VIBRATIONAL FREQUENCIES OF A TAPERED PILE

EMBEDDED IN AN ELASTIC FOUNDATION

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by

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THESIS

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ABSTRACT

The small amplitude, free transverse vibration of uniform piles embedded in and beams resting on elastic foundations have been studied. The results are used to model marine and offshore structures. The purpose of this thesis is to extend the research to include linearly tapered piles. The boundary conditions studied involved piles free at both ends (free-free) and pinned at both ends (simply-supported). A closed form solution was unattainable and a finite difference approach was used to attain a solution. An algorithm was developed and converted to a computer program. The computer program utilized the IMSL software package to determine the eigenvalues of the system. The first three eigenvalues are tabulated for various dimensionless parameters.

The accuracy of the program was checked by comparing results attained using the program under limiting conditions with published results. The solutions were achieved using a Digital VAX/VMS computer system and a CRAY-YMP computer.

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Chapter 1

INTRODUCTION

The purpose of this thesis is to study the small lateral vibrations of tapered piles partially supported in an elastic foundation and subjected to an axial force. The results are intended as design data for marine and off-shore structures and piles used in foundation support. The tapered piles can be various cross-sectional configurations such as solid or hollow, circular or square cross-sections. The boundary conditions applied in this study will be free at both ends (free-free) and pinned at both ends (simply-supported).

The problem of a uniform pile embedded in an elastic foundation and subjected to an axial force has been investigated by Valsangkar and Pradhanang [1]. Uniform beams on elastic foundations have been studied by Doyle and Pavlovic [2], while linearly tapered beams with various classical boundary conditions have been studied by Gorman [3], Mabie and Rogers [4]. Linearly tapered piles with non-classical boundary conditions have been studied by Lau [5]. The lateral vibrations of beams with various foundation models have been studied by Filipich, Laura, Sonenblum, and Gil [6] and by Pavlovic and Tsikkos [7]. The results of these studies have been extensively presented by their authors in tabular as well as graphical form.

Considerable emphasis has been placed on the study of beams which are linearly tapered and vibrate in the transverse direction. The linearly tapered beam maintains its cross-sectional configuration at all points along the centerline. The height and width at any point along the beams centerline can be expressed as a function of the displacement along the centerline.

This thesis is an extension of the work presented by Valsangkar and Pradhanang [1]. The study will include piles with linear tapers. The analysis of these piles will involve the derivation of the equation of motion and the eigenfrequencies for various taper ratios, elastic moduli, and axial forces. The programs used in this study were done on a VAX/VMS system and run on both a VAX system and a CRAY/YMP system.

The tapered pile under investigation is modeled as a Bernoulli-Euler beam. This model neglects rotary inertia and shear deformation. The foundation model used in the study is a Winkler foundation. There are three common foundation models in use, the Winkler model, the two parameter model, and the elastic continuum model. The complexity of the latter two models prompted the selection of the Winkler model. The Winkler model provides a reasonable approximation to the elastic support and is the conventional model in most engineering applications.

Chapter 2

A TAPERED PILE PARTIALLY EMBEDDED IN AN ELASTIC FOUNDATION AND SUBJECTED TO AN AXIAL LOAD

2.1 Governing Equations

The partial differential equations governing the small lateral vibration, as shown in Figure 2.1, of a Bernoulli-Euler beam which is partially embedded in a Winkler type foundation are

$$\frac{\partial}{\partial x^2}\left[EI\frac{\partial^2 v(x,t)}{\partial x^2}\right] + P\frac{\partial^2 v(x,t)}{\partial x^2} + ky + \rho A\frac{\partial^2 v(x,t)}{\partial t^2} = 0 \quad (2.1)$$

$$\frac{\partial}{\partial x^2}\left[EI\frac{\partial^2 v(x,t)}{\partial x^2}\right] + P\frac{\partial^2 v(x,t)}{\partial x^2} + \rho A\frac{\partial^2 v(x,t)}{\partial t^2} = 0 \quad (2.2)$$

