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DYNAMIC EFFECTS OF CLEARANCE

IN MECHANISMS

by

Upendra N. Sinha

A DISSERTATION

Presented to the Faculty of

The Graduate College in the University of Nebraska

In Partial Fulfillment of Requirements

For the Degree of Doctor of Philosophy

Department of Engineering Mechanics

Under the Supervision of Professor James C. Wolford

Lincoln, Nebraska

June, 1973

TITLE

Dynamic Effects of Clearance in Mechanisms

BY

Upendra Nath Sinha

APPROVED	DATE
James C. Wolford	June 12, 1973
G. M. Smith	June 12, 1973
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LIST OF FIGURES

NOMENCLATURE

$X_1, \dot{X}_1, \ddot{X}_1$	-	Displacement, Velocity, Acceleration of element 1
$X_2, \dot{X}_2, \ddot{X}_2$	-	Displacement, Velocity, Acceleration of element 2
B	-	Amplitude of harmonic displacement
B_1	-	F_o/K_2 static displacement of Harmonic forced motion
ω	-	Circular Frequency of harmonic forcing function
t	-	time in seconds
K	-	Stiffness of linear spring to replace compliance
K_2	-	Stiffness of external spring
C	-	Viscous damping coefficient at the compliance
C_d	-	Viscous damping coefficient
r	-	Clearance
ε	-	Amplitude of relative input motion
G	-	Gain of non-linear element, Clearance
ϕ	-	Phase shift of non-linear element, Clearance
M_1	-	Mass of element 1
M_2	-	Mass of element 2
ω	-	Resonant Frequency of impact pair
ζ	-	Non-dimensional damping ratio
ω_r	=	ω/ω_n
F_o	-	Amplitude of forcing function
λ	=	C/C_d
R_1	=	M_2/M_1
μ	=	K/K_2

ω_n^2	$= K_2/M_2$ for Model No. 2
ℓ	- general link
i	- No. of input link
ϕ_ℓ	- Angle between positive X-axis and link +ve CCW.
k	- Counter for input system position
$\phi_{i\ell}$	= Angle of input link i in position k
$\omega_{\ell k}$	- Angular velocity of link ℓ in position k
$\alpha_{\ell k}$	- Angular acceleration of link ℓ in position k
$g_{\ell k}$	- Angular velocity of link ℓ for a unit angular input velocity
	Angular acceleration of link ℓ for a unit angular input acceleration
$h_{\ell k}$	- Angular acceleration of link ℓ for a unit angular input velocity
$\bar{I}_\ell$	- Moment of Inertia of link ℓ about its centroid
$KE_{\ell k}$	- Kinetic Energy of link ℓ in position k
M	- Mass of link located at its centroid G
$V_{g\ell,k}$	- Linear velocity of centroid G of link ℓ in position k
$I_{\ell k}$	- Equivalent inertia of link ℓ located at input link in position k
a_ℓ	- length of link ℓ
$W(\ell)$	- Weight of link ℓ
CG1	- length of C.G from pivot of rotation of link 1
CG3	- length of C.G from pivot of rotation of link 3

ρ - length of C.G from pivot A on Crank and Rocker Mechanism

ρ - length of C.G from pivot B on Slider Crank Mechanism

$E(\rho, \gamma)$ - Polar coordinates of a general point in the coupler link

Clearance ratio = Clearance/Amplitude of forced Displacement

Clearance ratio = Clearance/Static Amplitude of forced motion

DYNAMIC EFFECTS OF CLEARANCE IN MECHANISMS

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University of Nebraska, 1973
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ABSTRACT:

Two models are studied to get a basic understanding of the dynamic force amplification due to clearance in mechanism joints. The describing function method used in control theory and applied by Freudenstein to the problem of mechanism clearance is shown to give a simple relationship between force amplification and parameters of the system such as clearance ratio, resonant frequency, and forcing frequency of the system. The system differential equations are also solved numerically and the results obtained by the numerical method correlate well with the relationship obtained with the describing function technique with two exceptions. The numerical analysis shows that the damping in the system has an effect on force amplification, which was found negligible when using control theory; also the force amplification factor minus 1 was found to be inversely proportional to a power of the forcing frequency which is greater than one but considerably less than two as determined by Freudenstein.

To permit a better understanding of the results, one clearance connection in a system is considered. However the methods used are applicable to some systems containing more than one clearance.

The idea of model analysis is extended to an offset slider crank and a crank and rocker mechanism with clearance at both the joints. Equivalent clearance at the joint under study is used to find the force amplification due to clearance.

CHAPTER I

INTRODUCTION

Clearances in mechanical joints and connections are very common and often inevitable. Sometimes clearances are provided in a mechanism to permit assembly of the parts made in mass production. For interchangeable assembly, the pins and bearings are made within certain tolerances and it may happen that the smallest pin is assembled in the largest hole causing the total clearance to be considerably larger than the nominal clearance the designer has recommended. As will be shown later in this study, large clearances cause an increased dynamic load and a mechanism having large clearances will deteriorate much faster than a mechanism with small initial clearances.

