

IMPLEMENTATION OF EVOLUTIONARY ALGORITHMS ON THE POWER INDUSTRY AND AVIATION SECURITY

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2012

Dedication

To my brother and my parents

For all their sacrifices, support and unconditional love throughout my life

PREVIEW

PREVIEW

IMPLEMENTATION OF EVOLUTIONARY ALGORITHMS IN THE POWER
INDUSTRY AND AVIATION SECURITY

by

ANUAR JESUS AGUIRRE NUÑEZ,
Bachelor of Science in Industrial Engineering

THESIS

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Chapter 1: Introduction

1.1 RESEARCH BACKGROUND

The present thesis is concerned in the use of metaheuristics to find excellent solutions for hard problems that our society is facing. A persistent rise in the importance of the optimization on the use of resources is one of the major trends of the human history. Since the industrial revolution that began in 1970s the industries began to try to use the resources in a more intelligent way. Industries and business always try get as most as possible outcome from what they are producing and/selling. In a remarkable profusion of applications, optimization has penetrated deeply into every area of human life, for example, industries, hospitals, home, and also in different growing commercial and service sectors making important changes on to them. Optimization is undoubtedly one most integral part of human contemporary life and a key part on human development in various sectors. The lack of optimization techniques causes a economical lost and a various steps behind of a comparable industry that uses it, due to the reduced commercial and industrial production. The primary function of an optimization technique is to create a reliable method to catch the extreme of a specific function by an intelligent arrangement of its evaluations (measurements).

Our society is highly dependent on time and cost-effective solutions, and two of the most important actual problems are the security on airports and the reliability on power systems, since air transportation and electricity are essential services. Therefore, having a reliable power system and an excellent airport security are very important factor for a good life quality. In order to provide constant electricity supply we need to design a component replacement schedule for the electric distribution systems. One of the key aspects in designing the component replacement schedule is the reliability. An unreliable power system infers in high cost to the electric company and most important to the customers.

According to SGI Federal (2003), the cost of a major power outage confined to one state can be on the order of tens of millions of dollars per day. One power outage that affects multiple states can cost over one hundred million dollars. A power grid consists in three main divisions, which are: the generating station, transmission network, and distribution network. The generation station is where the electricity is produced; the transmission network is the connection between the station where the electricity is generated and the distribution network. A bulk supply point is the point of connection between a distribution and a transmission network. The distribution network links a utility bulk transmission network and the retail customer.

For power systems, the number and duration of supply interruptions characterize the continuity of supply (Sand *et al.*, 2004). A reliable power system is a system that has the ability of supply electricity at any point of time. Sometimes in the electrical industry the term “availability” is used instead as “reliability”. One of the main focuses of this research is in the power distribution system. For the power industry most of the time the distribution system begins at the substation and ends with the final customer. Most of the customer electrical outages (up to 80%) occur due to a failure in the distribution system. Since the demand of electricity is increasing constantly it is basic for the power industry to have a very reliable electric distribution system. A component replacement analysis is the determination of when to change a specific component in the system, in order to minimize the impact of a failing component. The impact of a failure component can be determined in cost such as the cost of an outage. At each planning period two decisions should be taken: one is to either KEEP the component or REPLACE it for a completely new component. When a component is selected to continue is the system (keep it) the component must receive maintenance in order to continue its operations, as it wears out with the aging process.

In the power system we can divide the component replacement problem in two different categories: series replacement or parallel replacement. In a series replacement problem one or multiple

independent components, that interrupts the service when at least one fails, are considered to be replaced at a specific time. On the other hand, in a parallel replacement problem multiple interdependent components, that interrupts the service when a specific combination of components fail, are considered to be replaced at a specific time. When some constraints are added to the replacement problems (either series or parallel), the solutions search (KEEP or REPLACE) results in a complex combinatorial optimization problem.

