INVESTIGATION OF THE PRODUCER'S AND CONSUMER'S RISKS OF MILITARY STANDARD 781C WHEN THE TRUE PROCESS IS A NONHOMOGENEOUS POISSON PROCESS

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ABSTRACT

This research investigates the producer's and consumer's risks of MIL-STD-781C when the failure arrival process is nonhomogeneous Poisson, depicted by the power law model. The main objective is to study the behavior of the producer's and consumer's risks when the shape parameter of the model changes from 0.8 to 1.2, since these are the values that are most frequently encountered in qualification testing. This shape parameter represents the extent of nonhomogeneity in the process, the farther the shape parameter is from 1.0 in either direction the more nonhomogeneous the failure arrival process is. When the shape parameter is equal to 1.0, a homogeneous Poisson process is obtained. The source of those fluctuations in the shape parameters may be due to an improving system (shape parameter is less than 1.0) or a deteriorating system (shape parameter is greater than 1.0).

A methodology is developed using a Monte Carlo simulation that represents the same conditions and environment encountered in qualification testing situations in which a system is operated for a fixed period of time a then observe the number of failures occurring during that period of time.

Results indicate that plans with high risks (30%), and a high discrimination ratio (3.0) are very robust to fluctuations on this shape parameter. On the other hand, plans with a relatively low risk (10%) and a low discrimination ratio (1.5) are very sensitive to changes in the shape parameter.

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INTRODUCTION

1.1 STATEMENT OF THE PROBLEM

The U.S. Department of Defense (DoD) spends a large percentage of its budget in the acquisition of expensive weapon systems. Military Standard 781C (MIL-STD 781C) "Reliability Design Qualification and Production Acceptance Tests: Exponential Distribution" provides tests that insure that equipment purchased by the military services meets desirable levels of reliability. The plans accept or reject equipment that follows a homogeneous Poisson process (HPP) as far as the failure arrival process is concerned within a given level of producer's and consumer's risks. An HPP is a process for which the rate of occurrence of those failures , called ROCOF, also known as failure rate, is constant. In contrast, a nonhomogeneous Poisson process (NHPP) is a process in which the ROCOF is a function of time, and this is the major difference in the properties of both processes.

The producer's risk is defined as the probability of rejecting a system which actually meets the desired level of reliability while the consumer's risk is the probability of accepting a system which does not meet a specified reliability threshold.

As indicated in MIL-STD 781C, the time required to test a repairable system can be determined by choosing a given plan and its corresponding assumed parameters.

In other words, the time required to test a system can be determined given that the failure arrival process can be described by an HPP.

What are the consequences of using MIL-STD-781C if the failure arrival process is a NHPP? How valid would the predicted testing time be according to the standard ? Or from another point of view, for a given testing time, how different would the producer's and consumer's risk's be from those prescribed by the standard?

1.2 SCOPE AND ASSUMPTIONS

This paper will be limited to exploring the risks of fixed length tests as opposed to sequential test plans. The main difference between both types of plans is that in fixed length test plans, the length of testing is predetermined, whereas in sequential test plans the length of the testing as well as the number of failures is unknown, although the testing time tends to be shorter on the average than the testing time in the fixed length tests.

This study was also restricted to the power law model, which is only one example in the category of NHPP models. Other possible candidates for study could have been be the Cox-Lewis model and the Lewis and Shedler model.[8] *

1.3 PURPOSE

The purpose of this paper is to compare the producer's and consumer's risks of of the fixed length test plans of MIL-STD-781C with those obtained when the true underlying failure arrival process is a NHPP. The focus should on the robustness of these

* Number in brackets refer to the list of references

plans with respect to fluctuations of the shape parameter of the power law model. Recall that all risks specified in the standard are based on the assumption that the failure arrival process is an HPP.

1.4 EXPECTED CONTRIBUTIONS

This research offers the following contributions to the body of knowledge in repairable systems reliability:

1) Provides insight to practitioners into the selection of test plans for particular applications, that is, a practitioner may never look at a plan in the same way ever again. A plan with a high discrimination ratio is more convenient to apply than a plan with a low discrimination ratio when both plans have similar risks, specially if the value of the shape parameter is suspected of being much less or much greater than 1.0. In addition, the plan with the higher discrimination ratio consumes less testing time.

