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A STUDY OF KUMMER'S WORK

ON FERMAT'S LAST THEOREM

APPROVED :

William J. LeVeque

Dean of the Graduate School

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To

My Grandparents

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A STUDY OF KUMMER'S WORK

ON FERMAT'S LAST THEOREM

by

SHARAD CHANDARANA

THESIS

Presented to the Faculty of the Graduate School of

The University of Texas at El Paso

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THE UNIVERSITY OF TEXAS AT EL PASO

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NOTATION

ζ denotes a primitive p^{th} root of unity, $p \geqslant 3$, p prime.

$\lambda = 1 - \zeta$.

$K = Q[\zeta] = \{ a_o + a_1\zeta + \ldots + a_{p-2}\zeta^{p-2} \ : \ a_i \in Q \ , \ 0 \leqslant i \leqslant p-2 \}$

the cyclotomic field corresponding to p.

$A = Z[\zeta] = \{ a_o + a_1\zeta + \ldots + a_{p-2}\zeta^{p-2} \ : \ a_i \in Z \ , \ 0 \leqslant i \leqslant p-2 \}$

the ring of cyclotomic integers in K.

$A\alpha$ denotes the principal ideal generated by $\alpha \in A$.

ε denotes " belongs to " or " epsilon " in the appropriate context.

||| indicates end of proof.

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INTRODUCTION

This thesis presents a study of Kummer's work on Fermat's Conjecture, namely, $x^p + y^p = z^p$ has no non-trivial solutions in rational integers for $p > 2$; when p is a regular prime. A prime p is regular if it does not divide h, the class number of the cyclotomic field $Q[\zeta]$, ζ a primitive p^{th} root of unity. We assume the reader's acquaintance with the basic facts of algebraic number theory as they are exposed in [7] and [9]; and cohomology of finite groups as presented in [2] and [10].

We split the study into two chapters. In chapter I we first show that the equation $X^3 + Y^3 + Z^3 = 0$ has no solution in algebraic integers taking into account that $Z[\sqrt{-3}]$ is a unique factorization domain. Then assuming that p is a regular prime we divide Fermat's Conjecture into two cases : Case I :- $p \nmid xyz$ and Case II :- $p \mid z$. Transcending to the realm of ideals, we show that the conjecture holds in Case I for $p \geq 5$.

In chapter II using cohomology of finite groups we prove a crucial lemma due to Kummer and show that the conjecture holds in Case II.

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CHAPTER I

THE FIRST CASE

We say that the first case of Fermat's Last Theorem holds for the prime exponent $p > 2$ when there do not exist integers x, y, z such that $p \nmid xyz$ and $x^p + y^p + z^p = 0$.

In this chapter we shall show that the first case holds when $p \geqslant 3$ is a regular prime.

We begin with the following , where $A = Z\left[\sqrt{-3}\right]$.

<u>LEMMA I.1</u>. If $\alpha \in A$ and $\lambda \nmid \alpha$, then $\alpha^3 \equiv \pm 1$ (mod $A\lambda^4$).

<u>PROOF</u> : We have

$$\zeta = \frac{-1 + \sqrt{-3}}{2}$$

$$\lambda = 1 - \zeta = \frac{3 - \sqrt{-3}}{2}$$

and so $$\lambda^2 = \frac{3\,(\,-1 + \sqrt{-3}\,)}{2}$$

$$= -3\zeta$$

giving $$3 = -\zeta^2\lambda^2 .$$

Now, since $\lambda \nmid \alpha$, the congruence class of α is that of +1 or -1 i.e. $\alpha \equiv \pm 1$ (mod $A\lambda$). Suppose $\alpha \equiv 1$ (mod $A\lambda$) (the case $\alpha \equiv -1$ (mod $A\lambda$) is similar).

Then $$\alpha = 1 + x\lambda \qquad (\, x \in A \,)$$

Hence $$\alpha^3 = 1 + 3x\lambda + 3\,(\,x\lambda\,)^2 + (\,x\lambda\,)^3$$

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