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BEAMS UNDER LOAD IN PLANE OF CURVATURE.

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BEHAVIOR OF CURVED REINFORCED CONCRETE BEAMS

UNDER LOAD IN PLANE OF CURVATURE

By

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A THESIS

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ABSTRACT

Preliminary laboratory investigations were made to determine experimental techniques for predicting the flexural behavior of short radius reinforced concrete beams throughout the entire load range to failure. Twenty-four curved beams were fabricated with variations in the radius, amount of steel and the placement of the steel. During the loading of these specimens strains on the concrete surface and on the steel were monitored.

For those curved beams with steel near the extrados, a basic mechanics approach is developed for analyzing the experimental data obtained. A sample analysis is presented and the results of this and other beam analyses are discussed.

For those beams with steel near the intrados the expected concrete tension failure was confirmed. A theoretical development for determining radial reinforcement and its spacing is developed but no experimental verification was attempted.

The problems, with the loading system, that arose during the experimental investigation are discussed along with the modifications that were made. It is concluded that in order to obtain more significant results, further refinements in the loading system should be made. Means of obtaining these refinements are suggested in the conclusions.

General theoretical expressions for internal forces and moments are also developed. These expressions are based on using a second degree parabola to describe the stress-strain conditions on a cross section of the curved beam in flexure. Use of the expressions is discussed and the limitations presented.

TITLE

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William Peter Ilgen, Jr.

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NOTATION

A = area of concrete compression block

A_s = area of tension steel, square inches

A_{sb} = area of tension steel for balanced design, square inches

A_{sr} = area of steel reinforcement in radial direction, square inches

b = breadth of beam, inches

b' = effective breadth of beam, inches

C = concrete compression force, pounds

d = distance from outermost compression fiber to centroid of tension reinforcement, inches

ε_i = unit strain in concrete at intrados

ε_c = unit strain in concrete within the compression zone

ε_o = unit strain in concrete at maximum stress

ε_s = unit strain in tension steel

ε_u = unit strain in concrete at ultimate

ε_{yp} = unit strain in tension steel at yield

E_s = modulus of elasticity of steel

h = total depth of section, inches

k = ratio of the concrete deformation at the intrados to the sum of concrete deformation at the intrados and the steel deformation

k' = coefficient related to magnitude and position of internal compression force in concrete

M = bending moment, inch-pounds

P_r = machine load transmitted to right portion of beam, pounds

P_l = machine load transmitted to left portion of beam, pounds

R = radius to fiber location within section, inches

$\overline{R}$ = radius to centroid of the compression zone, inches

R_c = radius to fiber location within concrete compression zone, inches

R_i = radius to intrados, inches

R_n = radius to neutral axis before bending, inches

R'_n = radius to neutral axis after bending, inches

R_s = radius to steel, inches

R_{nb} = radius to neutral axis for balanced design, inches

σ_c = stress in concrete compression fiber, psi

σ_o = maximum concrete compression stress, psi

σ_s = stress in tension reinforcement, psi

σ_t = concrete tensile stress, psi

σ_{yp} = stress in tension reinforcement at yield, psi

T = steel tension force, pounds

$\overline{y}$ = distance to centroid of compression block from outermost compression fiber

$d\alpha$ = angle change of plane section after applying moment

β = angle between successive bars of radial reinforcement

Δ = reference angle used in determination of concrete tensile stress

$d\Delta$ = incremental change in angle Δ

$d\phi$ = angle included between faces of incremental segment of curved beam after applying moment

$d\theta$ = angle included between faces of increment segment of curved beam

ds = incremental length at neutral axis location

I

INTRODUCTION

Background and History

The analysis of a curved beam of one elastic material was developed by E. Winkler in 1858 and by H. Résal in 1862.[1] Although the approach each of these gentlemen took leads to an approximate solution, there is good agreement with the exact solution of the same problem which was first given by H. Golovin in 1881 and followed by C. Ribière and L. Prandtl.[1]

The approximate method is generally presented in mechanics of materials textbooks and is an elastic analysis of a homogeneous member based on the concept of conservation of plane sections from which it can be shown that the stress distribution of the normal stresses over any cross section follows a hyperbolic curve. Analysis using straight beam theory cannot logically be used because of this non-linear stress relationship.

