DETERMINATION OF RESONANT FREQUENCIES AND MODES IN SHIELDED DIELECTRIC CYLINDRICAL RESONATOR

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ABSTRACT

An analysis is made of the mode structure and resonant frequencies within a cylindrical millimeter-wave resonator. The vector Helmholtz equation is solved using variational finite-element (F.E.) techniques, in which the associated energy functional is analytically integrated in azimuth. The anisotropic permittivity is accounted for through a tensor transformation. A classical case of a microwave hollow-metal cavity is used to test the finite element formulation. A computer program is developed for the solution of the vector variational functional which is set into matrix form. An analytical solution is obtained to compare the results with the F.E. solution in order to eliminate the spurious modes. The measure of the angle between F.E. and analytical eigenvectors is used to identified all possible frequencies and modes from those spurious ones. A series of tables and graphs show how well F.E. results agree with the analytical solution and, hence, the validity of the proposed method.

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CHAPTER 1

Introduction

Recent advances in miniaturization of microwave passive components include the appearance of the low-loss, temperature-stable dielectric resonators. These devices are presently being used at millimeter-wave frequencies to replace waveguide filters and oscillators in environmentally demanding areas, such as space communications. Although the geometrical form of a typical dielectric resonator is relatively simple, its mathematical analysis defies exact solution of Maxwell's equation for the general modes. This is particularly true for hybrid modes with a large number of oscillations in the azimuthal direction. Such modes, i.e. "whispering gallery" modes[1], have been analyzed using a variety of approximate solutions[2] which include mode matching, perturbational methods, Green's functions, as well as, finite differences and finite elements. Of these, the most rigorous, general, and accurate is the finite element method[3]. However, the application of this method to the dielectric resonator is not without its problems and complexities.

The finite element method is chosen over the others because, once developed for a particular geometry, it can be set to a general form capable of contributing to the solution of similar problems.

A cylindrical resonator of radius b and length l as shown in Fig. 1.1 is used in the study of the transverse electric and transverse magnetic modes. A hollow cavity is formed in the volume enclosed by the metallic walls. The finite element method allows the solution of large-scale, complex electromagnetic field problems. Because the geometry of the hollow cavity, a three dimensional equations are obtained to account for the solution.

In 1985, Koshiba, Hayata, and Susuki[4] developed a two-dimensional finite element solution to the Helmholtz equation in the magnetic field intensity H for the rectangular dielectric waveguide modes. Their reduction of the problem from three to two dimensions was based on the assumption of an exponential dependence of the form $e^{j\beta z}$ in the approximating functions for $H(x,y,z)$, where β is the propagation constant. Integration of the variational energy functional associated with the Helmholtz equation along the longitudinal (or z) direction was rendered unnecessary by the choice of the exponential z dependence, and the placing of the tensor dielectric principal axis collinearly with that of the waveguide.

With the assumption of the ϕ dependence the problem was narrowed down to the solution of the homogeneous, three-dimensional, vector, anisotropic Helmholtz equation for the magnetic field intensity. It is analogous to the problem of

