ADAPTIVE REFINEMENT AND NOISE REDUCTION METHODS FOR SPARSE SIGNAL RECONSTRUCTION

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by

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ABSTRACT

The Iterative Re-Weighted (IRW) algorithm is tested as a solver for the maximum sparseness constraint underdetermined linear inverse problem. The pseudoinverse of the A-Matrix for this problem is formulated and tested using the QR decomposition and the SVD methods to compute it. It is confirmed that the IRW solver for the harmonic retrieval problem, using a fixed uniform grid of N frequencies, converges to a solution with the number of spectrum peaks equal to the number of sinusoids in the test signal.

To improve the resolution of the peaks obtained in the spectrum, we use an Adaptive refinement method because this is computationally more efficient than increasing the uniform frequency grid. The adaptive refinement method upon convergence will obtain a solution whose sparsity measure equals the number of components in the original signal at modest refinement depths. The stability of Adaptive refinement at higher depths of refinement can be problematic as shown with an example.

Two methods are formulated and tested to reduce the effect of noise when the signal is corrupted by additive white Gaussian noise (AWGN). They are, namely, the Regularization and the SVD Truncation methods. Regularization is successful in reducing the noise components in the IRW solution. When the number of sinusoids is known we use this method to find the component frequencies when the SNR is greater than 0 dB by experimentally identifying a range (λ_MIN to λ_MAX) of regularization

parameters that gives a correct number of sinusoids. We conclude that increasing the number of data samples increases λ_MAX and λ_MIN. By increasing the SNR level, the λ_MIN decreases and the λ_MAX increases. Increasing N decreases λ_MAX and λ_MIN. The SVD Truncation method is unsuccessful in removing the noise since by truncating the SVD values we cannot effectively control the number of components in the obtained solution, more work is needed in this approach.

We use Regularization and Refinement to form the composite methods namely Ref_Reg (first refinement then regularization) and Reg_Ref (opposite order) which are used to improve the resolution and to remove the noise components when a signal with off-grid frequency components is corrupted by noise. It is concluded the one that obtains the sparsity match with the smallest value of λ, and which attains the best quality solution, is the Ref_Reg approach.

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CHAPTER 1

INTRODUCTION

1.1 MOTIVATION

The motivations to my thesis comes from [1, 2, 3], where an inverse problem (of an underdetermined system) has been solved for maximally sparse solution using error diversity minimization techniques. Given a small data-set (sum of sinusoids) we find the least number of sinusoidal components that can become the exact fit to the data. The work is initiated by studying the Iterative Re-Weighted (IRW) Algorithm [1] and testing it with various examples. The IRW algorithm detects all "on grid" frequencies in noise free signal but fails to detect off the grid frequencies accurately in a noise free signal. Applications of sinusoidal modeling techniques including two-dimensional extensions include: spectral analysis, frequency-domain feature extraction, direction-of-arrival estimation, radar and ultrasound imaging, radar ranging, etc.

One simple way to detect off the grid frequencies is by refining (reducing the frequency step between the adjacent atoms) the whole Reconstruction matrix. This is computationally inefficient. This serves as a motivation for Adaptive Refinement, which is a computationally efficient method used to detect off the grid frequencies accurately. The effect of stability of the solution with increasing depths of refinement is studied.

In a real time situation the signal is generally corrupted by the noise in the channel and from the noise in the sensor. This gives the drive for a method which reliably

removes the noise components from the obtained signal data (noisy version) not affecting the “uncorrupted-signal” components. Regularization and Truncation are two methods used to remove the noisy peaks from the data. With the impact of known number of sinusoidal components in the signal we find the signal frequencies from noisy signal data. Observations are made on the effect of noise level and grid size and number of data samples on regularization parameter.

Adaptive refinement and regularization are two methods by which we improve the resolution and undo the effect of noise in a signal respectively. Both the methods are together used to form Composite methods that solve for the maximally sparse solution of an unknown signal corrupted by noise based on the measured data and assuming that the number of sinusoidal components in the signal of interest is known. “Refinement followed by Regularization” and “Regularization followed by Refinement” are two composite methods used to solve for off-grid frequencies corrupted by noise (Worst case signal). The composite methods are tested by simulating the algorithms on test examples.