It has been shown, however, that for a homogeneous beam as the ratio of the radius to the centroid of the beam to the depth of the beam, R/h, increases, the error caused by using a linear stress distribution decreases quite rapidly. For an R/h of 1 to an R/h of 10 the percent error decreases from 35% to approximately 3% respectively.[2] Once R/h>10, it seems reasonable to assume that

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[1]References indicated thus, see bibliography

straight beam theory could be used with accuracy sufficient for most purposes.

As stated above, the analysis of Winkler and Résal, as well as later work, is based on elastic theory. While this concept has dominated research and design since the early 1900's, ultimate load or plastic theory is gaining its proper place in the design of structures and structural members largely because of the experimental research of some persistent and imaginative individuals. This is not to say that the elastic theory is unimportant, but to fully understand the action of structural members throughout all phases of stress and strain up to collapse, the inelastic or ultimate theory must be considered. This is the area in which most research in reinforced concrete is currently being done.

Because it is difficult to determine precisely the stress distribution in a reinforced concrete beam in flexure, it is generally accepted that the stress-strain relationships determined from standard cylinder tests are close approximations of the conditions that exist in the structural member. These stress-strain relationships have been idealized by many over the years.[3] These idealized relationships range from rectangular configurations to those described by, in some cases, complicated mathematical expressions. A satisfactory expression would be one that describes the stress-strain relationship within the normal tolerances allowed for testing.

A search of the technical literature showed that research in the area of curved beams has been confined to cases involving loads

normal to the plane of curvature, or were extensions of Winkler's theory. No literature was found on short radius reinforced concrete beams subjected to loads in the plane of curvature and carried to ultimate.

Objectives and Scope

An experimental laboratory investigation involving concrete structural mechanics using anything but simple specimens loaded conventionally is likely to produce many problems. These problems relate to the control or measurement of the many variables inherent in specimen production and the development of instrumentation and loading systems. Even with ideal experimental controls and conditions, concrete by its very nature is inconsistent. Usually large numbers of specimens are necessary in order to establish reliable data. For these reasons an attempt has been made here to conduct preliminary experimental tests in order to establish an experimental method of investigating the flexural behavior throughout the entire load range to failure of short radius reinforced concrete beams using the basic mechanics assumptions of flexure theory. Experimental results are analyzed by comparing concrete compressive forces obtained by induced load and by strain. Load moment versus change in curvature relationships are also developed. Differences and deviations from expected values are discussed.

In addition, equations for calculating the ultimate moment for short radius reinforced concrete beams are presented. Development

of the equations is based on an assumed shape of the concrete stress-strain curve, in this case a second degree parabola, and on the assumption of maximization of the concrete compression block within the test specimen. Use of the equations involves a procedure to determine whether the beam is either under-reinforced or over-reinforced. Selection of the appropriate solution path is determined by which of the two conditions exists.

The results of the experimental and analytical procedures are discussed, and suggested changes for future work are listed.

II

ANALYTICAL AND THEORETICAL DEVELOPMENT

Basic Mechanics

In the development of analytical methods for predicting forces and moment for a curved reinforced concrete beam, three basic conditions are assumed to exist. These are:

1. The stress-strain relationships for the materials in the beam correspond to the stress-strain relationships obtained by standard axial tests of materials representative of those used in the beam.
2. Plane sections normal to the longitudinal axis of the beam before bending remain plane sections after bending occurs.
3. The equilibrium equations of statics must be satisfied.