The other important issue that this thesis addresses is the aviation security problem. For the threats of terrorism against aircraft during the past years has received more attention due to the security concerns. For this reason airport screening procedures all over the world and more in the United States have gone through important improvements in order to ensure safety. In 1996 the White House Commission on Aviation Safety and Security (CASS) was created due to the crash of the Trans World Airlines Flight 80 (TWA 800). The mayor contribution of the CASS was the recommendation of the deployment and use of new screening technologies and equipment, also the development of standards for training and testing the screening security device. In 1996 the United States congress passed the Federal Aviation Reauthorization Act of 1996. By the same time in 1997 the United States Congress passed the Omnibus Consolidated Appropriations Act. Finally in 2000, the United States Congress provided to the Federal Aviation Administration (FAA) a found of one billion dollars of airport security. One third of this founding was used to purchase and deployment of security equipment (Coughlin *et al.*, 2002). One of the major improvements in the airport security was reach by the use of Explosive Detection Systems, the implementation of Passenger-Baggage Matching and Automated Passenger Profiling. The Transportation Security Administration (TSA) has mainly focused in identifying potential threats by baggage screener training and the implementation of new procedures.

By finish of the 20th century, a system was implemented to improve the airport security by the identification of potential terrorists through the use profiles; this system is the Computer-Aided

Passenger Prescreening System (CAPPS). CAPPS helps the security personnel to focus their attention on the higher risk individuals. When a passenger is classified as not a risk a label of *nonselectee* is used over it, while in the other hand those found as a possible threat is classified as *selectee* (O'Harrow, 2002). According to previous research the implementation of CAPPS is not sufficient to warranty the total security and also it was found that some airports using CAPPS in order to select passengers for a more intense security process are less secure than other airports that use a random selections of passengers that go through inspection. For example, in 2002 Chakrabarti and Strauss developed Carnival Booth, an algorithm that uses a combination of statistical analysis and computer simulation, to demonstrate how a terrorist could defeat CAPPS. Chakrabarti and Strauss evaluated the efficacy of their algorithm and demonstrate that CAPPS is an inefficient method for airport security.

The breakpoint in the airport security was the terrorist attacks on September 11, 2001. Since 2001 aviation security changed to a uniform screening across the country with the law that mandates 100% checked baggage screening, eliminating the distinction between passengers' *selectees* and *nonselestees*. Even though, the TSA revisited the selective screening policies by the development of CAPPS II, a continuation of CAPPS, but on July 14, 2004, the TSA announced that CAPPS II would not be implemented. But, TSA announced that by the research done to implement CAPPS II, it was found that it is more effective to perform a more intense scrutiny of passenger that has a profile of "security risk" than increasing the security for all passengers. Poole and Passantino in 2003 said that is not cost-effective to implement a 100% checked baggage screening policy and proposed to create multiple levels of security for screening passengers.

CAPPS II only improve aviation security under some particular set of circumstances, so that it is recommended that CAPPS II be transitioned from security centerpiece to one part of many future components in aviation security (Barnett, 2004). In 2006 Martonosi and Barnett created a mathematical model that explores the effectiveness of airport passenger prescreening system against terrorist attracts

and noticed that CAPPS II may not improve the security of aviation. MITRE Corporation and Weiss developed in 2011 the Dynamic Airport Security Model. This model is a fast-time desktop simulation that accepts the airport layout, security procedure and threat vectors (path-weapon combinations) as the main inputs and with that information develop a model of the performance of the airport's security. Most of the airports today use many levels of security specially to check the baggage, using in most of them Explosive Detection Systems. There are many different types of Explosive Detection Devices that has different output. When an airport needs to find the best combination of levels and machines to use also results in a complex combinatorial optimization problem.

Since both problems previous described are combinatorial optimization problems this research focuses in the implementation of metaheuristics to solve those important issues that our society is facing. One of the most intense growing research area is the use of metaheuristics to solve combinatorial optimization problems. One of the main reasons for this is that the combinatorial optimization problems are very important for the scientific world, but are even more important for the industry. All the optimization problems try to find the best combination of the given variables to achieve an specific goal. For the combinatorial optimization problems the main goal is to find a discrete mathematical object minimizes/maximizes the objective function. The present research uses Genetic Algorithms to solve the component replacement schedule in power distribution systems problem and Memetic Algorithms to solve the design of an optimal baggage screening strategy.