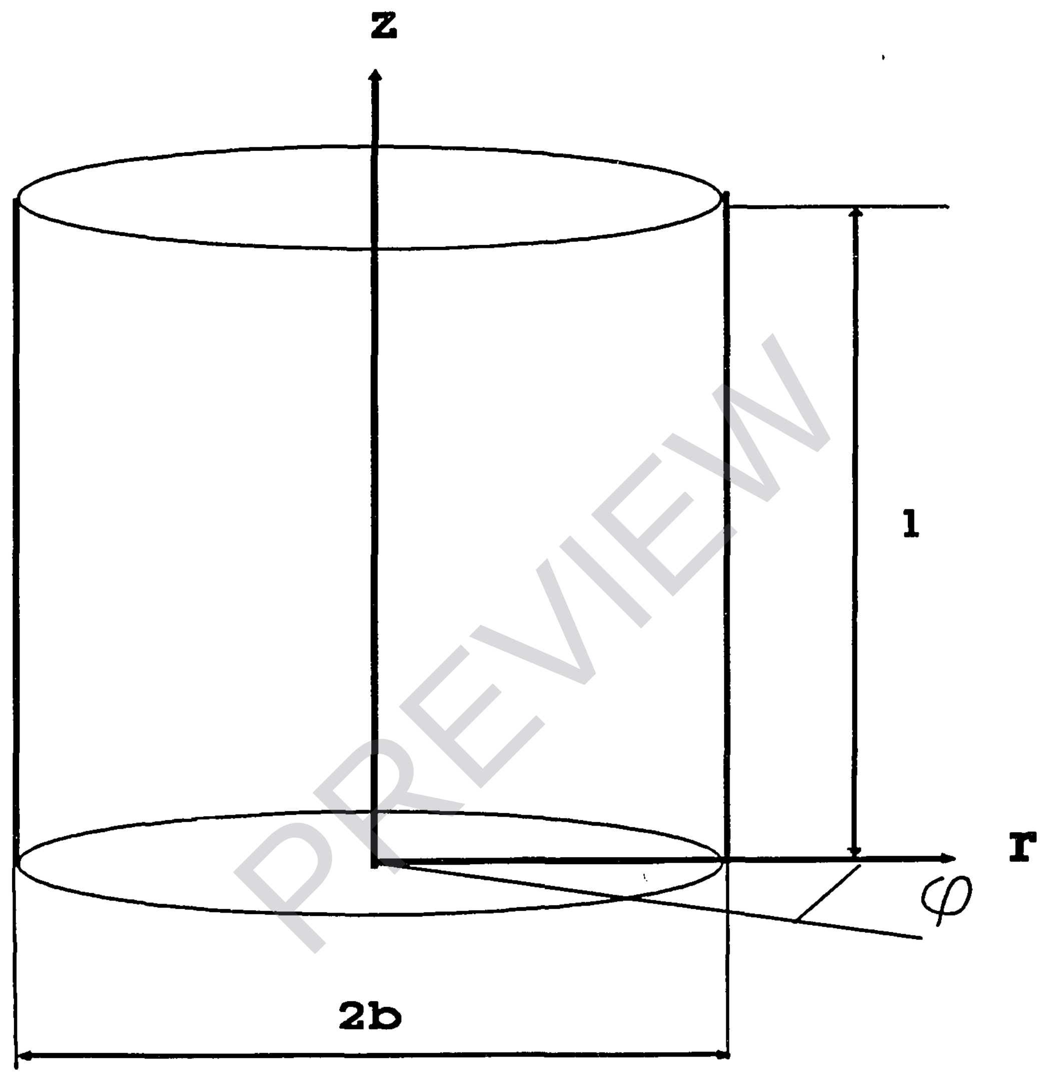

Fig. 1.1 Cylindrical Cavity Resonator Geometry Composed of Metallic Walls and Dielectric Interior.

modal vibration analysis in three-dimensional structures studied by mechanical engineers. A stationary, first variational formulation of the associated functional was obtained, and approximated with first-degree approximating expansion functions over quadrilateral elements, expressed in isoparametric coordinates. The resulting matrix formulation was integrated over the azimuthal variable to reduce the problem to the analysis of a two-dimensional r-z region. The extremization of the functional including the penalty function (to eliminate spurious modes) reduced the governing equation to an eigenvalue matrix. A computer solution to the matrix, subject to the boundary conditions imposed on the resonator metallic container, provided the structure and resonances of the natural modes.

The complete approach to the problem was divided into the following logical steps: derivation of the Helmholtz vector wave equation, finding the vector variational energy functional, specifying boundary conditions, general finite element formulation, tensor permittivity transformation, solution with the assumption of an exponential ϕ dependence, description of the finite element program, and the verification of the proposed method with the classical microwave cavity resonator.

Chapter two shows how the uncoupling of Maxwell's equations in the electric field and magnetic field intensities

leads to the homogeneous vector wave equation. In Chapter three the vector variational functional is discussed. This functional equation is based on a three-dimensional vector field which propagates within a lossless waveguide at frequency ω, and having a real wave number n. A modification to the functional is made to eliminate the spurious modes that will appear during the solution of the finite element equations. This modification is done through the incorporation of the penalty function, together with the Lagrange multiplier. This causes the spurious modes to move out of range from the initial true modes. In Chapter four, a set of boundary conditions are described to account for the metallic walls that are surrounding the cavity resonator. There are enforced (Dirichlet) and natural (Neumann) boundary conditions that have to be satisfied by the use of the vector variational energy functional. Enforced boundary conditions are exactly satisfied because the potential values at the boundary nodes are explicitly specified by setting known values to that boundary. On the other hand, natural boundary conditions are automatically satisfied when the contour integral in the functional term is dropped from the functional.

The general finite element formulation leading to an eigenvalue equation is obtained by minimizing energy functional and is discussed in Chapter five. To reduce

computational complexity the analysis is reduced from three to two dimensions by assuming an exponential dependence $e^{jn\phi}$. A general two-dimensional finite element discretization with matrix elements analytically integrated in azimuth is used to obtain the eigenvalue solution of this problem. The dimensional reduction required the transformation of the dielectric permittivity tensor from Cartesian coordinates to cylindrical coordinates; as discussed in Chapter six.

The integration over the functional to reduce the problem from three to two dimensions is explained in Chapter seven. Here it shows how the global matrices of the functional are set into a general format making possible the application of the programs for any kind of finite element (linear, quadratic,etc.) analysis. A complete description of the software developed for this purpose is described in Chapter eight. A series of subroutines are used to implement the solution of the functional which has been set into global matrices in order to solve for the eigenvalues. A subroutine from the International Mathematical and Statistic libraries (IMSL)[5] has been incorporated into the program to solve for the eigenvalues and eigenvectors. These computations allow the solution for the resonant frequencies, as well as, the structure of the magnetic modes. After the magnetic fields are computed at every node, solution of the electric fields is accomplished at every specific node.

