DIRECTED STRONGLY REGULAR GRAPHS

DMITRI IOURINSKI

Mathematical Sciences Department

APPROVED:

Dr. Art M. Duval, Chair

Dr. Piotr J. Wojciechowski

Dr. Vladik Kreinovich

Associate Vice President
for Graduate Studies

DIRECTED STRONGLY REGULAR GRAPHS

by

DMITRI IOURINSKI

THESIS

Presented to the Faculty of the Graduate School of

The University of Texas at El Paso

in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE

Department of Mathematical Sciences

THE UNIVERSITY OF TEXAS AT EL PASO

July, 1999

Acknowledgements

I would like to express my deepest appreciation and gratitude to the following people:

To Dr. Art Duval, who was the best possible thesis advisor, anyone can dream of.

To the committee members, who suggested many improvements.

To my parents and my wife for their support and understanding.

Contents

List of Figures

Introduction

Strongly regular graphs, graphs which possess a high degree of symmetry, were first introduced by R. C. Bose in 1963. These graphs are objects that posess high degree of symmetry. In 1986 Duval (see [**3**]) generalized the concept to directed graphs, and introduced tools necessary for the further development of the subject. In particular, he constructed several families of directed strongly regular graph. He also developed a set of necessary conditions on the parameters and made the list of all possible parameter sets for graphs with 20 or fewer vertices. For some of the parameter sets on this list, it is still unknown whether a directed strongly regular graph with those parameters exists.

In 1997 when Klin et al. (see [**5**]) constructed new directed strongly regular graphs, and showed that one of the parameter sets on Duval's list is not realized by any directed strongly regular graph. In 1998 Shaw (see [**7**]) constructed a new family of directed strongly regular graphs, as Cayley graphs of dihedral groups. He also extended the list of possible parameter sets up to 40 vertices.

Dihedral groups may be realized as semidirect products. We extend Shaw's construction to another, larger, family of semidirect products, namely the metacyclic groups. We thus find a new infinite family of directed strongly regular graphs.

Two major sources that were used for the results about directed strongly regular graphs are *A Directed Version of Strongly Regular Graphs*, Duval [**3**], and *Directed Strongly Regular Graphs*, Shaw [**7**].

This paper consists of three chapters. In Chapter 1 we give the definitions of several terms from the graph theory, introduce the concepts of strongly regular graph and its generalization to directed strongly regular graph and consider several examples of both directed and undirected strongly regular graphs. As a continuation of the review of necessary background, in Chapter 2 we will focus our attention on semidirect products of groups.

In Chapter 3 after giving an overview of the techniques first used by Shaw, namely the condition to whether Cayley graphs are directed strongly regular, we will construct a partition of $\mathbf{Z}_p$, that will later enable us to construct a subset S of a metacyclic group $C_p \times_\theta C_q$ whose Cayley graph is directed strongly regular graph. The graphs generated by such S are, in general, new, however they also include Cayley graphs of dihedral groups D_n, when n is odd.

CHAPTER 1

Directed Strongly Regular Graphs

The purpose of this chapter is to introduce the terminology and facts about directed strongly regular graphs that will be used later in the text. Since the paper is mostly concerned with the derivation of a new family of directed strongly regular graphs, it seems reasonable to start with some basic definitions and examples from the graph theory that will later enable us to make a transition to more specific matters.

1. Graphs

DEFINITION 1.1. A **graph** is a pair $G = (V, E)$ of finite sets satisfying $E \subseteq V \times V$; thus the elements of E are two-element subsets of V. The elements of V are called **vertices** of the graph G and the elements of E are the **edges** of the graph. We will assume that our graphs do not have **loops** (edges of the form $\{v_i, v_i\}$) and that they do not have **multiedges**, i.e., that there is no more than one edge between any two vertices. The number of elements in the finite set V is called **the order** of G.

A vertex v is called *incident* with an edge e if $v \in e$. Two vertices incident with an edge are called its *ends*, an edge $\{v_i, v_j\}$ usually written as v_iv_j. Two