1.2 OUTLINE

In Chapter 2 the ground work for the problem is framed. The Iterative Reweighted Algorithm as a solution of the problem is discussed and is tested with an example. We conclude that IRW algorithm detects correct frequencies only when the noise free signal data with all the components are “on-grid" is input to the algorithm. We form a new

method to solve for the solution using Singular Value Decomposition as opposed to IRW_QR (program) which uses QR decomposition. This program is called IRW_SVD.

In Chapter 3 we study the Adaptive Refinement method, the program is called IRW_REF. This method detects off-grid frequencies with improved resolution and computational efficiency. The stability of IRW_REF at higher depths of refinements is studied by applying the algorithm on an infinitely resolvable signal.

In chapter 4 we study methods which solve for signal frequencies when the signal data is corrupted by noise. First the solutions obtained by IRW when tested with noisy signals are studied. This gives an understanding of AWGN in frequency domain. The process of regularization is then explained. The algorithm is studied. With the impact of known number of sinusoids we solve for signal frequencies when the signal is corrupted by AWGN using regularization and truncation. The process of finding the range of λ over which the sparsity of the solution matches with the known number of components in the signal is called "Sparsity Match". We also find the minimum and the maximum value of λ called λ_Min and λ_MAX respectively for which the sparsity match occurs. REG_IRW (the program for Regularization) is tested with various examples. The effects of various parameters like N (grid size), M (data samples) and σ (noise level) on λ are studied. A noise reduction method based on Singular value decomposition is formed. This program is called SVD_TRUNC. It is tested with various examples. We discard this method because this doesn't offer a good control on the sparsity measure of the obtained solution.

In chapter 5 both the Regularization and Refinement put together are used to obtain signal frequencies for the worst case noisy signal. The method Reg_Ref first regularizes a signal till we obtain the required number of peaks and then refines to improve the solution. The Ref_Reg first refines the solution and then regularize the signal to obtain the sparsity match. Then both the methods are compared by applying on a same noisy signal data. It's concluded that the method that obtains the solution with least value of λ offers a superior quality of the solution. We see that least value of λ is more often attained with Ref_Reg than with Reg_Ref.

In Chapter 6 we summarize the work and make some suggestions for the future work.

1.3 SUMMARY

In this thesis, a model of data which confirms with the underdetermined system of equations is framed. This model is based on the DFT pair of a signal. We solve for the maximally sparse solution of the inverse ill-posed problem formed by the model of the data. The IRW method is studied; alternate methods (using SVD) are formulated and tested. The IRW methods works for a noise free case of signal. The IRW method is tested on the simple cases of Noise free signal with some constituent frequencies on the grid and some off the grid. It's observed that IRW retrieves on-grid frequencies accurately but fails to retrieve off-grid frequencies.

Adaptive refinement method is applied to retrieve off-grid frequencies accurately and efficiently (computational). This method retrieves both off-grid and on-grid frequencies of a noise free signal accurately. By studying the solution at various depths of refinement it's concluded that the algorithm looses stability at higher depths of refinement. With the increase in number of data samples (M) the stability is not guaranteed at higher depths of refinement.

When the signal is corrupted by noise regularization and truncation methods are used to undo (remove the noisy peaks) the effects of noise. Assuming we know the number of sinusoids in the original signal we can find the signal frequencies from noisy signal data. We see that a lower value of λ offers a better quality of solution. The SVD truncation method doesn't offer a flexible control on the sparsity measure of the solution and hence is discarded. It's observed that with increasing number of data samples (M) λ_MAX increases. It's observed that with increasing grid size (N) λ_MAX decreases. It's observed that with the increasing SNR (σ) λ _MAX increases and λ _MIN decreases.

Composite methods are used to detect off-grid signal frequencies in the noisy signal. These methods use both adaptive refinement and regularization to detect off-grid frequencies in noisy signals. "Refinement followed by Regularization" and "Regularization followed by Refinement" are two composite methods used to solve for off-grid frequencies corrupted by noise. It's concluded that the composite method that obtains solution at lower value of λ obtains a better quality fit of the solution.

