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Searching for chaotic components in financial time-series

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SEARCHING FOR CHAOTIC COMPONENTS IN FINANCIAL TIME-SERIES

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ABSTRACT

Searching for Chaotic Components in Financial Time-Series. Enright, Arthur J., DPS Pace University, 1992, 102pp. Major Professor: Maurice R. Larrain, Ph.D.

This study attempts to distinguish between random and chaotic motion in several U.S. Treasury issues as reflected in 596 secondary market week-end interest rates covering the period January, 1980 to May, 1991. The Treasury series studied are the 3 month and 1 year Bills, the 1 year and 10 year Notes, and the 30 year Bond. The series are rescaled to a value range of .0000 to 1.000 and compared to 596 random data values and 596 values of the Logistic function which is known to be chaotic in nature. The Treasury series as well as the Logistic and random data series are reformulated into three dimension series using the Packard-Takens method and are evaluated using Lyapunov Exponent and Correlation Dimension measures. The investigation focuses on phase-space portrait differences rather than an extensive quantitative analysis. The central hypothesis of non-randomness in the Treasury series behavior is strongly supported, but the evidence for chaotic behavior, while indicative, is ambiguous. An extensive introductory section on chaotic behavior is also included.

CONTENTS

PREFACE

With few notable exceptions such as Larrain, 1990; Murphy, 1990 and Larrain and Pagano, 1989, the vast majority of the research literature concerning chaotic behavior in Economics and Finance is highly mathematical in content. This orientation effectively precludes many practitioners of financial analysis from access to, or benefitting from the knowledge gained through the research. Fortunately, the use of computers required by the nature and complexity of the research has provided computer algorithms which may be used to perform analyses of financial data without the need for an in-depth manipulation and presentation of the underlying mathematical theory.

Furthermore, chaotic systems are often characterized by "strange attractors" which are visually as well as topologically recognizable by graphic displays or "portraits" produced by these algorithms. These may be interpreted as qualitative measures of the chaotic nature of a system which complement the quantitative measures produced by the same algorithms.

It is our intention to synthesize and demonstrate a visually oriented approach to distinguishing between random and chaotic components in financial time-series. This approach is also intended to facilitate an understanding of chaotic behavior and its possible influence in financial analysis for a wider group of financial professionals who may lack the mathematical sophistication or computer expertise required to utilize the research directly. As a result, the mathematical content is minimized and the emphasis is focused on pictorial representation and the utility of Information Systems techniques in financial analysis.

The financial time-series considered are evaluated only with regard to the nature of the random-like behavior of the series. No attempt has been made to explain the behavior in terms of Economic theory.

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Chapter 1

INTRODUCTION and CHAOS PRIMER

The increasing availability of powerful computational resources over the last twenty years has greatly facilitated the investigation of phenomena which previously was not feasible. One of the most promising of these advances has been the progress made in the study of nonlinear dynamics or "chaotic" systems. The major advance in this area is the realization that relatively simple deterministic nonlinear relationships can give rise to complex random-like behavior patterns and, conversely, some apparently random behavior may be attributed to or closely approximated by a simple deterministic nonlinear system.

Perhaps the most significant aspect of chaotic systems is the ubiquity of such systems. In addition to its origins in meteorology, physics and mathematics, chaotic behavior has been recognized as significant in the interactions of biological populations, the feedback control of electrical circuits, the response of cardiac

cells to electrical impulses, the evolution of chemical reactions, the buildup of armaments in competing nations, and of particular importance here, the movement of some economic prices (Jensen, 1987).