To verify the finite element program, a classical microwave cavity resonator is treated and comparisons are made of the frequencies and modes between the analytical and finite element solutions. Chapter nine describes the development of different programs used to accomplish this task. Analytical solutions are programmed to obtain the exact solutions for the resonant frequencies, as well as, the value of the magnetic fields in each direction at every node from the discretization. The electric field E was then obtained numerically from Maxwell's equation. By comparing the difference in frequency between analytical and finite element values a range is set, thus making the program check only those values close enough for identification. The program then compares the angle between finite element and analytical eigenvectors, where true modes are identified and spurious modes are eliminated from the finite element results. Finally, a discussion of the results and conclusions are included to show how all the theory and software developed are valid and in a general format, so that it can be applied to many other cylindrical cases.

In Appendix A a full explanation about the derivation of the vector variational functional is described in detail. Appendix B shows how the tensor permittivity values are transformed from cartesian to cylindrical coordinates. Derivation of the matrix to calculate the electric field is

explained in Appendix C. Appendix D shows how the different patterns appear for the transverse electric (TE) and transverse magnetic (TM) modes when certain conditions are imposed. A software designed for the finite element computation is included in Appendix E.

CHAPTER 2

Helmholtz vector wave equation

2.1 Introduction

The governing laws in electromagnetic field problems can be expressed by a single set of equations called Maxwell's equations. The Maxwell relations as applied to the problem of the resonator deal with a linear, anisotropic, homogeneous, lossy dielectric medium that is charge free. Solutions of the electric and magnetic field for a given boundary-value problem can be obtained by either Maxwell's equations or from the wave equations. Maxwell's equations are first-order, coupled, partial differential equations while the wave equation is an uncoupled second-order partial differential equation. Because of the geometry of the cylindrical cavity resonator, the cylindrical coordinate system is used in the solution of the wave equation in the magnetic field intensity vector *H*.

2.2 Derivation of Helmholtz Wave Equation

Electromagnetic fields associated with boundary-value problems must satisfy Maxwell's equations or the wave equation. In this case of an anisotropic and lossless materials, the current-free phasor form of Maxwell's equations generalize to the following expressions,

$$\nabla \times E = - j\omega\mu_0 H \ , \tag{2.1}$$

$$\nabla \times H = j\omega\epsilon_0 \ [\ K\ ] E \ , \tag{2.2}$$

where ω is the radiant frequency, μ_o is the permiability of free space, ϵ_o is the permittivity of free space, and $[k]$ is the tensor dielectric constant of the material inside the cavity. The tensor dielectric constant is related to the tensor permittivity of the material $[\epsilon]$ by the relation

$$[\ K\ ] = \left[\ \frac{\epsilon}{\epsilon_o}\ \right] \ . \tag{2.3}$$

Taking the curl of both sides of (2.2) gives

$$\nabla \times [\ K\ ]^{-1} \nabla \times H = - j\omega\epsilon_o \nabla \times E \ . \tag{2.4}$$

Substituting (2.1) into (2.4) leads to the homogeneous vector Helmholtz wave equation, namely

$$\nabla \times [\ K\ ]^{-1} \nabla \times H = j\ \omega\ \epsilon_o\ (\ -j\ \omega\ \mu_o\ H) \ , \tag{2.5}$$

or

$$\nabla \times (\ [\ K\ ]^{-1} \nabla \times H\ ) - k_o^2\ H = 0 \ , \tag{2.6}$$

where

$$k_o^2 = \omega^2\ \epsilon_0\ \mu_0 \ . \tag{2.7}$$

Here, k_o is called the propagation constant of the medium.

2.3 Spurious Modes.

The solution of the vector Helmholtz using finite elements is generally plagued by the presence of spurious, divergent vector fields. These nonphysical modes are present because the functional for anisotropic materials is not self-adjoint. They may be eliminated within the spectral region of interest through the use of a Lagrange multiplier (i.e. penalty function) in the variational formulation[6]. Other commonly-adopted techniques are also available for the elimination of such solutions[7]. Chapter three explains how this term is added to the functional.

CHAPTER 3

Vector Variational Energy Functional

3.1 Introduction

Using the finite element (F.E.) method for the solution of the electromagnetic fields in the cylindrical cavity resonator requires the variational formulation of the energy functional. A three-component equation is formulated for the solution of the wave equation in (2.6). The solution of the functional is normally set in terms of $\boldsymbol{H}$ because of the discontinuity of the electric field $\boldsymbol{E}$ latter at the dielectric interfaces.

3.2 Variational Formulation

A three-component vector field with frequency ω and an azimuthal mode number n, may be assumed to be of the form

$$H\,(\,r,\phi,z) = H\,(\,r,z\,)\;e^{jn\phi}\quad, \tag{3.1}$$

while the functional for (2.6) is known to be[4]

$$F\{H\} = \int_{\Omega} \{(\nabla \times H)^{*} \cdot ([K]^{-1}\,\nabla \times H) - k_o^2\, H^{*} \cdot H\}\, d\Omega\;. \tag{3.2}$$

When (3.2) is set into matrix form a serious difficulty arises during the computation of the eigenvalues representing the normalized frequencies associated with the number of oscillations in ϕ. An indeterminate number of spurious modes