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Doublet Splitting in the Nuclear 1P Shell Due to a Spin-Orbit Interaction

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Doublet Splitting in the Nuclear 1P Shell

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ABSTRACT

The second order contribution of a two-body spin-orbit interaction to the doublet splitting in N^{15} and He^5 nuclei has been estimated. Initially an estimate was obtained by replacing the energy denominators in the Schrödinger perturbation formula for the second order energy shift by a single denominator. This permitted the use of closure and the resulting expression contains only ground state matrix elements. Appropriate choice of the single denominator yields a rigorous bound on the exact second order splitting. For a short range force, the two-body terms so obtained seemed unreasonably large. For this reason a second estimate was made by adding the three- and four-body terms calculated by the single energy denominator method to the exact two-body terms, computed with the Bolsterli-Feenberg technique. The exact two-body terms, while reduced from those of the first estimate, are still strikingly large.

The potential parameters used in making the above estimates were, in each case, chosen in two ways. One was to require that the potential resemble, as closely as possible, the Gammel-Thaler phenomenological spin-orbit potential. The other was to require that the splitting obtained by first order perturbation theory be equal to the experimental splitting. With the potentials determined in each fashion both estimates of the second order

contribution to the splitting were larger than and opposite in sign to the first order result. The conclusion is drawn that it is unwarranted to attempt to account for the role which the two-body spin-orbit interaction plays in the doublet splitting by a perturbation expansion.

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1. Introduction

Many of the gross characteristics of atomic nuclei are in considerable accord with the predictions of a simple shell model. At first glance the success of the shell model is rather surprising. The model was originally applied to nuclear structure problems because of some similarities between the systematics of atoms and those of atomic nuclei. The shell model had been quite successfully applied to the atoms themselves. By reason of this success the techniques of this model were well known to physicists and it was an easy matter to attempt a description of the nuclear problem with it. This was done despite the fact that the model had slight apparent justification in terms of what was known about nuclear forces.

The simplest shell model Hamiltonian to meet with any particularly good fortune in describing the properties of nuclei consists of the kinetic energies of the nucleons, a single particle central potential and a single particle spin-orbit interaction of the type

$$U_{L.S} = \sigma(r)\ \bar{l} \cdot \bar{s} \qquad 1.1$$

The nucleons are assumed to occupy single particle states correlated only by the Pauli principle and the mode of coupling of their angular momenta. With this sort of Hamiltonian one can predict the ground state spins and parities of most nuclei. The principal exceptions occur

in nuclei with large quadrupole deformations, where the assumption of spherical symmetry breaks down.

In the atomic case such a Hamiltonian has a very strong basis. The atomic electrons move in a force field predominantly determined by the nucleus of the atom. The interaction between electrons is well approximated by assuming each electron to move in a central potential determined by the average charge distribution of its fellows. Corrections to this central field approximation are, in general, small effects. In the nuclear case there is no massive center of force and the interparticle interactions are the entire story. The single particle potentials which are used to describe the nucleus are only averages of the manifold interactions between the particles. That such an averaging should produce an accurate picture of nuclear characteristics is somewhat surprising in view of the short-range character of nuclear forces. Considerable work has been done toward a justification of this averaging.[1] It is not, however, our purpose here to review these works and we will abandon the point with the remark that the justification lies primarily in the fact that the Pauli principle largely inhibits the scattering that would otherwise take place and hence has a smoothing effect on the nucleon orbits.

1. Elliott, J. P. and Lane, A. M., Handbuch der Physik 39 241 Springer Verlag

The effect of the spin-orbit force is to split the states with total angular momentum given by

$$j = \ell \pm \tfrac{1}{2} \qquad 1.2$$

It is noted, on the basis of experimental observation,[2] that the strength of the single particle spin-orbit interaction needs to be increased as a given shell is filled. This strange single particle spin-orbit potential with a variable strength hopefully may be replaced, at some expense in computational simplicity but with a saving in grief over justifiability, by a two-body interaction with a constant strength.

There exist within the two body nucleonic interaction two well established forces which are quite capable of producing a separation of the $j = \ell + \tfrac{1}{2}$ and $j = \ell - \tfrac{1}{2}$ levels. The first is the tensor force

$$U_T = v_T(r_{12})\, T_{12}\, S_{12} \qquad 1.3$$

where S_{12} is the tensor operator

$$S_{12} = \frac{(\vec{\sigma}_1 \cdot \vec{r}_{12})(\vec{\sigma}_2 \cdot \vec{r}_{12})}{(\vec{r}_{12} \cdot \vec{r}_{12})} - \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{3}$$

T_{12} contains the isobaric spin dependence, and $v_T(r_{12})$ is the radial form of the interaction. The second is the spin-orbit force,

$$U_{L\cdot S} = v_{LS}(r_{12})\, T_{12}\, \vec{\mathcal{L}}_{12} \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) \qquad 1.4$$

where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the spins of particles 1 and 2,

2. Ajzenberg, F. and T. Lauritsen, Reviews of Modern Physics 27, 77 (1955)

$\hbar \vec{\ell}_{12}$ is the relative orbital angular momentum, T_{12} is the isobaric spin dependence and $v_{LS}(r_{12})$ is the radial form of the interaction.

The presence of the tensor force in the nuclear interaction, suggested by Wigner[3] on the basis of invariance considerations, is affirmed by the fitting of the quadrupole moment of the deuteron.[4]

The existence of the spin-orbit force, 1.4, is a direct consequence of a Dirac equation with a central potential. For a central potential, weak compared to the rest energy of the particles involved, the Dirac equation may be treated in an approximation which displays a spin-orbit force as a correction to the non-relativistic Schrödinger equation. This correction term, the so-called Thomas term, was for some time rejected as a possible explanation of the doublet splitting on the grounds that it was too weak. In the case that the nucleon-nucleon interaction contains a strong repulsive core, the spin orbit interaction introduced by the Thomas term becomes very large in the neighborhood of the core. It is true that the expansion in terms of V/Mc^2, which yields the Thomas term as a first order contribution, fails to converge rapidly for a potential with such a core. Nonetheless it is quite appealing to assume that the spin-orbit force

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3. Wigner, E. Proceedings of the National Academy of Sciences 27, 282 (1941)
4. Rarita W. and J. Schwinger Physical Review 59, 436 (1941)
   59, 556 (1941)

obtained in a more complete meson-theoretic treatment of the two nucleon problem[5] would be quite strong in the neighborhood of a repulsive core, strong enough, perhaps, to produce an important doublet splitting. The existence of such a strong spin-orbit interaction, whether Thomas-like or not, is lent considerable credence by the work of Gammel and Thaler[6] in fitting nucleon-nucleon scattering data.

The isobaric spin dependence of the tensor force is determined by the fact that a strong tensor interaction between particles with odd spatial symmetry leads to a poor comparison with the nucleon-nucleon scattering experiments and unreasonably large doublet splittings. The isobaric spin operator for the tensor force is thus taken to be the singlet projection operator,

$$T_{12} = \frac{1}{4}\left(1 - \vec{\tau}_1 \cdot \vec{\tau}_2\right) \qquad 1.5$$

This operator singles out antisymmetric isobaric spin states, the tensor operator, S_{12}, singles out symmetric spin states and thus, since the two particle state must be overall antisymmetric, the isobaric spin singlet operator effectively selects states with even orbital symmetry.

On the other hand, the two body spin-orbit force

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5. Breit, G. Proceedings of the National Academy of Sciences 46, 746 (1960).
6. Gammel, J. L. and R. M. Thaler Physical Review, 107, 1337 (1957).