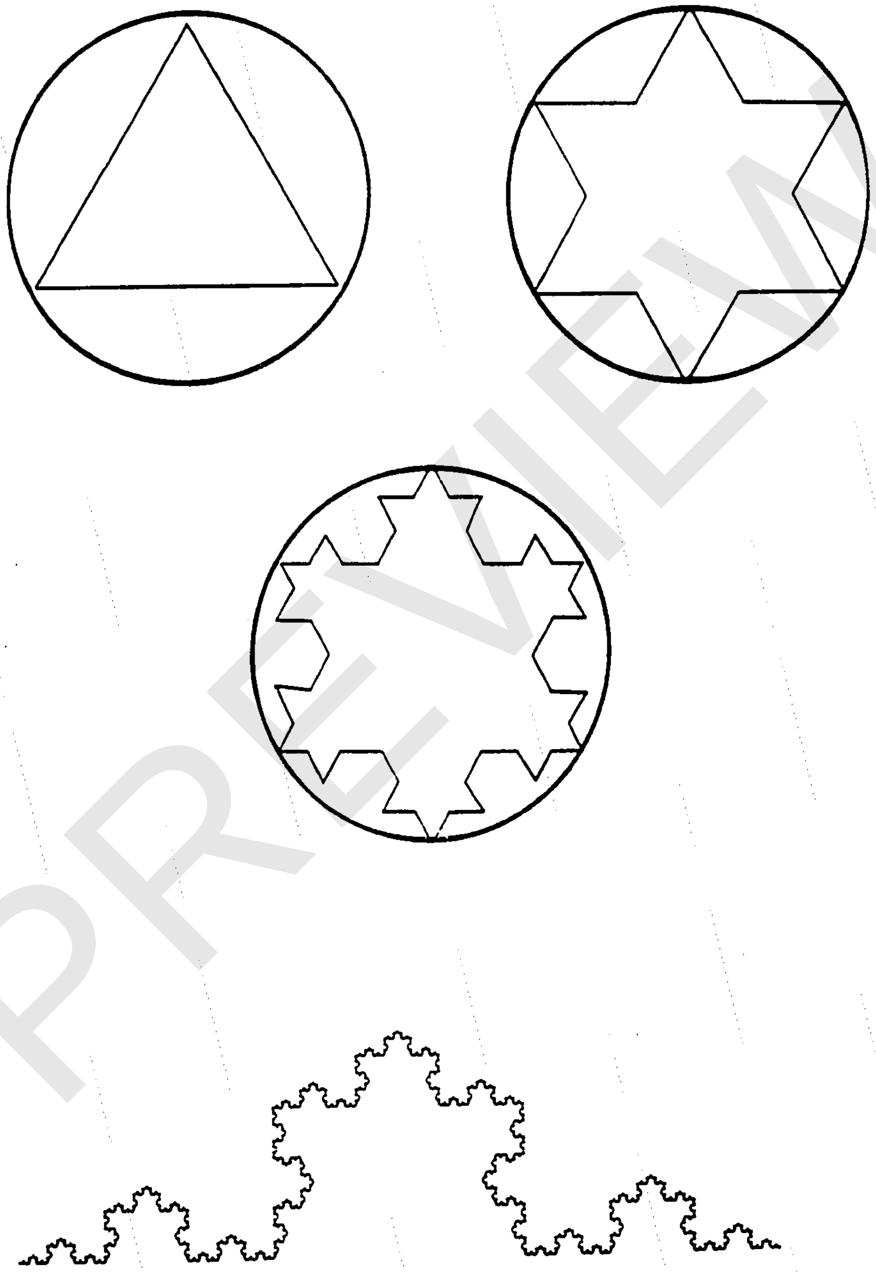
Several characteristics of chaotic systems which are particularly significant to this dissertation are; the recursive nature of dynamic relationships, the self-similarity of chaotic systems under scaling, and the applicability of proven analytic methods to identify chaotic behavior in any set of consecutive observations. A characteristic of chaotic behavior related to self-similarity is the concept of a "fractional" or "fractal" dimension (Mandelbrot, 1982), widely considered to be a broad measure of the irregularity or roughness of a curve, surface, volume or any multidimensional system. A frequently quoted example of this characteristic is the increasing measured length of a rocky coastline as the measuring unit is decreased to encompass ever smaller indentations in the coastline. Fractal dimensions may also be visualized as a measure of how densely the curve or system occupies the metric space in which it lies (Barnsley, 1988, pg. 172).

The "Snowflake Curve" shown in the Figure 1. which follows, is an example of a fractional dimension. This curve was first described by Baron Helge von Koch in 1904 and has a dimension of about 1.26. The Koch "Snowflake Curve" may be generated by starting with an equilateral triangle of some unit length on each side. A similar triangle of one-third the size is "attached" to the center third of each of the sides of the original triangle. The result of this first "transformation" is a "Star of David" with six connected equal triangles. This process is continuously performed on each of the triangles resulting from the previous transformation. After several such transformations, the curve takes on the outline of a symmetrical snowflake.

Each successive transformation increases the length of the previous result by four-thirds and, if continued indefinitely, the length of the resulting outline approaches infinity but remains contained within the finite circle which circumscribed the original triangle.

Figure 1.

