NUMERICAL ANALYSIS OF THE COMPLETE EIGENVALUE SOLUTION OF RIDGED WAVEGUIDE.

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by

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THESIS

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ABSTRACT

In this thesis the complete solution of the ridged waveguide is presented. The derivations of these solutions are the work of James P. Montgomery. His solution was obtained by the formulation of an integral eigenvalue equation which is subsequently solved numerically by application of the Ritz-Garleking method. The significance of the eigenvalue spectrum is discussed and the modes are classified. Equations are given for the electric and magnetic fields. Also the orthogonality property is tested for each case given. Numerical results are also presented.

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CHAPTER 1

Introduction

Many problems encountered today have demostrated the necessity of a complete knowledge of a waveguide's eigenvalue spectrum. One such problem is the radiation of waveguide elements in an array enviroment where higher order modal resonances may create so-called blind spots. Another one is the approach in designing waveguide-to-microstrip transition, where a single-ridged waveguide is a natural transition from waveguide to microstrip. By lowering the height of the waveguide gradually and smoothly, the EM fields are forced to concentrate in a region confined between the ridge and the bottom of the guide.

People who have been involved in this area like Cohn (1) who published in 1947 a paper about the ridged waveguide eigenvalues obtained by using transverse resonance to formulate trascendental equations. The discontinuity susceptance between parallel plate waveguides obtained by Whinney and Jamieson (2) was employed in the calculations. In 1955 Hopfer (3) extended Conh's work to other aspect rattios by inclusion of a first-order correction factor. Hopfer used a quasi-static solution for the discontinuity susceptance betwwen parallel plates obtained by Marcuvitz (4). In 1966 Pyle (5) extended Hopfer's work by publishing accurate data for any aspect ratio.

Additionally, Pyle included an analysis of error effects. Each of these previous investigations was primarily aimed at the solution for the TEn0 eigenvalues. Little was said about the fields of ridged waveguide until 1961 when Getsinger(6) formulated approximate field equations by assuming a TEM mode at the gap and matching only the electric field. Getsinger used the eigenvalues obtained by Hopfer.

In order to perform a complete study of the ridged waveguide this thesis formulates an integral eigenvalue problem. The homogeneous equations are solved numerically by applying the Ritz-Garleking method (7),(8) to yield a generalized matrix eigenvalue problem. The generalized eigenvalue problem is solved by applying a solution obtained by Barlow and Jones (9). The result of this research is the ability to investigate any eigenvalue with its associated field and the approximate error introduced by numerical solution. This ability in turn permits analyis of complex structures requiring such knowledge.

CHAPTER 2

TE Eigenvalues Solution

The geometry of the ridged waveguide is ilustrated in Fig. 1. A symmetry plane (S) is assumed about the y coordinate axis. No symmetry has been assumed about the x coordinate axis. Thus,the eigenvalues problem is subdivided into four cases:

1) TE to z fields with a magnetic wall at s.
2) TE fields with a magnetic wall at the symmetry plane s.
3) TM fields with a magnetic wall at the symmetry plane s.
4) TM fields with an electric wall at the symmetry plane s.

