4.35: Themes and Codes for Maria's Reasons.	170
Table 5.1: Actions Most Frequently Enacted.	179
Table 5.2: Number of Actions and Situations Observed.	181
Table 5.3: All Themes and Codes of Teachers' Reasons for Differences in their Responses and Actions.....	183
Table 5.4: Raw Scores and Percentages of the TCKS, T1, T2, T3, and KtAS.....	187

PREVIEW

List of Figures

Figure 1.1: Interaction of Knowing-to and Knowing-about (Mason & Spence, 1999, p. 145).....	8
Figure 2.1: Educational System in Mexico.....	22
Figure 2.2: Domains of Mathematical Knowledge for Teaching (Ball et al., 2008, p. 403).	38
Figure 2.3: Theoretical Framework for the Study on Cognitive Types of Teacher Content Knowledge (Tchoshanov, 2011, p.146).....	40
Figure 2.4: The Network of Pedagogical Content Knowledge (An et al., 2004, p.147)	43
Figure 2.5: Teachers' Knowledge: Developing in Context (Fennema et al., 1992, p. 162).	46
Figure 2.6: TPACK Framework and its Knowledge Categories (Koehler et al., 2009).	47
Figure 3.1: A Mixed Methods Sequential Explanatory Design QUAN--->QUAL	59
Figure 4.1: TCKS Mean Score Percentages of Results by Cognitive Type.	93
Figure 4.2: Pathways for Analysis of the Classroom Observations.	102
Figure 4.3: Triangulation Process of the KtAS Survey and the Classroom Observations.	111
Figure 4.4: Individual Interview	139

List of Illustrations

Illustration 3.1: Examples of TCKS Items.....	66
Illustration 3.2: Example of KtAS Item and Ranking Format.	69
Illustration 4.1: KtAS Item Removed From Analysis.	89

PREVIEW

Chapter 1: Introduction

1.1 OVERVIEW

This study examined the mathematical teacher content knowledge of Mexican middle school teachers in Mexico. A second type of teacher knowledge, “knowing-to act”, was also analyzed in order to determine if any relationship existed between mathematical teacher content knowledge and “knowing-to act”. This study provided an understanding and description of this relationship. A mixed methods study was developed in which an explanatory sequential research design was used. During the quantitative phase, two surveys were administered, one that measured the mathematical teacher content knowledge, and the second one explored the teachers’ “knowing-to act”. Then, looking for a deeper understanding of this relationship, a qualitative phase was implemented. And interview and class observation data were collected in order to enrich the interpretation of information provided in the quantitative phase.

This chapter introduces several important aspects of this research. The rationale for conducting this study will be presented in the research problem section. In addition, the purpose of the study and the description of the specific research site will be provided. Also, reasons for the importance and relevance of conducting this study in Mexico at the middle school level will be discussed. An introduction to the conceptual framework and philosophical foundations that were used as lenses for conducting this study will be presented. Additionally, a definition of terms and necessary background information will be provided.

1.2 RESEARCH PROBLEM

Teacher knowledge is a critical topic for teaching, learning and culture. The process of teaching and learning is a decisive aspect to shape culture. Therefore, to be focused on components of this process such as teacher knowledge is relevant for the educational field. Since the knowledge possessed by teachers has an impact on students’ learning, it is an educational issue worthy of study. In the past twenty-five years, a growing number of studies focusing on teacher knowledge have been conducted (e.g. Shulman, 1986; Taylor, 2011; Tchoshanov, 2011).

However, teacher knowledge is very broad, and it includes different kinds of knowledge. The criticism and classification of the different types of knowledge that a teacher should possess in order to teach mathematics effectively become relevant for teachers, teacher education, scholars, and policy makers. These classifications and conceptualizations of teacher knowledge will allow pre-service and in-service teachers to become aware of the knowledge that they need to possess as the knowledge base for teaching mathematics. Based on research focusing on this worthy issue, policy makers and teacher education programs can make decisions about how teachers should be prepared in order to help students to learn mathematics.

In the field of mathematics education, scholars have addressed only some kinds of knowledge and their components, as it is discussed in chapter two (e.g., An, Kulm, & Wu, 2004; Davis, & Simmt, 2006; Tchoshanov, 2011). Some of the categorizations of teacher knowledge in mathematics are mathematical teacher content knowledge (e.g. Tchoshanov, 2011), pedagogical content knowledge (e.g. An et al., 2004), knowledge of curriculum (e.g. Ball, Thames, and Phelps, 2008; Shulman, 1986), and “knowing-to act” (e.g. Mason, 1998), among others. The complex nature of the mathematical knowledge for teaching challenges investigators to research and define with precision each kind of teacher knowledge. In addition, interactions among these kinds of knowledge are crucial as a part of the knowledge base for teaching mathematics and, as such, some scholars have recognized and studied it (e.g. An et al., 2004; Koehler and Mishra, 2009). Therefore, further research is needed about the nature of the interaction among specific kinds of teacher knowledge. Additionally, knowing what kinds of knowledge have a direct influence on teaching practices could be used to enhance teacher education programs and teaching practices. This study aimed to provide insights that will provide awareness to teacher education programs and policy makers to make important decisions in regards to what teachers need to know for teaching mathematics in an effective manner.