Koch "Snowflake" Curve Evolution



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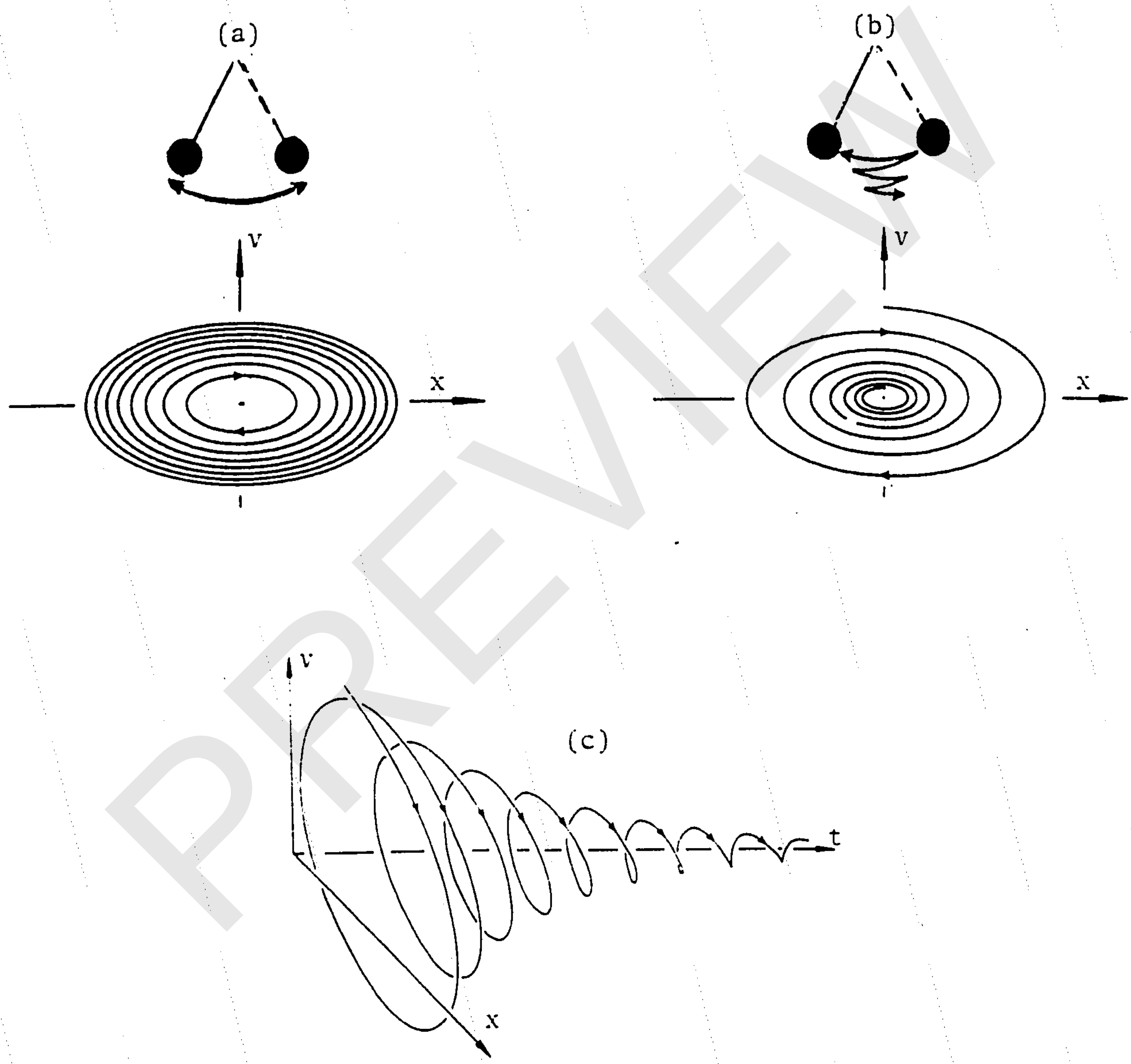
Chaotic systems typically are recognizable from patterns formed in a multi-dimensional "phase-space." A two dimensional phase-space pattern may be generated by plotting the velocity, (v) versus displacement, (x) for an unforced, frictionless pendulum as depicted in the following Figure 2a. With zero velocity and displacement located at the center of a four quadrant cartesian coordinate grid, the pattern formed by the motion of the pendulum will be an ellipse. Depending on the initial displacement of the pendulum, it will continuously trace the same elliptical path in the phase-space. Each different initial displacement will result in a different elliptical path with increasingly larger initial displacements resulting in increasingly larger ellipses. If the pendulum system is not "frictionless" as in Figure 2b., the phase-space trajectory will be a single elliptical spiral of decreasing displacement and velocity, eventually settling to the center point of zero displacement and velocity. Figure 2c. adds time, (t) as a third dimension in phase-space for the pendulum of Figure 2b.

Figure 2.