CHAPTER 2

IRW REVISITED

2.1 INTRODUCTION

In this chapter we form the problem, study the A-Matrix and discuss IRW method as a solution to the problem. We study the IRW algorithm and test it with various examples. The process of finding pseudo inverse with the help of QR decomposition is understood. Also an alternate method of finding the Pseudo inverse of the A matrix, by using the SVD decomposition is examined. The results produced using both the methods are observed. We also observe that Pseudo inverse using QR decomposition takes lesser number of computations [2] and hence is computationally efficient.

2.1.1 Problem Formulation

The problem that we solve is essentially "given a small set of samples (of sum of sinusoids) we find the maximally sparse representation of the signal in frequency domain that has produced this data". We model the signal (figure 1) and its Discrete Fourier transform to be points in the R^N space, where N is the DFT length. The given data (figure 2) is modeled to be a point in R^M space, where M is the number of samples.

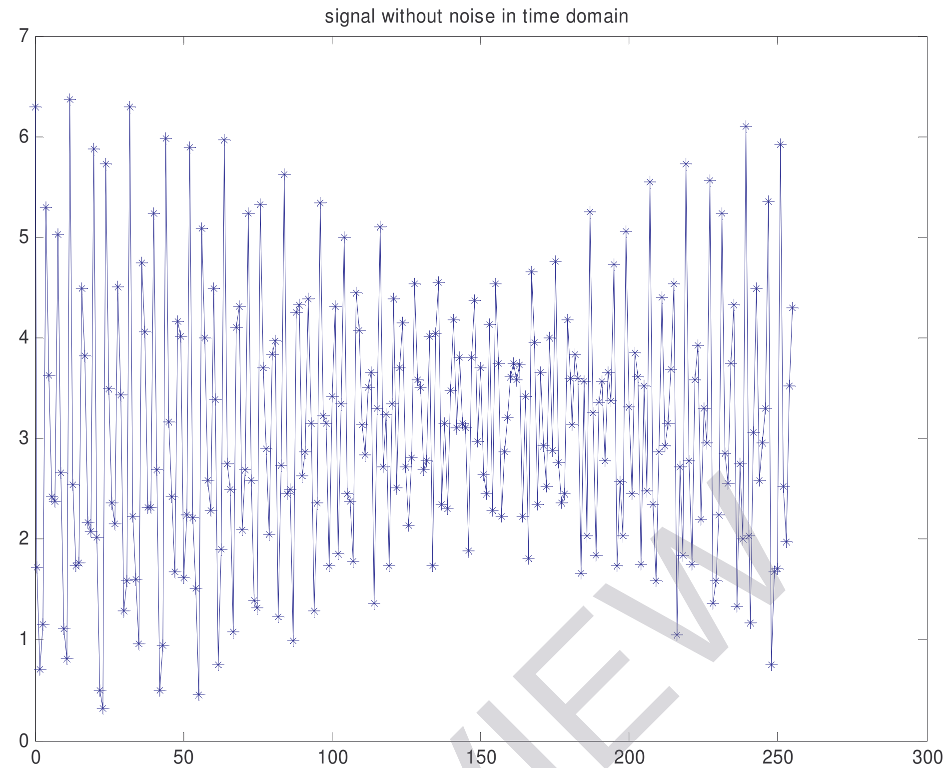

Figure 2.1: An example signal generated using 4 sinusoids of freq.'s 0.0977, 0.2520, 0.5029, and 0.7505

In figure 2.1 we generate a test signal with four frequencies 0.0977, 0.2520, 0.5029, 0.7505. This is an example signal. We collect some contiguous samples of this "signal" and this forms the "signal data". A signal data of figure 2.1 is shown in figure 2.2. This is 25 contiguous samples of signal shown in figure 2.1.

To determine the N-point DFT of the signal, we need N points of data but we have only M-points (M<N) of signal data, hence we model the data to be an underdetermined system of equations and of all the solutions (representations of the signal in frequency domain) of the system we seek for the maximally sparse solution.