Memetic Algorithms (MA's) and Genetic Algorithms (GAs) are a class of search and optimization methods inspires by the evolutionary adaptation in nature. GA's were introduced in early 1970s by Holland, but were implemented in optimization problems until late 1980s by Goldberg. While the combination of an Evolutionary Algorithm (EAs) with a local search makes a "Memetic Algorithm" (Moscato, 1989). In both cases simple GAs and MAs are global optimization methods, since the first

trial solutions run by them are based on global information that is then utilized by the search process. In both cases the GAs and MAs the optimization mechanism are the following:

- GAs and MAs uses a population of chromosomes, where each chromosome represents a trial solution of the problem.
- For each chromosome its fitness value is evaluated, where the fitness value is the criterion of the optimization problem evaluated.
- For each generation, the chromosomes that gave the fitness value are selected to undergo through a series of evolutionary operators to produce new chromosomes (sometimes called offspring).
- The better chromosomes then evolve by the optimization process, and by then the fittest chromosome is the optimized solution.

As motioned before the solutions of the optimization problem are codified chromosomes (Krasnogor and Smith, 2005). GAs and MAs uses evolution operators, and in for them they are the same two evolution operators, the crossover operator and the mutation operator. GAs and MAs begin to work with a given population (set) of random solutions in form of chromosomes, where the fitness value of each chromosome is determined by the evaluation of the objective function (Kamepalli, 2001). The evolutionary operators help the best chromosomes to interchange information to produce new chromosomes. After the evolutionary operator process the new set of solutions are then evaluated and used to make a continuous evolution. The process is repeated for a specific number of generations to obtain a optimum solutions, or at least a near to optimum solution to the combinatorial optimization problem.

Sometimes, classical GAs is not aggressive enough to solve some combinatorial optimization problems and should be enhanced with local search methods (Mendoza *et al*; 2009). The difference between a GA and a MA is the use of a local search operator extra for the MA. A local search helps the MA to locate a local optimum at each iteration more efficiently than GAs (Garg, 2009). The term

‘Memetic Algorithms’ was adopted in the late 80’s in order to denote a new family of metaheuristics that combines tightly separated families such as evolutionary algorithms (Moscato *et al*; 2004).

1.2 RESEARCH OBJECTIVES

The present work shows how evolutionary algorithms can be used in order to solve hard combinatorial problems. Two different industries with highly different demands were chosen to demonstrate the functionality and flexibility of evolutionary algorithms to solve combinatorial problems. For the component replacement scheduling model for the power distribution system over finite planning horizon a Genetic Algorithm was used. A model is developed for a radial configuration and for a complex configuration. Radial configuration is the most commonly used in the power industry but that tendency is moving towards a complex configuration. The main objective of the research focus in the power industry is to find the replacement schedule that minimizes the total cost, subject to budget constraints.

In order to minimize the total cost for the component replacement schedule, another objectives should be addressed. For example, first of all a generalized formulation for the main goal in which all the issues are examined should be done. Most of the issues that should be addressed are the objectives and constraints that are commonly encountered in real life.

In the other hand using a Memetic Algorithm solves the second problem addressed by this research. A model was developed for different number of levels, from two up to ten. For space constraints ten is the maximum number of levels allowed for a common airport. The main objective of the second half of the present research is to find the best configuration of machines and levels that minimizes the total cost.

1.3 PROPOSED THESIS LAYOUT

In chapter 2 metaheuristics approaches and exact optimization methods were presented. In the first section of the chapter describes what is a combinatorial optimization problem. The second section of the chapter gives a brief explanation of some exact optimization problems. The exact optimization methods explained are: Linear Programming, Integer Programming, and Dynamic Programming. The third section of the chapter explains what a metaheuristic is and how it works. Different types of metaheuristics are explained such as Tabu Search and different evolutionary algorithms. The evolutionary algorithms explained are: Ant Colony Optimization, Particle Swarm Optimization, Monkey Algorithm, Genetic Algorithm and Memetic Algorithm. Since the Genetic Algorithm and Memetic Algorithm are the evolutionary algorithm used in the present research one section of the chapter is completely dedicated to them.

Chapter 3 gives a main description of the component replacement schedule for the power distribution system and how the power system works. In the first section it is explained what is component replacement analysis. The second section explains the concepts of electricity generation, electricity transmission and the distribution systems. Each stage of the power system network is explained in detail. The third section of the chapter is focused in explain the distribution system, since it is the main topic of the first half of the research. The third section explains how the distribution network works, which factors and components are involved, and finally the main importance of the distribution system.

