

# ELECTROHYDRODYNAMICS OF ELECTROSPINNING PROCESS

by

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DISSERTATION TITLE

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# **ELECTROHYDRODYNAMICS OF ELECTROSPINNING PROCESS**

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University of Nebraska, 2000

Advisor: Yuris Dzenis

The emerging technology of manufacturing of polymer nanofibers by the electrospinning method is addressed in this dissertation. Rapidly growing interest in the electrospinning process is based on a large number of current and potential applications of polymer nanofibers. Major subprocesses of the electrospinning process, i.e. jet initiation, steady state spinning, and jet instabilities, are studied analytically in this dissertation. Jet initiation is treated as a problem of shape evolution of a charged liquid meniscus. A closed form solution for the meniscus shape is obtained and analyzed and a criterion for jet initiation is derived. A steady state electrospinning model is developed. A governing equation for the jet radius is derived from the general coupled electrohydrodynamic equations. Non-linear rheological behavior of the polymer fluids is taken into account. An asymptotic solution for the long jets is obtained and analyzed. Stability of a rectilinear jet is analyzed by modeling axisymmetric and non-axisymmetric jet radius fluctuations. An equation for the critical initial jet radius is derived. It is shown that the initial jet radius does not depend on the size of the capillary tube. Bending instability of an electrospun jet is considered. A general model is formulated taking into account viscoelastic properties of a polymer fluid. Governing equations for the viscoelastic jet bending are derived and analyzed. Numerical

simulations of the kinetics of bending instability development are performed. A criterion of bending instability is derived. Experimental studies of the electrospinning process are also conducted. Experimental observations are compared with the theoretical predictions. The electrospinning process zone, starting at the point of the first bending instability and extending to the nanofiber collection area, is experimentally studied for the first time. Spatial distributions of the mass flow rate, current density, and jet segment diameters are obtained and analyzed. The theoretical and experimental results provide better understanding of the complicated electromechanical process and can be used for the design of improved electrospinning devices.

PREVIEW

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Table 1.2 Jet flow modes of electrostatic atomization process

Table 1.3 Properties of atomization, conventional spinning, and electrospinning

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$A_{n,m}$	constants in the expression for electric field potential, here $n=0$ and $n=2$ as $m$ is an index in Chapter 2
$A_1, A_2$	functions of wave vector for the expression for correlation function of jet radius fluctuations in Chapter 3
$B_m$	constant in Chapter 2
$C$	concentration of polymer solution
$C_{ij}$	ground capacitance matrix in Chapter 2
$C_{ij}^l$	lumped capacitance matrix in Chapter 2
$\vec{D}$	electric displacement
$D_R$	jet draw ratio, $D_R = R_0^2/R^2$
$\vec{E}$	electric field vector of magnitude $E = \sqrt{(\vec{E} \bullet \vec{E})}$
$E_s$	magnitude of electric field induced by surface electric charges
$F_\eta$	viscoelastic force
$F_e$	mutual charge repulsion force
$G$	shear modulus of the Maxwell fluid

- $I_0$  constant total electric current of electrospinning process
- $\tilde{I}_f(z, t)$  Langevin sources of electric charges(molecular electric current) averaged over jet cross section in Chapter 4
- $I$  the unit tensor
- $J_0(\dots)$  zero-order Bessel's function
- $K_0(\dots)$  zero-order modified cylindrical function or modified Hankel function
- $L$  jet segment semilength in material (Lagrangean) coordinates in Chapter 5
- $L_0$  characteristic initial segment semilength in Chapter 5
- $M_w$  polymer molecular weight in Chapter 6
- $N_B$  inverse Bond number,  $N_B = 4\gamma/(\epsilon\xi E^2)$  in Chapter 3
- $N_\delta$  dimensionless parameter,  $N_\delta = \epsilon E \delta_\phi/(2\gamma)$  in Chapter 4
- $N_{v\sigma}$  convection-conduction number,  $N_{v\sigma} = \epsilon Q/(\pi\sigma R^3)$
- $N_R$  Reynolds number of the power-law fluid,
- $$N_R = \frac{Q_0^2 \rho_m}{2\pi^2 \sqrt{3}^{(m+1)} \cdot \mu R_0^4} \cdot \left[ \frac{Q_0^2 \rho_m}{2\pi E I_0 R_0^2} \right]^m$$
- $N_w$  Weber number,  $N_w = \rho_m Q_0^2/(16\pi^2 \gamma R_0^3)$
- $N_E$  effective Euler number,  $N_E = 4\epsilon \rho_m Q_0^2/(R_0^2 S^2)$
- $N_Q$  dimensionless parameter,  $N_Q = \epsilon Q_0 E/(I_0 R_0)$  in Chapter 4