Pendulum Phase-space Portraits



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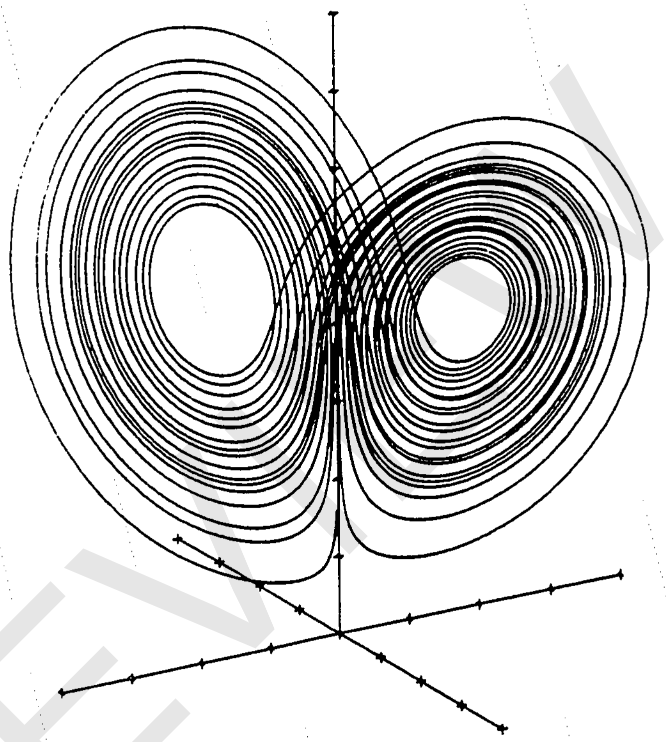
In dynamical systems terminology, these elliptical trajectories would be considered to be "attractors" since once the initial displacement was established, future positions and velocities of the pendulum are known with certainty and are constrained to the appropriate elliptical trajectory. When a chaotic system is concerned, the analogous phase-space pattern is known as a "strange attractor" and future "positions" in the phase-space pattern are, at best, only probabilistically predictable. The existence of a strange attractor as depicted in the following Figures 3a. and 3b. indicates the presence of a chaotic system. Conversely, as is the case for the preceding pendulum systems, the presence of a "point" or "periodic" attractor generally precludes the presence of a chaotic system.

While strange attractors such as those depicted in Figures 3a. and 3b. are generated by mathematical equations, attractors generated by experimental observation of a natural process such as a financial time-series are not likely to produce such exotic portraits, and consequently require quantitative analysis to confirm the presence of chaotic behavior.

Figure 3.

Chaotic "Strange" Attractors

(a)
Lorenz
"Butterfly"
Attractor
(3 dimensions)



(b)
Ueda
Attractor
(2 dimensions)

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The two most widely used analytical techniques are measurements of a fractional dimension, and the largest Lyapunov exponent associated with the data. Several methods are used to measure the fractional dimension of a chaotic system, but the majority of these are most applicable to systems of mathematical equations rather than a series of experimental observations. The method which has proved most efficient for time series was developed by Grassberger and Procaccia (1983), and is known as the "correlation dimension." This technique measures the density of points in a cross-section of an attractor relative to a specified cross-sectional area. Chaotic attractors tend to show a pattern of wide separation of points relative to non-chaotic patterns. The chaotic pattern also tends to be consistent as the cross-section area is magnified, exhibiting the characteristic self-similarity of chaotic systems.

In addition to the existence of a "strange attractor", and a fractional dimension, the analytic measure known as the Lyapunov exponent has proved a powerful tool to identify the presence of chaotic motion in a system.

The existence of a positive Lyapunov exponent in a system identifies the system as having a chaotic component in its behavior (Brock, 1986; Jensen, 1987). The Lyapunov exponent measures the rate at which two nearby points in a system diverge as the system evolves. A characteristic of chaotic systems is a high sensitivity to initial conditions in determining later values of the system. In contrast to non-chaotic systems in which a small difference in initial conditions tends to remain constant or even converge to the same later value, chaotic systems exhibit a growing difference in later values as the system evolves.

This high sensitivity of chaotic systems to slight differences in initial conditions has been dubbed the "Butterfly Effect" and is attributed to a question posed by the meteorologist Edward Lorenz in relation to his weather studies at a meeting of the American Association for the Advancement of Science in 1979, "Does the flap of a butterfly's wings in Brazil set off a tornado in Texas ?" (Gleick, 1987, Notes on Sources, pg. 322.)