2) Provides insight to theorists into the subtle differences and similarities between the homogeneous Poisson process and the Exponential distribution as the appropriate model of MIL-STD-781C. Without a doubt, the appropriate underlying model of the standard is the HPP. Notice that the producer's and consumer's risks, in the simulation output in appendices E and F, match the risks specified in the standard when the shape parameter is 1.0.

3) Demonstrates the use of Monte Carlo simulation which culminated in the development of a broad view of the producer's and consumer's risks when the failure arrival process is a NHPP, depicted by the power law model.

Chapter 2 presents a review of the topics related to this research. Chapter 3 discusses the research methodology, and validation of the simulation algorithm that generates the sequence of failures used in this research. Chapter 4 presents the results of the simulation and analysis and Chapter 5 provides summary and conclusions and suggests directions for further research.

CHAPTER 2

BACKGROUND

2.1 OVERVIEW

This chapter covers the most important topics related to modeling the reliability of repairable systems. First, a discussion on the theory of nonhomogeneous Poisson processes is presented. Second, a discussion of the power law model, which is one of many candidates for study within the class of nonhomogeneous Poisson processes, is introduced. Third, a definition of the producer's and consumer's risks of MIL-STD-781C is presented as well as the introduction of the fixed length test plans and its parameters. An example on how to use the standard to determine the required testing time for a given set of parameters, is also given. And last, a discussion of the difference in reliability analysis modeling between repairable and nonrepairable systems is introduced.

2.2 NONHOMOGENEOUS POISSON PROCESS (NHPP)

A nonhomogenous Poisson process is a stochastic point process and is defined as follows: The counting process $\{N(t), t \geq 0\}$, where N(t) = number of failures in the period (0,t], is said to be an NHPP if

1. $N(0) = 0$
2. $\{N(t), t \geq 0\}$ has independent increments, that is, the number of failures in an interval is not influenced by the number of failures which occurred in any strictly earlier interval (i.e., no overlap)

3. The number of failures in any interval of length $(t_2 - t_1)$ has a Poisson distribution with mean = $\int_{t_1}^{t_2} ROCOF(t)dt$. That is, for all $t_2 > t_1 \geq 0$

$$P\{N(t_2) - N(t_1) = j\} = \frac{e^{-\int_{t_1}^{t_2} ROCOF(t)dt}\{\int_{t_1}^{t_2} ROCOF(t)dt\}^j}{j!}$$

where:

mean = E(N(t)) = expected number of failures in (0,t] = λt^{γ}

ROCOF(t) = rate of occurrence of failures = d(E(N(t))/dt = $\lambda\gamma t^{\gamma-1}$

$\lambda = (1/\theta)^{\gamma}$

θ	Mean time between failures (MTBF) of exponential distribution
P{}	probability function
j	dummy variable representing the number of failures
!	factorial symbol
λ	scale parameter of power law model
γ	shape parameter of the power law model
t	period of time
t_1	beginning of interval
t_2	end of interval

2.3 THE POWER LAW MODEL

The power law[8] model is one of the many NHPP models and has a ROCOF as follows:

$$ROCOF(t) = \lambda\gamma t^{\gamma-1}$$

where:

ROCOF(t) rate of occurrence of failures as a function of time

t accumulated time on item

λ scale parameter

γ shape parameter

Figure 1 shows a plot of the ROCOF(t) vs. testing time, t. The purpose of this figure is to show the general shape of the ROCOF for certain ranges of the shape parameter of the power law model. It can be seen that if the shape parameter is 1.0 then the ROCOF is not a function of time and the model is equivalent to a homogeneous Poisson process (HPP in Figure 1). Similarly, for the case of a deteriorating system ($\gamma > 1$) the ROCOF is increasing with time (NHPP2 in Figure1), and for the case of an improving system ($\gamma < 1$) the ROCOF is decreasing with time (NHPP1 in Figure 1).

2.4 MIL-STD-781C, PRODUCER'S AND CONSUMER'S RISKS

MIL-STD 781C consists of a set of plans that establishes the amount of risk that the U.S. government and the contractors incur when materiel is accepted or rejected during the acquisition process.