Chapter 4 explains the model developed to solve the component replacement scheduling. Chapter 4 gives a detail explanation of the methodology and formulation necessary to solve the problem. Section 2 gives a literature review of the application of similar metaheuristic to find an optimal solution in different problem domains. Section three presents the application of Genetic Algorithms in different component replacement problems. Section four present the Non-Homogenous Poisson Process (NHPP)

method. NHPP is used to calculate the aging process of the components of the distribution system. Section four also present the different formulation sued to calculate the total cost of schedule. The total cost is the summation of the maintenance cost, the unavailability cost and the purchase cost. Also, presents the Genetic Algorithm developed. Two configurations were solved for the component replacement analysis. A algorithm is developed for the radial configuration and the complex configuration. Section 1 present the algorithm developed for the radial configuration, with a detailed explanation of the steps involved to find the optimal solution. The second section explains the difference between the formulations to find solve the radial and the complex configurations. The third section of the chapter gives an example of the complex configuration and the best schedule obtained for it.

Chapter 5 gives a main description of the aviation baggage-screening problem. The first section gives an introduction to the baggage-screening problem along with a literature review. The second section explains the basic concepts of the baggage-screening problem.

Chapter 6 explains the methodology to solve the problem and the model developed. In section one the problem is analyzed and shows the mathematical model of the problem. The second section describes the model assumptions. Section three explains the Memetic Algorithm and Genetic Algorithm used. Section four gives an example of the baggage-screening problem and explains the results.

Chapter 2: Heuristic Optimization Methods

2.1 INTRODUCTION

The main focus of this chapter is to introduce and understand the different methods of optimization techniques. The different optimization methods are explained briefly with an special emphasis in the explanation of the Genetic Algorithms and Memetic Algorithms. First a brief description of what a combinatorial optimization problem is presented. Afterwards the exact optimization methods are described, with special emphasis in Linear Programming, Integer Programming and Dynamic Programing. Then, the metaheuristic method is explained. Tabu Search, Ant Colony Optimization, Particle Swarm Optimization, Monkey Algorithm, Genetic Algorithm and Memetic Algorithm are explained.

2.2 COMBINATORIAL OPTIMIZATION PROBLEMS

When a best solution is found over a set of feasible solution we say that the problem solved was optimized. All problems that could be optimized try to find the best combination of values that minimize or maximize the objective function. The generalized optimization theory covers a large area of mathematics. Optimizing is to find “best value” of the objective function.

In recent years the term “Combinatorial Optimization” (CO) has emerged. CO is used to describe the areas of mathematical programming concerned to the solution of optimization problems that has a combinatorial structure. Humanity has faced optimization problems since ancient ages; the main problem with optimization problems is that sometimes they have an infinite number of solutions. In recent years it has been found that most of the recent optimization problems have a finite number of solutions, even though the set of solutions is pretty big. Combinatorial problems are focused on the techniques and theory of the problems that have a finite number of solutions and the main goal is to find the best solution, where the set of feasible solutions is discrete or can be reduced to discrete. Software

engineering, mathematics and Artificial intelligence are some important applications of the operations research and computational complexity theory related to combinatorial optimization. Most of the real world problems such as scheduling, assignment, routing, packing, cutting, network design are combinatorial optimization problems. The most common combinatorial optimization problems are:

- **Traveling Salesman Problem:** For a set of different cities, which will be the best path to follow if all the cities must be visited and must return to the start point. The main objective is to minimize total distance traveled.
- **Facilities Layout Problem:** A set of facilities that need to be laid on the space of a factory. The main objective is to maximize the benefit of the location.
- **Vehicle Scheduling Problem:** A set of cars that need to visit a number of locations. The main objective is to minimize the distance traveled, with the constraint of vehicle capacity.
- **Transportation Problem:** For a set of warehouses to a set of factories design a distribution system. The main objective is to minimize the transportation cost for each factory and warehouse demand.

The last examples are the most commonly hard combinatorial optimization problems in real world, but they are not the only and most important real-world combinatorial optimization problems. There are two different types of techniques to solve combinatorial optimization problems. One technique is the exact optimization methods such as, Linear Programming, Integer Programming and Dynamic Programming. The second category of techniques is the metaheuristic optimization such as Genetic, Monkey and Memetic Algorithms. Other examples of metaheuristics are the Particle Swarm Optimization, Tabu Search and Ant Colony Optimization.