This sensitivity to initial conditions is graphically illustrated in the following Figure 4a. which depicts the divergence of two initially nearby points in a classic chaotic system, the Logistic map:

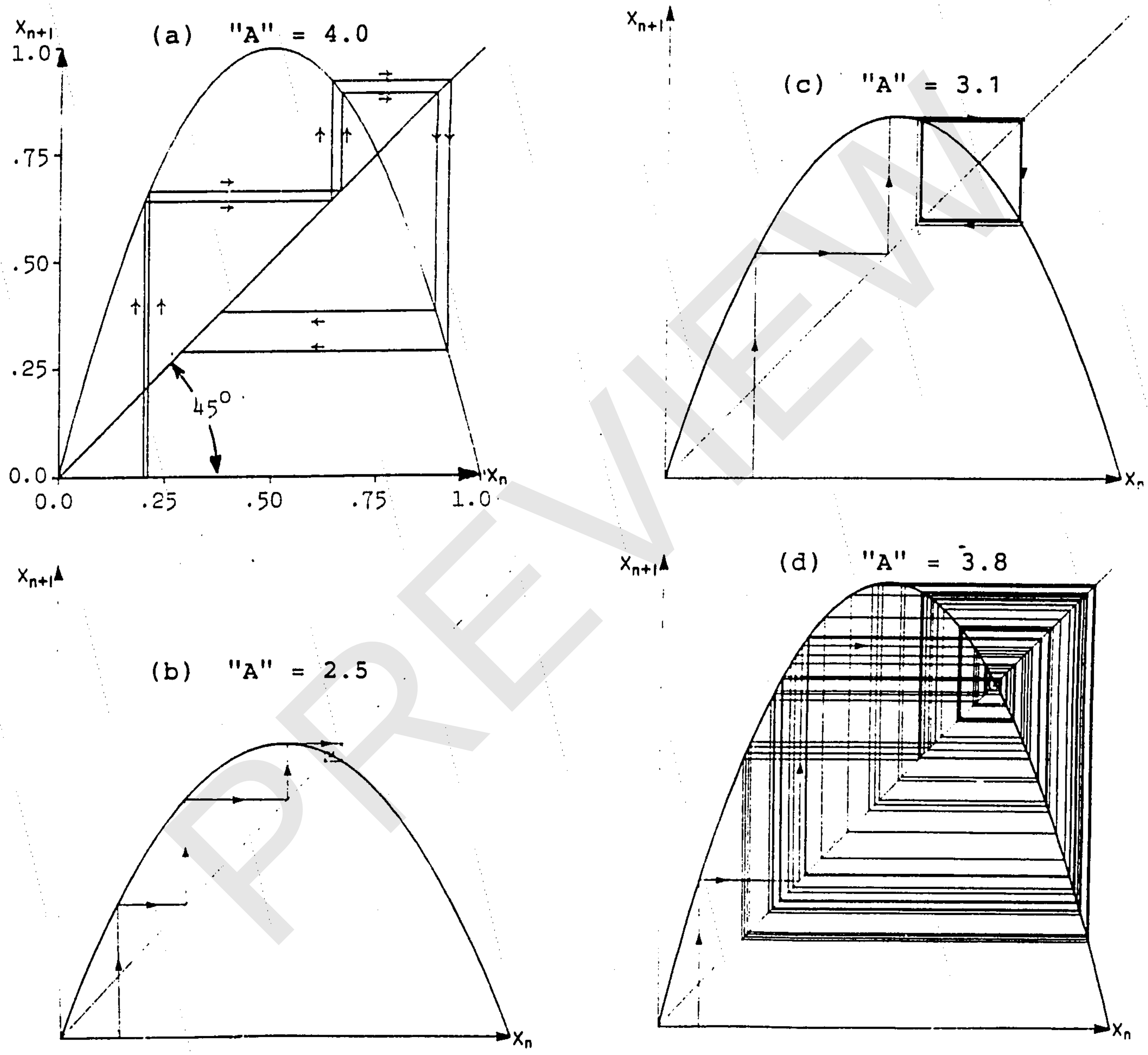
$$X_{n+1} = F(X_n) = A(X_n - X_n^2).$$

For initial values of X_n between 0.0 and 1.0, this function generates a parabola with a maximum value corresponding to $X_n = .5$ and proportional to the value of the parameter "A". Since successive values of X_n are equal to the previous value of X_{n+1}, lines drawn horizontally from the parabola intercept to a reference 45 degree line and then vertically back to the parabola trace the evolution of the function and the rapid divergence of the initially nearby points.

The Logistic function exhibits chaotic behavior for values of "A" between 3.7 and 4.0, which is the region shown in Figure 4a. and Figure 4d. For "A" values below 3.0, the Logistic function evolves rapidly to a constant value as depicted in Figure 4b. For "A" values between roughly 3.0 and 3.4, the function oscillates between two values as shown in Figure 4c.

Figure 4.

Logistic Function Diagrams



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