Based on the need for research about interactions among different kinds of mathematical teacher knowledge, two types of teacher knowledge were explored in this study: mathematical teacher content knowledge and “knowing-to act”. Mathematical teacher content knowledge is the

first type of knowledge that was studied. There is an assumption in the mathematics education field that to some extent, the higher the level of mathematics studied by teachers the higher pedagogical skills they develop (Sorto, Marshall, Luschei and Carnoy, 2009). Research on the area of teacher knowledge (e.g. Sorto et al., 2009) shows that this assumption is not always correct, and this study added to this discussion. Sorto and colleagues (2009) showed that mathematics teachers in Panama and Costa Rica at the seventh grade are proficient in the content, but they exhibited a deficient teaching of mathematics. As reported in the previous study, a weak mathematical teacher content knowledge primarily affects mathematical instruction, which can in turn cause students have poor opportunities to learn (Tchoshanov, 2011), and student frustration and a negative attitude towards mathematics (Sorto et al., 2009). Moreover, in mathematics it is very important that teachers be able to make connections among mathematical concepts in order to help students to make sense of mathematics.

The second type of knowledge that was studied is “knowing-to act”. “Knowing-to act” is the process in which “knowledge [that] enables people to act creatively rather than merely react to stimuli with trained or habituated behavior” (Mason and Spence, 1999, p.136). According to Mason and Spence (1999), there is an absence of “knowing-to act” that leads mathematics teachers not to be able to respond creatively in the moment even when they possess mathematical and pedagogical content knowledge. Consequently, this absence may limit the learning opportunities that teachers can offer. Research on this kind of knowing may help teachers develop the active knowledge needed to respond creatively in the moment.

Around the world, there is interest in researching teacher preparation programs to provide educational policy makers relevant information such as which types of teacher knowledge have a stronger impact on student learning (Tchoshanov, 2011). Several international studies focused on teacher preparation in countries such as Russia, USA, Singapore, Finland, etc. (e.g. TEDS-M Study, 2012). Research shows that most teacher preparation programs in the United States focus mainly on content. Fewer programs emphasize pedagogical knowledge, fewer still attempt to include content and pedagogy, but these programs have courses that focus primarily on one area

or another (Davis & Simmt, 2006). American teacher education programs often use a coarse distinction between mathematical knowledge and instructional knowledge. Mathematics courses are offered by the mathematics department, and the instructional courses are offered by education departments (Davis & Simmt, 2006; Sorto et al., 2009). In most American universities, teacher preparation programs have few courses that integrate the content and the pedagogy needed to teach that subject matter (Davis & Simmt, 2006). This problem was also pointed in Grossman (2008), he mentioned that teacher preparation programs do not provide enough preparation to teachers in order to be ready to teach in mathematics classrooms.

In Mexico, this phenomenon is also identified. For example, an analysis of the degree plan for mathematics middle school teacher preparation program (DGESPE, 2010) at “the normal school” (La normal superior), which is the main institution for teacher preparation in this country, shows that mathematics teacher preparation in Mexico fits with the patterns found in the United States’ teacher preparation programs (Davis & Simmt, 2006) as will be shown in the next chapter. It is critical for teachers to be prepared with content, pedagogy, and the blend of both content and pedagogy, among other important aspects for teaching mathematics (Shulman, 1987; Ball et al., 2008). Teacher preparation programs need to provide the support that teachers need in order to help their students to make sense of the topics, instead of only providing teachers with tools to present sequences of instructions.

1.3 PURPOSE OF THE STUDY

The intent of this study was to measure middle school mathematical teachers’ content knowledge and its relationship with teachers’ “knowing-to act” ability. An explanatory sequential mixed methods design was used, and it involved collecting quantitative data first and then explaining the quantitative results with in-depth qualitative data. In the quantitative phase of the study, two surveys were administered in order to collect data from 70 mathematics teachers in middle schools in the Mexican borderland to assess whether their mathematical content knowledge was related to their “knowing-to act”. The qualitative phase was conducted as a

follow up to the quantitative phase to help explain the quantitative results. In this explanatory follow-up, the plan was to explore the knowing-to enacted during mathematics instruction.

The research was conducted in a Mexican city located on the border with the United States. Since I am Mexican and I live in the Mexican borderlands, it placed me in a privileged condition to conduct research from the cross-cultural community perspectives that have shaped me. My education -from elementary school to a Master's degree- was earned in Mexican public schools and universities. Thus, I was familiar with the social context of public education in Mexico. My bachelor's degree is in computer science engineering and my Master is in mathematics education. Thus, my education focused on mathematics that allowed me to be a mathematics teacher in one University of that country. Therefore, my background enhanced my ability to do research in my own country.