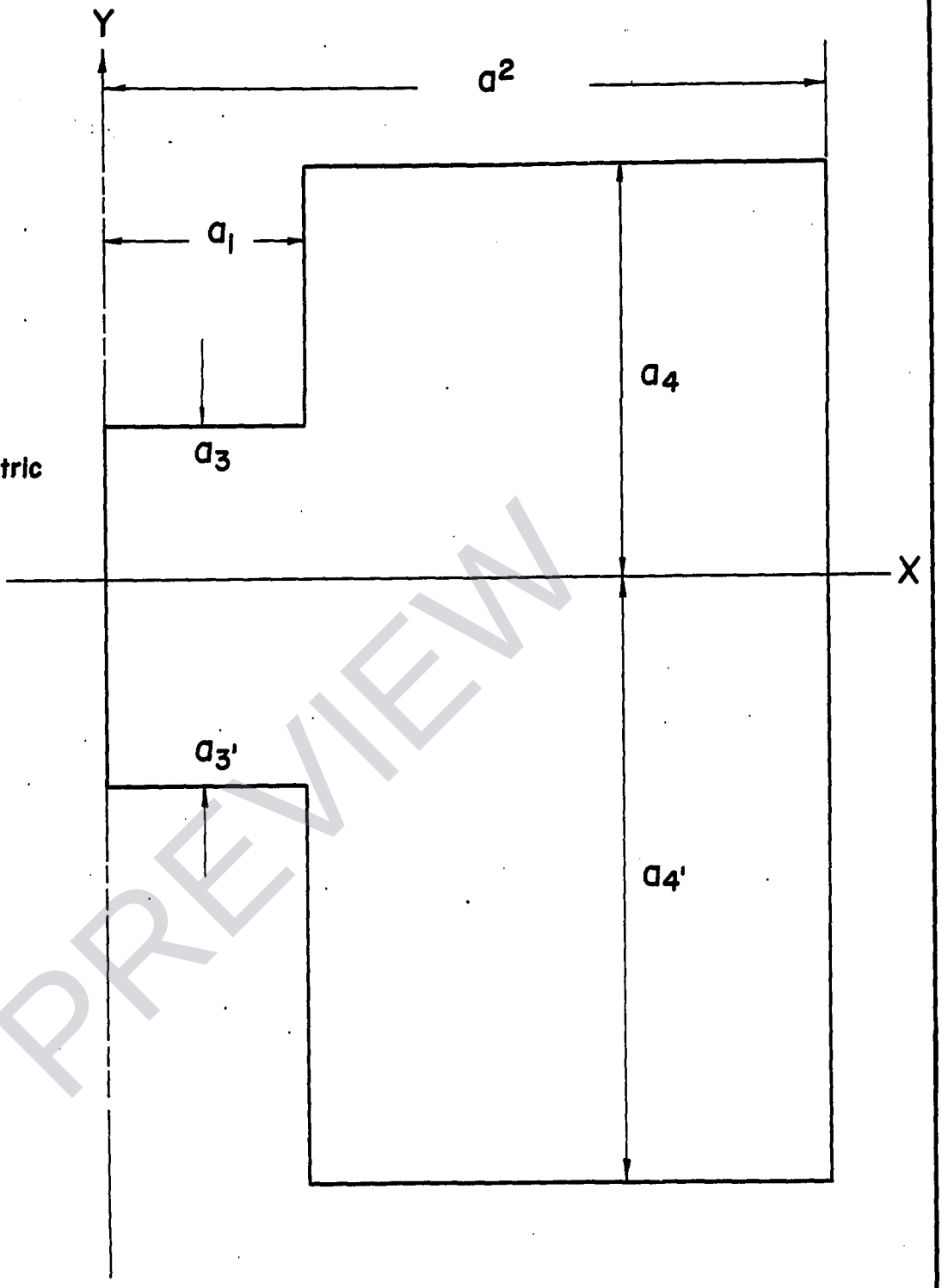
The body of this chapter will be devoted primarily to the fields with a magnetic symmetry plane.

The TE fields are derivable from a hertzian potential of the form:

$$\Pi_h = g(r_T) \cdot \not{\phi}(z)\, \hat{a}_z \qquad (2.1)$$

where: g(rt)=function of the transverse coordinate vector.

$\not{\phi}(z)$=is a function of only the z-coordinate.



$2a_1 = s$

$2a_2 = a$

$a_4 + a_{4'} = b$

$a_3 + a_{3'} = d$

Fig. 1.

Geometry of Ridged Waveguide

Both scalar function must satisfy the wave equation:

$$\nabla^2 g(r_T) + K_T^2 g(r_T) = 0 \qquad \text{(a)}$$
$$\phi''(z) + \gamma^2 \phi(z) = 0 \qquad \text{(b)} \qquad (2.2)$$

where: kt= waveguide eigenvalue

γ = propagation constant

$$\gamma = \begin{cases} (K_o^2 - K_T^2)^{1/2}, & K_o > K_T \\ -j(K_T^2 - K_o^2)^{1/2}, & K_T > K_o \end{cases} \qquad (2.3)$$

Where the scalar potential functions are related to the electric and magnetic fields by the equations:

$$\vec{E}(r_T) = -j\omega\mu \phi(z) \nabla g(r_T) \times \hat{a}_z \qquad (2.4)$$

$$\vec{H}(r_T) = \phi'(z) \nabla g(r_T) + K_T^2 g(r_T) \phi(z) \hat{a}_z \qquad (2.5)$$

Often only the transverse components of the field are equired, this enable us to simplify the last two equations s:

$$\vec{E}_t(r_T) = \nabla g(r_T) \times \hat{a}_z \tag{2.6}$$

$$\vec{H}_t(r_T) = Y\hat{a}_z \times E_t(r_T) \tag{2.7}$$

where: $Y = \gamma/\mu\omega$

The TE mode boundary conditions are:

$\hat{a}_n \cdot \nabla g(r_T) = 0$ for metallic boundary.

$g(r_T) = 0$ for magnetic boundary.

If we apply these equations to the geometry,we are able to ivide the potential into two regions.

Region 1 (0 < x < a1):

$$g_r(r_T) = \sum_{n=0}^{\infty} \eta_{1n} \sin(K_{x1n} X) \cdot \cos\left(\frac{n\pi}{a_3 + a_3'}(y - a_3)\right) \tag{2.8}$$

$$K_{x1n} = \begin{cases} \left(K_T^2 - \left(\frac{n\pi}{a3+a'_3}\right)^2\right)^{1/2}, & K_T > \frac{n\pi}{a3+a'_3} \\ -j\left(\left(\frac{n\pi}{a3+a'_3}\right)^2 - K_T^2\right)^{1/2}, & K_T < \frac{n\pi}{a3+a'_3} \end{cases} \quad (2.9)$$