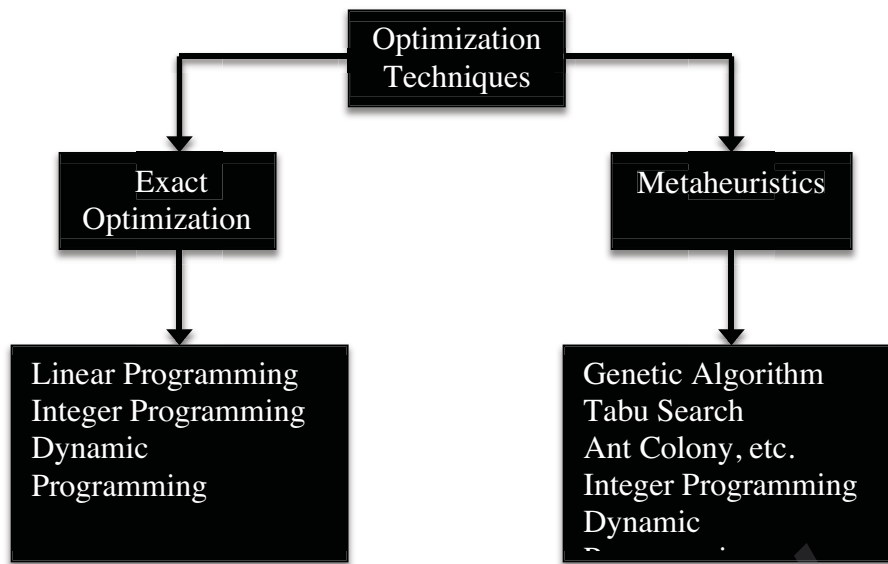


Figure 2-1: Optimization Techniques to solve Combinatorial Problems

2.3 EXACT OPTIMIZATION

The main advantage of the exact optimization methods is that they do guarantee to find the best solution over the set of feasible solutions. The main disadvantage is the run-time to find the optimal solution. The run-time for the exact optimization methods increases exponentially as the size of the problem increases. Most of the time only small optimization problems can be solved to prove the optimality of the solution found.

2.3.1 Linear Programming

Linear programming problem is a problem that has linear constraints and a linear function subject to the constraints that needs to be either maximized or minimized. In operations research the most used optimization technique is Linear Programming (Zoints, 1974). A constraint is a formula that limits the feasibility of a solution; the constraints must be represented as inequalities and equalities. LP takes all the constraints related to the same situation and finds the BEST combination for the main objective satisfying the given constraints. To solve a linear programming problem the objective function

and the constraints must be linear. An example of a linear programming problem is the problem that a farmer faces when he has a limited amount of money to buy seeds to plant and has only 10 acres to use, he wants to maximize its profit but each different seed would need a different amount of space and will give a different profit, so which is the best combination of seed to maximize profit. Linear Programming is used everyday for the organization and allocation of resources. Since linear programming is extensively used in economics, linear programming is one of the most important optimization techniques.

Simple Linear Programming problems are solved graphically by plotting the inequalities/equalities formulas that represent the constraints of the problem. The constraints form a bounded area in a x - y plane. The area bounded is called the feasible region, and the corners of the feasible region are the candidates of the optimal solution. To find the value of the corners a pair of lines that intersect must be solved in point of the intersection. Once the coordinates of the intersection points were found those values are tested in the optimization equation, in which we are trying to find to minimum or maximum value. Figure 2 shows the graphic representation of the linear programming method, where the area in blue is the feasible region.

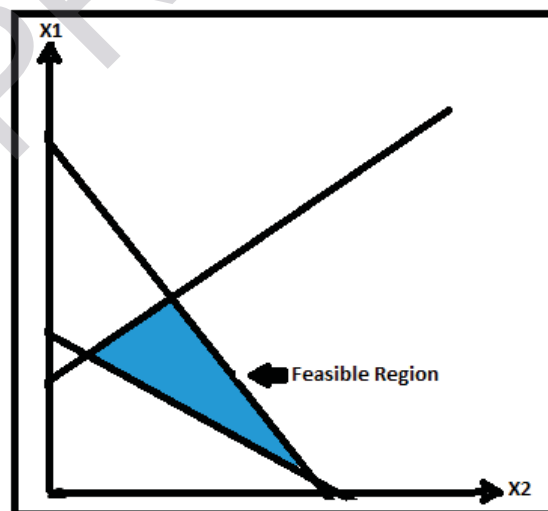


Figure 2-2: Graph of the bounded region of a Linear Programming problem