A COMPARISON OF ALTERNATIVE OPTION PRICING MODELS: SYSTEMATIC VS. MIXED SYSTEMATIC/DIVERSIFIABLE JUMP DIFFUSION

by

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This study proposes a new alternative option pricing model that includes two independent jump diffusions. Unlike the existing single-jump model, a two-jump model can be constructed using theory that simultaneously allows for diversifiable as well as systematic jump risk. We compare three option pricing models: a two-jump, a single-jump and a no jump stochastic volatility model, respectively. We find that, as a whole, jump models perform better than no-jump model in terms of pricing error, hedging performance and dynamic trading profits. However, within the jump models, the two-jump model and single-jump model vary in their relative performance depending on the test criteria. Our findings support the inclusion of jump diffusion in option pricing and in some cases the distinction between systematic and diversifiable jump plays a role in pricing, hedging and dynamic trading with options.

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CHAPTER ONE

1.1 Introduction and the Purpose of Research

Over the past three decades, the most notable advance in parametric option pricing models was the addition of stochastic volatility and a single jump diffusion to the Black and Scholes (1973) model. Since most stock returns deviate from a normal distribution, option pricing models should be revised to account for return skewness and leptokurtosis. In particular, Johnson and Shanno (1987), Melino and Turnbull (1990), Amin and Ng (1993), and Ball and Roma (1994) argue that the constant volatility model must be misspecified. Bates (1996), Bakshi, Cao, and Chen (1997), Craine, Lochstoer, and Syrtveit (2000) extend the Black-Scholes model to incorporate stochastic volatility and a single jump diffusion process.

Our study extends the single jump model by dividing the jump process into non-systematic as well as systematic components. Theoretically, shocks to the asset's returns may have both economy-wide (systematic) and firm specific (non-systematic) origins. Consequently, unlike index options, whose volatility reflects shocks to the market as a whole, much of individual stock option volatility is related to idiosyncratic events.

We compare three alternative option pricing models using individual firm equity options: (1) the stochastic volatility-no jump model (Wiggins (1987), Hull and White (1987), Johnson and Shanno (1987), Melino and Turnbull (1990), Stein and Stein (1991), Amin and Ng (1993), Heston (1993), and Ball and Roma (1994)), (2) the stochastic volatility-single jump model (Merton (1976), Bates (1991), Bates (1996), Bakshi, Cao, and Chen (1997), Scott (1997), Hilliard and Reis (1999), and Craine, Lochstoer, and

Syrtveit (2000)), and (3) our new two jump model. For brevity, we denote the three models SV, SVJ and SVJJ, respectively. This study differs from previous works in one key respect. Our focus is on American options. Most studies have used European options, because the no-early exercise feature of European options leads to closed form solutions for the option's price. In practice however, most individual equity options are of the American variety and any benefit to modeling the jump process is particularly relevant if it affects the options actually used by traders. We address two research questions. The first is whether or not option pricing models that include a jump diffusion process (systematic and/or non-systematic) more closely match observable American stock option prices. The second is whether the distinction between a systematic and a non-systematic jump influences the pricing, hedging performance, and trading performance of the option model.

For theoretical rigor, we derive a closed form solution for the European version of our SVJJ model. The derivation illustrates how the non-systematic jump diffusion can be combined with the existing single systematic jump diffusion model based on Merton (1976). Because of the early exercise feature of American options, no closed form solution exists for the American option's price. We use discrete-time processes and simulation to fit the model to individual American stock options.

Our findings show that as a whole, the SVJJ and the SVJ models perform better than the SV model with regard to pricing error, hedging effectiveness, and dynamic trading strategy profits. Thus, jump models appear to outperform the no-jump alternative stochastic volatility model.

The comparison between the two jump models, SVJ and SVJJ, is less definitive. The two jump and one jump models vary in their relative performance depending on the evaluation criteria (pricing vs. hedging, vs. dynamic trading) and the particular equity being tested. However, the SVJJ model seems to dominate the other two models in terms of the dynamic trading profits. Unlike the pricing error and the hedging effectiveness, the dynamic trading profits only depend on identifying whether the market option prices are under or overvalued rather than on the magnitude of the deviation.

1.2 Literature Review

The development of modern option pricing theory can be classified into three categories. The first category includes the Black-Scholes-based models. After Black and Scholes (1973) introduced the risk-neutral pricing method, there was a wave of option models that relaxed many of the simplifying assumptions made by Black and Scholes.[1] Several articles include Hull and White (1987), Wiggins (1987), Heston (1993), and Stein and Stein (1991). Merton (1973) showed that the Black-Scholes model can be derived under weaker assumptions.[2] Galai (1977), Finnerty (1978), and Black and Scholes (1972) tested the pricing efficiency of the option market on the basis of the Black-Scholes model. Galai (1978) and Bodurtha and Courtadon (1986) further test the pricing properties of the Black-Scholes model using put-call parity.