Each plan is comprised of a set of parameters as follows:

θ_0 upper test limit MTBF

θ_1 lower test limit MTBF

α producer's risk = The probability that a test plan will reject when $\theta \geq \theta_0$

β consumer's risk = The probability that a test plan will accept when $\theta \leq \theta_1$

T Total test duration. The life units of test accumulated among all test items.

c acceptable number of failures

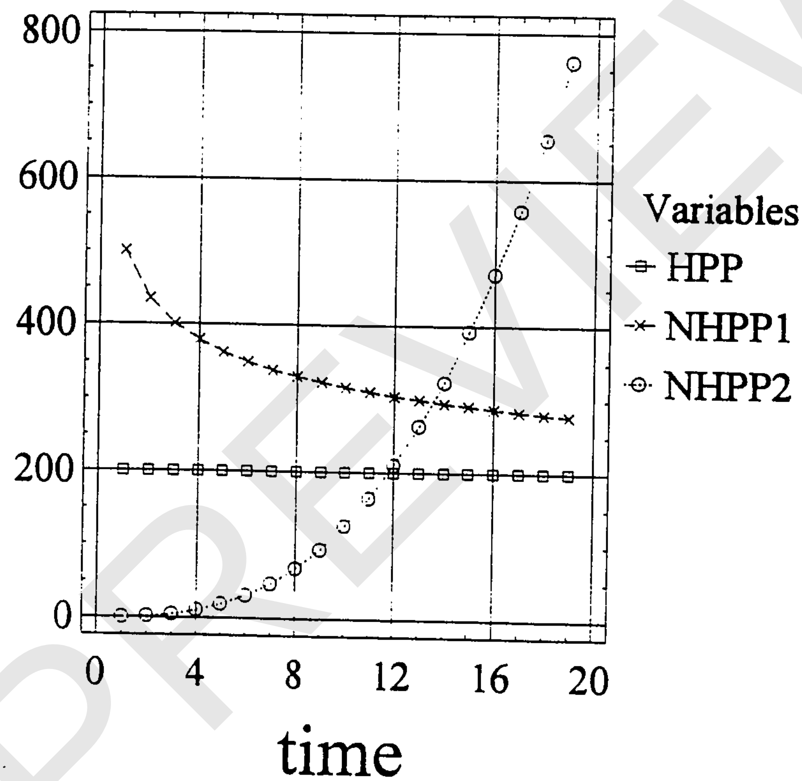

FIGURE 1: RATE OF OCCURRENCE OF FAILURES VS. TIME

DR (θ_0/θ_1) = discrimination ratio = ratio of the upper test limit MTBF to the lower test limit MTBF

The probability of acceptance for a test plan that follows an HPP is, in general, the probability of experiencing an acceptable number of failures, which for this application is determined using the summed Poisson

$$P\langle accept|\theta\rangle = \sum_{k=0}^{c}(((T/\theta)^k)e^{-T/\theta})/k!$$

where:

θ MTBF for the exponential distribution

c acceptable number of failures

k subscript(dummy) variable

T Total time accumulated among all items

! factorial symbol

$\sum$ summation symbol

The definition of the risk, α, requires that the probability of acceptance be $1-\alpha$ when $\theta = \theta_0$.

This is represented as:

$$P\langle accept|\theta = \theta_0\rangle = 1-\alpha = \sum_{k=0}^{c}(((T/\theta_0)^k)e^{-T/\theta_0})/k!$$

The definition of risk, β, requires that the probability of acceptance be β when $\theta = \theta_1$.

This is represented as:

$$P\langle accept | \theta = \theta_1 \rangle = \beta = \sum_{k=0}^{c} (((T/\theta_1)^k) e^{-T/\theta_1}) / k!$$

For given values of α, β, θ_0, and θ_1, the above equations may be solved simultaneously for T and c. Since c is a discrete variable, an exact solution may not be possible.

Example: How to determine the required testing time for a communications system using MIL-STD-781C.

The test of a communications system is to be based on a lower test MTBF (θ_1) of 100 hours, and an upper test MTBF (θ_0) of 300 hours.

Determine:

How many hours of test are required for design qualification prior to a production decision if it is desired a producer's risk (alpha) and consumer's risk (beta) of (nominally) 10 percent each?

Solution:

Assumption: System follows an HPP

The test planning parameters are:

$\theta_0 = 300$

$\theta_1 = 100$

DR = 300/100 = 3.0

alpha = α = 0.10

beta = β = 0.10