Educational research in Mexico has been limited due to the lack of research funding (Reyes, 2013). In Mexico, there are 1804 educational researchers; this number is too small in comparison with the large Mexican educational system and the educational issues that have existed for several decades (The Organization for Economic Co-operation and Development-Center for Educational Research and Innovation, *Revisión Nacional de Investigación y Desarrollo Educativo, Reporte de los examinadores sobre México*, 2004). An attempt to compare the number of educational researchers in Mexico with the number of researchers in the area of education in the U.S. was done. However, information about it was not found.

Furthermore, the educational researcher preparation is very limited, only 14 PhD programs and 51 master's programs are recognized by the National advising board of graduate studies CONACYT in Mexico (CONACYT, 2013). The absence of institutional conditions that allow the conduct of rigorous research also contributes to the limited opportunities for conducting educational research in Mexico (Barriga, 1998). In addition, an existing gap of research at the elementary and middle school levels that analyzes specific mathematical knowledge and instructional skills required for teaching effectively has been identified (National Mathematics Advisory Panel, *Report of the Task Group on Teachers and Teacher Education*,

2008b). Of great relevance is to point out that in Mexico, research on the middle school level is critical because this level has had the greater dropout rate from the basic education for several years. Basic education in Mexico consists of preschool, elementary and middle school (Reimers, 2001; Blasco, 2003, Secretaría de Educación Pública, 2010).

The target population of this study is mathematics middle school Mexican teachers currently teaching in Mexico on the border with United States. A relevant feature of the participants that were part of the research is that they did not belong to any “minority group” such as the perceptions that in the United States people have towards Mexican people (Orellana, 2001). The participants are Mexican origin teachers who lived in Mexico. They did not see themselves as “minorities” as most Mexican people are often considered by the American society (Oropesa & Landale, 2009) and specifically in the educational research. Therefore, becoming aware of the socio-cultural context where these participants lived and taught was not the same context typically known by people and educational researchers in the United States is critical for the readers.

This study added to current knowledge that may help to restructure mathematics teacher education programs, and hopefully impact the teaching and learning process in Mexican middle schools. According to the Ministry of Public Education in Mexico (Secretaría de Educación Pública, 2004), teaching and learning mathematics in middle schools focus on three purposes: to develop mathematical skills, promote positive attitudes towards mathematics, and acquisition of mathematical knowledge. As it will be demonstrated in Chapter 2, few studies in Mexico focus on teacher content knowledge and its relationship with teaching practices at the middle school level. Based on these needs and the limitations of educational research in Mexico at the middle school level in regards of mathematics, doing research in Mexico is critical and essential. In addition, conducting research in Mexico will allow for comparison studies with other countries, such as the United States or Russia, where studies of teacher content knowledge have been conducted (e.g. Tchoshanov, 2011).

The broader educational purpose of this study focused on understanding how mathematical teacher content knowledge influences teachers' instructional decisions to act in particular ways. I used the quantitative component of this study to look critically at the interrelationship of the mathematical teacher's content knowledge and the teachers' "knowing-to act" at the moment of instruction. The qualitative component explored in depth the findings of the quantitative component, specifically the relationship between these two types of teacher knowledge, in order to provide an understanding and description of the teachers' "knowing-to act" influenced by the mathematical teacher content knowledge.

1.4 CONCEPTUAL FRAMEWORK

This paper analyzed the mathematical teacher content knowledge and teachers' "knowing-to act" (Mason and Spence, 1999) possessed by Mexican in-service teachers in classrooms at the middle school level in Mexico. This research drew on the Shulman's teacher knowledge model (Shulman, 1986, 1987). Shulman (1986) distinguished three categories of teacher knowledge: a) content knowledge; b) pedagogical content knowledge; and c) curricular knowledge. He defined content knowledge as "the amount of organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p.9). This category of knowledge includes knowledge of facts, connections, models, generalizations and how the understanding of this content knowledge is structured and generated. This study focused on this category of content knowledge, specifically on the mathematics content that teachers need for teaching effectively. Several scholars have focused on this category in mathematics (e.g. Ball, Thames, & Phelps, 2008; Rowland, 2008). However, few studies have examined the cognitive types of teacher content knowledge (e.g. Tchoshanov, Lesser, & Salazar, 2008) and its relationship with student achievement (e.g. Tchoshanov, 2011). This research examined the three different cognitive types of mathematical teacher content knowledge: cognitive type 1 (which refers to the teacher content knowledge of facts and procedure); cognitive type 2 (the knowledge of concepts and connections); and cognitive type 3 (knowledge of models and generalizations).