The greatest advantage of the Black-Scholes pricing method is that it separates option pricing from investor preferences. In addition, all the model's parameters, except

[1] Black and Scholes assumed that the variance of the stock return is constant; that the short term interest rate is known and constant; that the stock returns are lognormally distributed; and that the Sharpe-Lintner-Mossin capital asset pricing model holds.
[2] It is not assumed that interest rates are nonstochastic and that investors have homogeneous expectations.

the asset's return volatility, are observable in the market. Only the asset's return volatility requires estimation, thus modeling the appropriate volatility behavior is critical to proper option pricing.

There has been a great deal of research in optimally estimating the volatility parameter for option models. Galai (1977),Whaley (1982), Melino and Turnbull (1990), Bates (1996), Bakshi, Cao, and Chen (1997), Craine, Lochstoer, and Syrtveit (2000) are examples. One such approach finds the volatility that equates the market price with the option's price. This volatility is typically referred to as the stock's implied volatility. For example, Bhattacharya (1987) uses the implied volatility to test whether or not option prices contain information not already embodied in the contemporaneous stock price and finds that while option prices contain additional information not reflected in stock prices, the information is not sufficient to cover trading costs.

A critical limitation of the Black-Scholes model is the assumption that the volatility remains constant over time. Empirical regularities such as volatility clustering and the volatility smile suggest that the Black-Scholes model is misspecified with respect to return volatility behavior. Figures 1 and 2 illustrate these two typical volatility distortions. Figure 1 illustrates the problem with the Black-Scholes model and moneyness. Theoretically, given the same expiration date for the same option, the implied volatilities must be identical regardless of the moneyness of the option. In most option markets, however, the implied volatility from the Black-Scholes model displays the skewed pattern shown in Figure 1. The model's option price is too high relative to the market's option price for extreme out-of-the-money and in-the money options.

Figure 1

This is a typical example of the volatility smile. The moneyness represents the ratio of strike price to stock price and the implied volatilities represent standard deviations backed out from the Black-Scholes model.

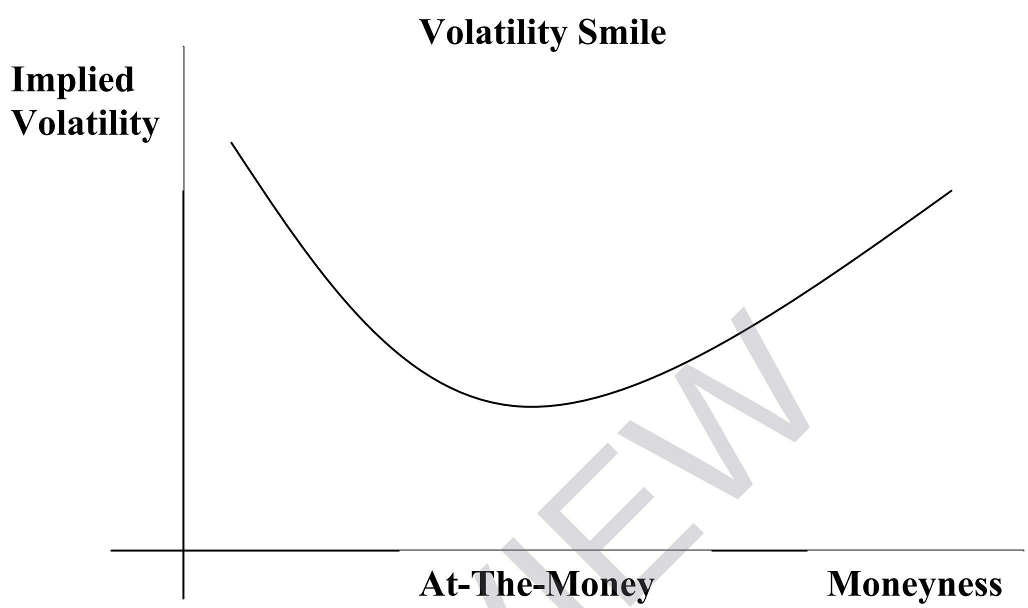
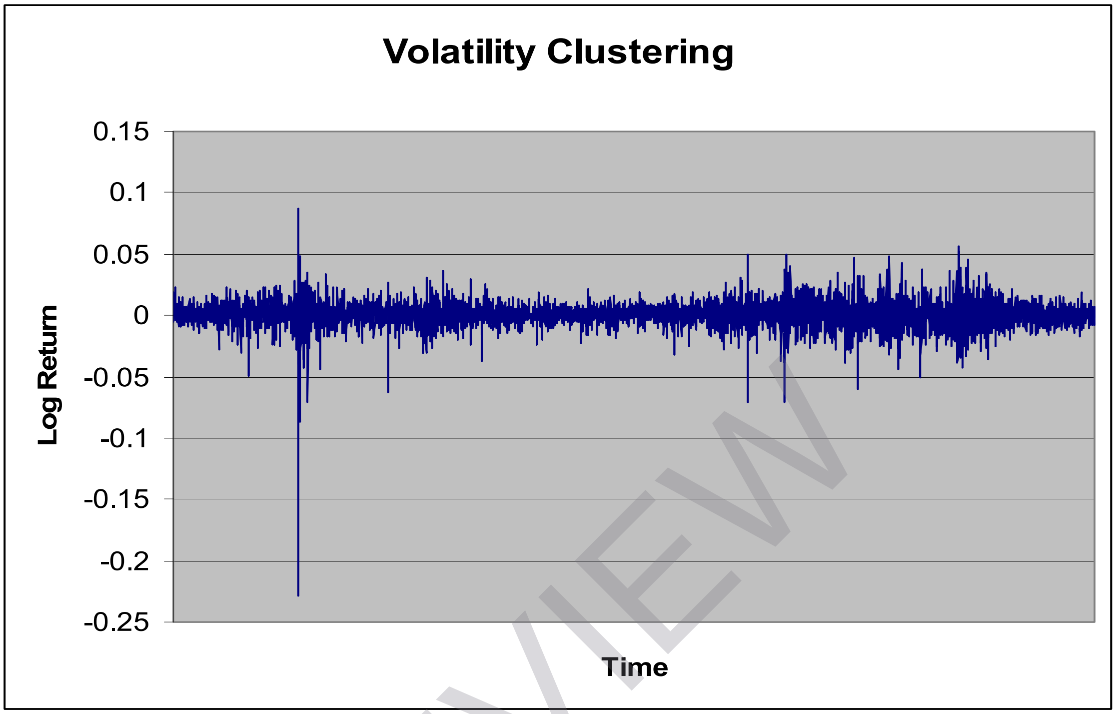