In a material as nonhomogenous as concrete, stress-strain relationships are dependent upon properties of cement, aggregate, and water; mix proportions, placing, curing, and age of concrete; the size and shape of the specimen; rate of loading; and stress distribution throughout the specimen.[4] If the stress-strain values as obtained from axial tests of standard 6 x 12 cylinders are to be used as representative of the concrete in a structural member, these physical and material differences between test cylinders and test specimens must be kept to a minimum. Many of the differences

between conditions in test cylinders and structural members can be eliminated, but differences in size and shape of specimens or differences in stress distribution cannot. The elimination of size and shape differences is impractical. Clark, Gerstle, and Tulin[5] have pointed out, however, that the stress-strain curves for cylinders and specimens subjected to various stress distributions or gradients are nearly identical up to the point of maximum stress but differ as to the values of ultimate strain. This would indicate reliability of axial stress-strain curves at least up to the point of maximum stress.

The stress-strain relationships for samples of the steel obtained in a standard axial tension test can be used with greater confidence, than those from concrete compression tests, as representative of the stress-strain relationships in the reinforcing steel of a concrete structural member.

In most non-rigorous analyses involving flexural theory, the fundamental hypothesis is made that plane sections perpendicular to the longitudinal axis of the beam before bending occurs remain plane sections after bending occurs. The truth of this hypothesis in the case of pure bending has been rigorously proved by Timonshenko and Goodier.[1]

The third condition, that of equilibrium, is a basic fact. Ideally, the calculated total opposing forces at any cross section of the beam in pure bending should be equal in magnitude and opposite in sense to the external forces as required by equilibrium. However,

differences will exist between analytical calculations based on the assumptions of point application of loads, frictionless supports, weightless members, and completely fixed joints and the actual conditions in a structural system.

Stress, Strain and Moment

A linear strain distribution for curved beams from the compression face to the tensile steel does not exist, thereby complicating the mathematical approach to the problem. However, total longitudinal deformations of various fibers are proportional to the distance of each fiber from the neutral axis or neutral surface. Using the assumption of conservation of plane sections, the total deformation relations in an incremental portion of the beam can be made.

Using deformation relationships, stress-strain properties for concrete and steel and equilibrium, the strain, stress and moment conditions within the beam can be formulated.

From Fig. 1 and similar triangles OAB and OCD

$$\frac{\varepsilon_i R_i d\theta}{\varepsilon_s R_s d\theta} = \frac{k\ (R_s - R_i)}{d\ (1 - k)} \qquad (1)$$

where ε_i and ε_s are unit strains of the extreme concrete compression fiber and in the steel respectively, and $d = R_s - R_i$.