where

E is the modulus of elasticity,

I is the area moment of inertia of the pile cross section

P is the axial force

k is the elastic coefficient of the foundation

v is the transverse deflection

ρ is the material density of the beam

A is the area of the beam cross section

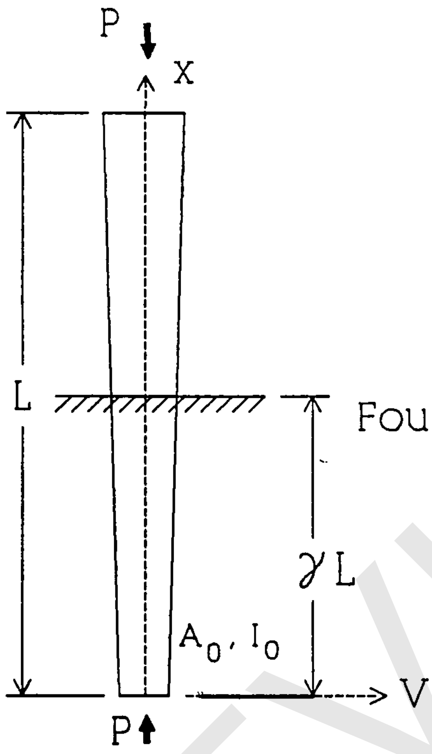

Figure 2.1 Schematic diagram of embedded pile

Equation (2.1) governs that portion of the beam which is supported by the foundation and Equation (2.2) governs that portion of the beam which is outside of the foundation. The governing equations are applicable under various boundary conditions.

The boundary conditions for the free-free and simply supported cases follow respectively:

At x=0:

$$EI\frac{\partial^2 v(x,t)}{\partial x^2} = 0 \quad ; \quad \frac{\partial}{\partial x}\left(EI\frac{\partial^2 v(x,t)}{\partial x^2}\right) = 0 \qquad (2.3a\text{-}b)$$

At x=L:

$$EI\frac{\partial^2 v(x,t)}{\partial x^2} = 0 \quad ; \quad \frac{\partial}{\partial x}\left(EI\frac{\partial^2 v(x,t)}{\partial x^2}\right) = 0 \qquad (2.3c\text{-}d)$$

At x=0:

$$v(x,t) = 0 \quad ; \quad EI\frac{\partial^2 v(x,t)}{\partial x^2} = 0 \qquad (2.4a\text{-}b)$$

At x=L:

$$v(x,t) = 0 \quad ; \quad EI\frac{\partial^2 v(x,t)}{\partial x^2} = 0 \qquad (2.4c\text{-}d)$$

2.2 Equations of Motion with Non-dimensional Variables

Applying the method of separation of variables one can assume a solution of the form

$$v(x,t) = V(x)\,T(t) \qquad (2.5)$$

Substitution of Equation (2.5) into Equations (2.1) and (2.2) renders the following equations

$$\left(\frac{d}{dx^2}\left[EI(x)\,\frac{d^2V}{dx^2}\right] + k(x)\,V + P\frac{d^2V}{dx^2}\right)\left(\frac{1}{\rho A(x)\,V}\right)$$

$$= -\frac{1}{T(t)}\;\frac{d^2T(t)}{dt^2} = \omega^2 \tag{2.6}$$

and

$$\left(\frac{d}{dx^2}\left[EI(x)\,\frac{d^2V}{dx^2}\right] + P\frac{d^2V}{dx^2}\right)\left(\frac{1}{\rho A(x)\,V}\right)$$

$$= -\frac{1}{T(t)}\;\frac{d^2T(t)}{dt^2} = \omega^2\ , \tag{2.7}$$

where ω^2 is a constant, which represents a natural frequency of vibration of the system. Equations (2.6) and (2.7) can be rewritten in the following set of equations.