Backlash or clearance in machinery is essentially a cost problem. If the performance warrants a very small clearance, selective assembly is the solution. For example, coupling bolts for large rotating machines like steam-turbines and turbo-generators are individually fitted to avoid amplification of high torques transmitted through the coupling.

The cost of improved dynamic performance can be assessed and compared with costs due to breakage, wear, service failures, noise, and decreased customer acceptance.

Clearance or backlash implies a freedom between two sections of a system such that the driven section can freely move some amount forward from the drive position before encountering a "stop". If, under idle conditions, the driven part of the system is partly moved forward from the drive position, the sudden application of an external force to the driven section will cause it to slow down without immediate restraint from the remainder of the system. When contact between the two parts of the system takes place, the difference in velocity between two sections can cause excessive shock forces or torques and vibration in the system before the two parts are brought together in speed.

The potential sources of backlash in a power transmission system are gears and couplings. Loose fits and wear in these components can provide dangerous sources of shock loading on the system.

When clearance is present in a mechanism, the motion, even in the case of small oscillations, is generally accompanied by impact between the elements of the kinematic couple due to the interruptions in the kinematic chain.

Thus, the problem of dynamic analysis become essentially

non-linear in character and linear equations are not applicable in describing the motion as a whole but apply only for seperate intervals between collisions.

LITERATURE SURVEY

The analysis of the effect of clearances on the accuracy of the output motions of mechanisms and of the dynamic effects of clearances has been the subject of a number of investigations.

Garret and Hall [1] developed equations which define a mobility band for linkages. This mobility band defines the output motion error due to clearances in the joints and tolerances in the link lengths. Garret and Hall also considered the problem statistically.

Mabie and Corderman [2] presented charts which could be used to design a linkage with an output within specified accuracy limits. Several combinations of tolerances, bearing clearances, and link-length ratios were considered.

Lakshminarayana and Narayanamurthi [3] considered individual loop closure equations rather than the input-output relationship of the complete linkage to determine the influence of tolerances, and they suggest their method as a convenient procedure for complex linkages.

The dynamic effects of clearances in the mechanical chains of control systems was studied by Tustin [4] .

Numbers in parentheses refer to the Bibliography at the end of the dissertation.

He built a model of motor driving a load through backlash. In this model the displacement of the motor produces equal displacement of the load, but only after taking up a definite clearance in the direction of drive. The fundamental motion of the load is determined when a simple harmonic force, F cosωt, is applied to the motor. The assumption made in the analysis is that the motion of the center of mass is unaffected by mutual forces and collision between the two masses. Motion with periodic impact is considered, but only on the assumption that the coefficient of restitution of the velocity on impact is zero, and that after impact both masses move together for a certain time after collision. A graphical method of solution was developed.

Satyendra [5] presented an extension of Tustin's work and employed an analytical rather than a graphical method of harmonic analysis. The system he used consisted of a mechanical linkage in which the load member was subjected to a restraining force. Describing functions were deduced analytically that represent the load response for finite values of inertia, clearance, and Coulomb friction between the load member and the external surface.

Freeman [6] and Thomas [7] investigated the effect of speed dependent friction on the stability and limit cycle phenomenon using the describing function technique.

Some studies made for a vibro-impact mechanism system

are worth mentioning here although they were applied to the dynamics of vibration dampers based on impact action. Feigin [8] studied the dynamics of a two mass system coupled through a clearance. The motion of the masses is accompanied by intermittent impact under the action of an external harmonic force or a periodic sequence of impulses. Partial elastic impact takes place at the stops so that the reflected velocity is a given fraction of the impact velocity. Mapping in the phase plane is explained. The technique used is one advocated by Besapalova [9] in his study of the stability of vibro-impact mechanisms and is called the point transformation technique. It is shown that periodic motion of the system is possible only with a period which is a multiple of the period of the exciting force. The behavior of the system is characterized by two parameters, mamely, the impact velocity ratio and a non-dimensional measure of clearance. It is shown that a steady-state condition with one impact per half period exists when the period is an odd multiple of the exciting force period. When the impact of masses is entirely inelastic, an arbitrary number of complex steady-state conditions exist in which sliding motion is included.

Masri and Gaughey [10] studied the stability of a mass constrained to oscillate with clearance in a container having mass. Analytical and experimental studies of an

impact damper were made in which periodic (2 impacts/cycle) solutions were obtained and stability boundaries were determined. It was assumed that the time of impact was small in comparison with the period of motion. The elasticity of the compliance was considered and was replaced with a very hard spring. To simulate different coefficients of restitution on impact, a viscous damping was provided. The results of experiments and analog computer solutions were presented.