Figure 2

We use the most recent five years of data from S&P500 log returns as an example of volatility clustering.



Consequently, the implied volatilities are high for extreme values of moneyness. The resulting volatility smile reflects the degree of misspecification of the option pricing model. Figure 2 illustrates the problem of assuming a constant volatility. It is clear that volatilities cluster over time, that is volatilities are heteroskedastic. In addition, most stock returns exhibit skewness and leptokurtosis that are not consistent with the moments of the normal distribution assumed by the Black-Scholes model.

Recognizing the limitations of a constant volatility, the option literature moved to the second category of option models: stochastic volatility models. The stochastic volatility based-models aim to address the underlying skewness and leptokurtosis of the stock return distribution. Time-varying volatility can be modeled independent of whether or not a closed form option pricing model can be derived. Wiggins (1987) derived an option partial differential equation (PDE) under stochastic volatility by constructing a hedge portfolio whose return was assumed to be uncorrelated with that of the stock. In addition, with some special assumption[3] governing the underlying asset, the PDE was solved numerically. Melino and Turnbull (1990) also derived a partial differential equation under stochastic volatility and solved it using numerical methods. Johnson and Shanno (1987) use a Monte-Carlo method to model stochastic volatility by assuming that the random shock of some asset exactly mimics that of the variance of the stock. Hull and White (1987) handled stochastic volatility by solving the option price using a series expansion under the assumption that the volatility is independent of the underlying stock return. Amin and Victor (1993) generalize the option formula with time varying volatility. However, their formula was limited because it was not preference free. Stein and Stein (1991) derive a closed form solution for the option assuming that the stock price volatility

[3] The underlying asset is assumed to be the market portfolio.

follows an autoregressive process. Heston (1993) uses a risk-neutral probability measure and the characteristic function to derive a closed form solution of the option price with stochastic volatility. His method focuses on the square-root process and a consumption-based price of volatility risk. Unlike prior models his model had no restrictions for the correlation between the volatility shock and underlying asset return shock and was also preference free. Ultimately, his derivation became the foundation for option pricing with jump diffusion processes. Using a similar technique, Ball and Roma (1994) develop an option pricing model using the moment generating function for the average variance.