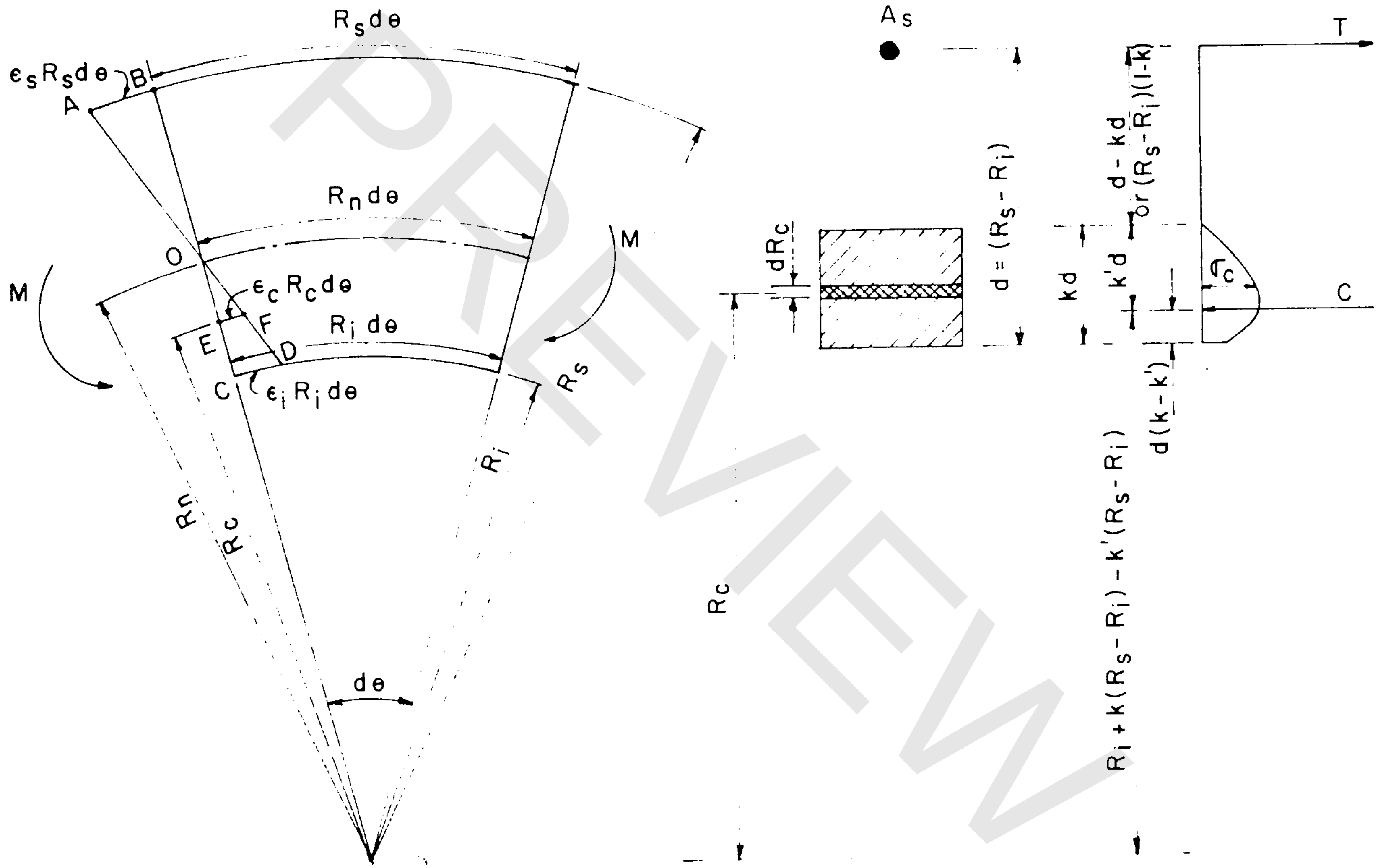

Fig I Incremental Portion of Curved Beam Under Pure Moment.

Solving for k this becomes

$$k = \frac{\varepsilon_i R_i}{\varepsilon_s R_s + \varepsilon_i R_i} \quad . \tag{2}$$

An equation to determine the unit strain at any point in the compression zone of the cross-section can be developed in much the same manner as Equation (1). Using similar triangles OEF and OCD

$$\frac{\varepsilon_c R_c d\theta}{\varepsilon_i R_i d\theta} = \frac{R_i + k(R_s - R_i) - R_c}{k(R_s - R_i)} \quad . \tag{3}$$

Solving for ε_c, this becomes

$$\varepsilon_c = \frac{\varepsilon_i R_i}{R_c} \left[\frac{R_i + k(R_s - R_i) - R_c}{k(R_s - R_i)} \right] \quad . \tag{4}$$

If the beam is subjected to pure moment, the equilibrium of total tension and compression forces requires that C = T. From Fig. 1

$$C = \int_{R_i}^{R_i + k(R_s - R_i)} \sigma_c b dR_c \quad , \tag{5}$$

$$T = A_s \sigma_s \quad , \tag{6}$$

$$A_s \sigma_s = b \int_{R_i}^{R_i + k(R_s - R_i)} \sigma_c dR_c \quad . \tag{7}$$

A necessary part of the moment calculations involves the distance between C and T and therefore requires the determination of the centroid of the compression block. In the case of curved