$$\frac{d}{dx^2}\left[EI(x)\,\frac{d^2V}{dx^2}\right] + k(x)\,V + P\frac{d^2V}{dx^2} - \omega^2\rho A(x)\,V = 0 \tag{2.8}$$

$$\frac{d}{dx^2}\left[EI(x)\,\frac{d^2V}{dx^2}\right] + P\frac{d^2V}{dx^2} - \omega^2\rho A(x)\,V = 0 \tag{2.9}$$

$$\frac{d^2T}{dt^2} + \omega^2 T = 0 \tag{2.10}$$

2.3 Equations of Width and Height

In the case of a pile tapered in two directions, as shown in Figure 2.2, the width of the pile b at a distance x from

the embedded end can be determined by using the rule of similar triangles. The use of this rule produces the following equation

$$\frac{b - b_0}{b_1 - b_0} = \frac{x}{L} \,, \tag{2.11}$$

where

b is the width at some displacement x along the centerline

b_0 is the width at the supported end

b_1 is the width at the unsupported end.

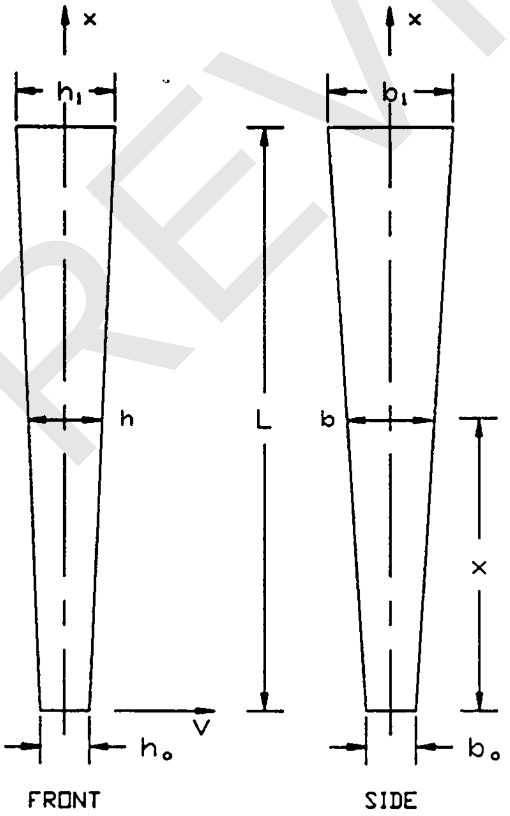

Figure 2.2 Diagram of Pile Parameters

Solving Equation (2.11) for b presents the following mathematical relationship

$$b = b_0 + (b_1 - b_0)\left(\frac{x}{L}\right) . \qquad (2.12)$$

The same procedure can be applied to the height, h, resulting in the following mathematical expression

$$h = h_0 + (h_1 - h_0)\left(\frac{x}{L}\right) . \qquad (2.13)$$

2.4 Area, Moment of Inertia and Elastic Parameter Equations

An expression for the area moment of inertia as a function of displacement was determined by implementing the following equation

$$I(x) = \frac{1}{12} b(x) h(x)^3 . \qquad (2.14)$$

Substitution of Equations (2.12) and (2.13) for b(x) and h(x) allow Equation (2.14) to be rewritten in the following manner

$$I(x) = \frac{1}{12}\left[b_0 + (b_1 - b_0)\frac{x}{L}\right]\left[h_0 + (h_1 - h_0)\frac{x}{L}\right]^3 . \qquad (2.15)$$

A similar procedure was used in determining the Area as a function of displacement along the centerline of the pile. The following equation was used in determining the variation in area with position.

$$A(x) = b(x) h(x) \qquad (2.16)$$

After making the appropriate substitutions of b(x) and h(x) the equation becomes

$$A(x) = \left[b_0 + (b_1 - b_0)\frac{x}{L}\right]\left[h_0 + (h_1 - h_0)\frac{x}{L}\right] . \qquad (2.17)$$

The procedure for determining the expression for the elastic parameter as a function of displacement is the same as was used in determining the expressions of the area moment of inertia and the cross-sectional area.

$$k(x) = k_0 b(x) , \qquad (2.18)$$

where k_0 is the basic subgrade modulus [8].
The expression for the elastic parameter becomes the following

$$k(x) = k_0[b_0 + (b_1 - b_0)] \qquad (2.19)$$

making the following substitutions

$\alpha = h_1/h_0$ and $\beta = b_1/b_0$.