Although stochastic volatility option pricing models represent a theoretical and empirical improvement over constant volatility models, they still do not accommodate discrete jumps or "extreme events" that occur in financial markets. Bakshi, Cao and Chen (1997) and Roger, Lochstoer and Syrtveit (2000) conclude that incorporating random jump is required to capture characteristics of financial return distribution and improve pricing error. The existing stochastic volatility models rely on a sub-martingale process for the underlying stock price[4]. These models can not accommodate the discontinuous jumps that one observes in actual stock prices. The last category of option pricing models attempts to integrate asset price jumps into the stochastic volatility models. The price change in the Black-Scholes model and its stochastic volatility variants are assumed to be continuous, and provided that we take an infinitesimal time difference, the magnitude of the random shock converges to zero. However, the stock market crash of 1987 provides a striking illustration that stock price movements can take large discontinuous jumps. A poisson jump diffusion was added to the stochastic

[4] In the sense that in general, a rate of return for a risky asset is expected to be positive, under the true probability measure, the underlying stock price is more likely to become a sub-martingale that, on the average, increases over time. (See Neftci (2000))

volatility model to accommodate these market price discontinuities. The poisson jump diffusion process allows the random shock to keep its nonzero magnitude over any infinitesimal time increment. The jump diffusion model was originally described by Merton (1976). However, his model did not gain popularity because the jumps were assumed to be diversifiable, and an explicit form was not provided.

After Bates (1991) derived a closed form solution for a pure jump diffusion model with constant volatility, many additional option models with jumps were introduced. Bates (1996), Bakshi, Cao, and Chen (1997), Scott (1997), Hilliard and Reis (1999), and Craine, Lochstoer, and Syrtveit (2000) all incorporate a jump process. In particular, jump risk can be priced under a risk-neutral measure in a representative agent model with time-separable power utility. Hilliard and Reis (1999) applied the jump process suggested by Bates (1991) to commodity futures and option prices. Bates (1996), Bakshi, Cao and Chen (1997), Scott (1997), and Roger, Lochstoer and Syrtveit (2000) all incorporated stochastic volatility as well as a single jump diffusion into their option pricing models. In addition they illuminate why a jump diffusion is needed theoretically and empirically.

Merton's single jump model and the jump models that followed it all assume that the jump process is systematic (non-diversifiable). From a theoretical standpoint, these models are most appropriately applied to a diversified portfolio's return rather than an individual asset's return. For an individual asset, any discrete price shock may have a non-systematic as well as a systematic component. The chief contribution of this dissertation is to develop a two jump option pricing model with stochastic volatility and both systematic as well as unsystematic jumps.

The two jump process is a clear delineation from existing single jump models. It overcomes the theoretical deficiency resulting from the absence of the non-systematic shock to an individual asset's return and subsumes the single jump model as a special case.

CHAPTER TWO

2.1 Model Description

While the empirical analysis of our study focuses on American options, for theoretical rigor we derive the European version of our model as well. The American and European options have the same pricing mechanism for the underlying asset but differ only with respect to the early exercise feature of American options. The early exercise feature of the American options complicates the modeling and as a result no closed form solution exists for the American option. In this chapter, we derive a comprehensive closed form solution of the European versions for the stochastic volatility model, the single jump model and our two jump stochastic volatility model. The American options need to be solved numerically. We outline the discretization procedure necessary for the underlying stock return and volatility processes to price the American options numerically.

2.1.1 European Option

Merton (1976) introduced the first discontinuous jump model. For analytical tractability, he assumes that the jump is non-systematic and consequently no risk adjustment is necessary. Subsequently, researchers have assumed that the jump should be a non-diversifiable shock related to a macro-economic event. With time-separable power utility and the general equilibrium model proposed by Cox, Ingersoll, and Ross (1985), the risks associated with the jump and volatility could be priced analytically.

Bakshi, Cao and Chen (1997) showed that adding a discontinuous jump component to the option pricing model greatly improves the model's fit in terms of skewness and lepto-kurtosis. We examine whether the type of jump, non-systematic or systematic, further improves the option model's fit to the data.

In theory, if the underlying asset is an individual stock, the jump component will be affected by non-systematic shocks as well as systematic shocks to the return series. On the other hand, in Bates (1991) and Bakshi, Cao, and Chen (1997), the assumption of a single systematic jump seems to be reasonable only because they tested their models using the S&P 500, a well diversified portfolio. For the more general case, including individual assets and portfolios, the single systematic jump assumption seems counter to the systematic/non-systematic risk distinction made elsewhere in the pricing literature.

We denote our new alternative model SVJJ (Stochastic Volatility and systematic Jump and nonsystematic Jump). Combining the systematic and nonsystematic jumps with stochastic volatility allows the model greater flexibility to fit the empirical data. The SVJJ contains the two existing models, the SV (Stochastic Volatility) and the SVJ (Stochastic Volatility and single systematic Jump), as special cases.

Proceeding with our two jump structure, we begin with the following two stochastic differential equations for a non-dividend paying European option.

$$\frac{dS}{S} = (\mu - \lambda_1 \overline{k}_1 - \lambda_2 \overline{k}_2)dt + \sqrt{\upsilon}\, dW_1 + k_1 dq_1 + k_2 dq_2 \tag{1}$$

$$d\upsilon = (\theta - \beta\upsilon)dt + \sigma_\upsilon \sqrt{\upsilon}\, dW_2 \tag{2}$$

$$Cov(dW_1, dW_2) = \rho dt$$