Barkan and Touhy [11] analyzed the behavior of four-bar linkages when subjected to external impact. They used the energy balance criterion to find the effective moment of inertia for the entire linkage and the equation of motion of a quasi-rigid four-bar linkage was developed. A simple, but limited analytical solution of the problem was presented.

Artobolevskii [12] reduced all forces and masses to a crank rotating about a fixed axis, and introduced the concept of a characteristic criterion. This criterion is the ratio of what he defines as the instantaneous power of the Jerk to the instantaneous power of the kinetic energy, in the position of the linkage considered. The larger the value of the criterion, the more important is the role of Jerk in the machine and the greater are the additional dynamic loads on the links and the kinematic pairs of the machine. Detailed expressions for the characteristic cri-

terion under different assigned parameters are presented.

Goodman [13] studied the dynamic effects of backlash in a film transport mechanism and an oscillating mixer drive. He formulated an equivalent dynamic system, or box-car diagram as he called it. All masses, spring gradients, coefficients, forces, torques, displacements, and backlash are referred to the one point of most interest in the mechanism. The experimental results of two models are presented to show that the analysis gives a qualitative description of the effects of backlash. This was an intermediate step towards setting-up the piecewise-continuous equation of motion, which can be solved by analog or digital computer.

Vorob'eva and Suzdal'tsev [14] deduced a simple computation formula for the determination of the approximate value of the shock forces and the permissible clearances in the kinematic couples of mechanisms. The dependence of the impact force on various parameters is shown. The impact force S for a mechanism scheme with n+1 masses connected in series and an elastic connection of the last component with rigidity C is determined by the approximate dependence:

$$S = (1+\varepsilon)\left[2PC\mu(l_1 + l_2 + l_3 \cdots + l_n)\right]^{\frac{1}{2}}$$

Where: P is the motive force,

ε is the coefficient of mass,

C is the rigidity of the elastic connection of the last component,

$l_1+l_2+l_3+\ldots+l_n$ is the sum of clearances,

μ is the coefficient of restoration under shock.

The proposed computation formula is recommended to designers when it is necessary to obtain approximate values of the impact forces or their dependence on the parameters of the mechanism scheme.

Kozhevnikov and Leninskii [15] studied the effect of clearance on the dynamics of a heavily loaded machine, a rolling mill. This machine is characterized by a cyclic loading due to insertion and withdrawal of the sheet being rolled. The dynamic analysis is effected in two stages, first for an idealized scheme without any clearance, and then with clearance. The rates of the impact of the elements in the system, arising in the moving or sliding joints, are determined. Results of experiments carried out with an actual rolling mill are presented.

A similar study was done by Rieger and Smally [16]. A computer method was developed for calculating transient torque values as a function of time following the shock load application. A term "torque amplification factor" is defined as the ratio of peak transient torque at the point with clearance to steady torque without clearance. Numerical

results of torque amplification were presented with varying degrees of backlash or clearance. The effect of clearance at different points in the complex geared system were presented. Test results obtained under actual rolling conditions were compared with the numerically calculated results.

Yudin and Sergeyev [17,18] studied the dynamic effects of clearance in a slider crank mechanism with the clearance at the crank. The condition under which the kinematic chain is unlocked are determined. Different variants for the position of the connecting rod and the point of contact are illustrated. Their computer solution revealed a substantial amplification of reaction force at the clearance joint. The reaction force reaches a maximum value at those positions where the position error of the mechanism and the angular velocity of the crankpin motion around the surface of the crank bearing are a maximum in absolute terms. The minimum value of the reaction force occurs when the position error goes through zero.

Takaki [19] studied the dynamic effects of higher-order harmonics caused by nonlinearity of the bearing clearances of a vehicle engine. An idealized five-bearing crankshaft and a vibration model of the five-bearing cylinder block were used in the study. A Fourier analysis was made of the forces on the bearing portion and also of the vertical acceleration of the cylinder block. The results indicated that the force tramitted includes a great propor-

tion of higher order terms. By decreasing the clearance the effect of third, fourth and sixth order harmonics are reduced. The effects of other parameters like flywheel mass, engine rpm, gas pressure, and first order inertia forces were also discussed.

Kobrinskii [20] introduced a mechanical model of a two-mass system with clearance at the joints and studied the analytical solution of the equation of motion. Impact was considered to be rigid body at various coeffients of restitution. The stability analysis was based on perturbation technique.

Dubowsky and Freudenstein [21,22] studied Kobrinskii's model but included the surface compliance and allowed unilateral as well as bilateral impact on the surfaces. This results in dynamic coupling, stress, and time duration of contact. The latter affects the frequency of oscillation and the nature of impact. The dynamic response was found analytically and numerically. A technique like the describing function method used in control theory was applied to study the stability and the limit cycle criterion. The model analysis was further extended to a slider crank mechanism with clearance at the crankpin. A loop equation of equilibrium was developed to find the reaction forces at the joint.

This short list by no means exhausts the collection of works which discuss the effect of clearance on the be-