Region 2 (a1 < x <a2):

$$g_2(r_T) = \sum_{n=0}^{\infty} \eta_{2m} \cos\left(K_{x2m}(x-a_2)\right) \cdot \cos\left(\frac{m\pi}{a4+a'_4}(y-a_4)\right) \quad (2.10)$$

$$K_{x2m} = \begin{cases} \left(K_T^2 - \left(\frac{m\pi}{a4+a'_4}\right)^2\right)^{1/2}, & K_T > \frac{m\pi}{a4+a'_4} \\ -j\left(\left(\frac{m\pi}{a4+a'_4}\right)^2 - K_T^2\right)^{1/2}, & K_T < \frac{m\pi}{a4+a'_4} \end{cases} \quad (2.11)$$

Sustituting eqs. (2.8) and (2.9) in eqs.(2.6) and (2.7) we can have the electric field for region 1:

$$\vec{E}_1(r_T) = \sum_{n=0}^{\infty} -\eta_{1n} \left[\left(\frac{n\pi}{a3+a'_3}\right) \sin\left(K_{x1n}x\right) \cdot \sin\left(\frac{n\pi(y-a_3)}{a3+a'_3}\right) \hat{a}_x + K_{x1n} \cos\left(K_{x1n}x\right) \cdot \cos\left(\frac{n\pi(y-a'_3)}{a3+a'_3}\right) \hat{a}_y \right] \quad (2.12)$$

In the same way for region 2:

$$\vec{E}_2(r_T) = \sum_{m=0}^{\infty} -\eta_{2m} \left[\left(\frac{m\pi}{a4+a'_4}\right) \cos\left(K_{x2m}(x-a_2)\right) \cdot \sin\left(\frac{m\pi(y-a_4)}{a4+a'_4}\right) \hat{a}_x + K_{x2m} \sin\left(K_{x2m}(x-a_4)\right) \cos\left(\frac{m\pi(y-a_4)}{a4+a'_4}\right) \hat{a}_y \right] \quad (2.13)$$

t this point we need to find the aperture electric field in region 1 and 2 using Fourier series. We can define a Fourier serie for a periodic function f(x) that satisfies the Dirichlet conditions and with a fundamental period T as:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos\frac{2n\pi x}{T} + b_n \sin\frac{2\pi n x}{T} \right]$$

where:

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(x)\, dx$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cdot \cos\frac{2n\pi x}{T}\, dx$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cdot \sin\frac{2n\pi x}{T}\, dx$$

Assuming the electric field changes only in the y direction it is possible to define a periodic function Egap'(y). At x=al the boundary conditions require that the tangential electric and magnetic fields be continuous. Also,it is required that the tangential electric field vanish on the conducting walls of the ridge. The boundary conditions are applied at a cut off condition ($\gamma = 0$).

Matching the electric field results (eqs. 2.12,2.13) we get at x=a1 that:

Region 1:

$$\vec{E}_1(r_T) = -\eta_{10} K_{x10} \cos(K_{x1n} a_1)\, \hat{a}_y$$
$$-\sum_{n=1}^{\infty} \eta_{1n} K_{x1n} \cos(K_{x1n} a_1) \cdot \cos\left(\frac{n\pi(y-a_3)}{a_3+a_3'}\right) \hat{a}_y$$
$$-\sum_{n=1}^{\infty} \eta_{1n} \frac{n\pi}{a_3+a_3'} \sin(K_{x1n} a_1) \cdot \sin\left(\frac{n\pi(y-a_3)}{a_3+a_3'}\right) \hat{a}_x \quad (2.14)$$

Then the a0 component can be defined as:

$$a_0 = \eta_{10} K_{x10} \cos(K_{x10} a_1) = \frac{1}{a_3+a_3'} \int_{-a_3'}^{a_3} E_{GAP}(y')\, dy'$$

Also the an component is:

$$a_n = -\eta_{1n} K_{x1n} \cos(K_{x1n} a_1) = \frac{2}{a_3+a_3'} \int_{-a_3'}^{a_3} E_{GAP}(y') \cos\left(\frac{2n\pi(y'-a_3)}{a_3+a_3'}\right) dy'$$

$$-\eta_{1n} K_{x1n} \cos(K_{x1n} a_1) \cdot \left(\frac{a_3+a_3'}{2}\right) = \int_{-a_3'}^{a_3} E_{GAP}(y') \cos\left(\frac{2n\pi(y'-a_3)}{a_3+a_3'}\right) dy' \quad (2.15)$$

For region 2:

$$\vec{E}_2(r_T) = \eta_{20} K_{x20} \sin(K_{x20}(a_1-a_2))\, \hat{a}_y$$
$$+\sum_{m=1}^{\infty} \eta_{2m} K_{x2m} \sin(K_{x2m}(a_1-a_2)) \cdot \cos\left(\frac{m\pi(y-a_4)}{a_4+a_4'}\right) \hat{a}_y \quad (2.16)$$
$$+\sum_{m=1}^{\infty} -\eta_{2m} K_{x2m} \cos(K_{x2m}(a_1-a_2)) \cdot \sin\left(\frac{m\pi(y-a_4)}{a_4+a_4'}\right) \hat{a}_x$$