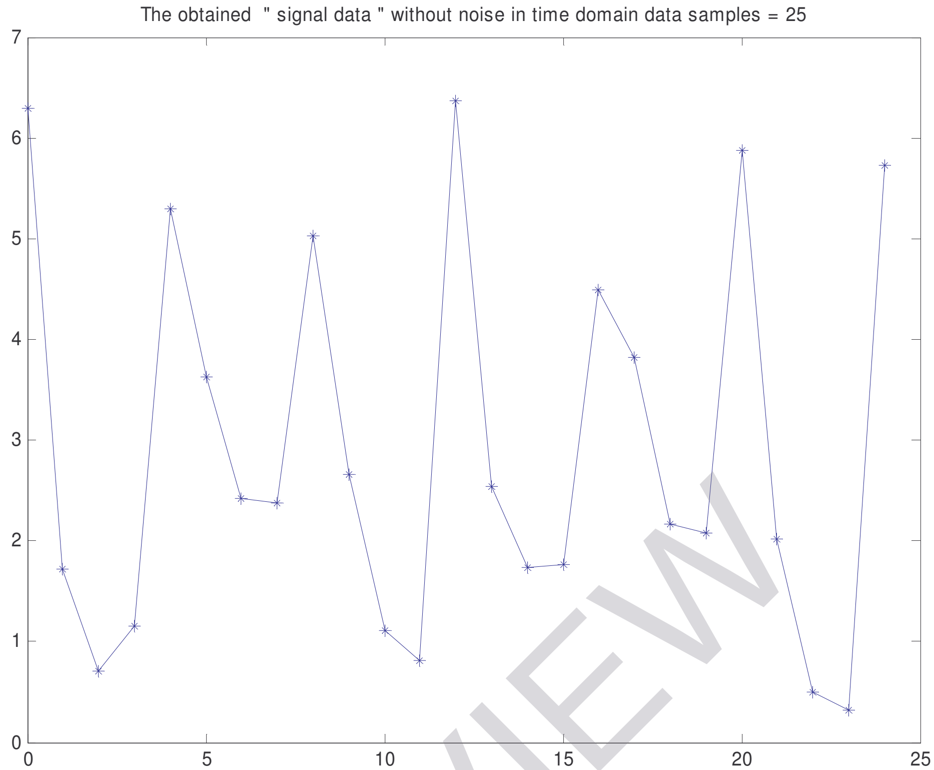

Figure 2.2: An example Signal data (25 contiguous samples beginning at origin from figure 2.1)

We know the DFT pair of a signal is represented by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \qquad k = 0,1,\ldots\ldots\ldots\ ..,\ N-1 \tag{2.1}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N} \qquad n = 0,1,\ldots\ldots\ldots\ ..,\ N-1. \tag{2.2}$$

The DFT pair of equations can be represented in Matrix form [4] as

$$X = F\ X \tag{2.3}$$

$$x = \frac{1}{N} F^{H} X \tag{2.4}$$

Where, x is the obtained signal N-vector

X is the N vector DFT of the signal x

And F is the DFT matrix given by

$$F = \begin{bmatrix} 1 & 1 & \cdots\cdots\cdots & 1 \\ 1 & e^{-j2\pi/N} & \cdots\cdots\cdots & e^{-j2\pi(N-1)/N} \\ . & . & \cdots\cdots\cdots & . \\ . & . & \cdots\cdots\cdots & . \\ . & . & \cdots\cdots\cdots & . \\ 1 & e^{-j2\pi(N-1)/N} & \cdots\cdots\cdots & e^{-j2\pi(N-1)(N-1)/N} \end{bmatrix} \tag{2.5}$$

Since we have less than the required number of data samples we truncate the above matrix form so that it can be applied to our case of underdetermined system of equations. The representation of the obtained data in the matrix form is given by

$$x = \frac{1}{N} T F^{H} X \tag{2.6}$$

Where T is the truncation matrix given by $T = [I_{MXM} \quad 0_{(MXN)-M}]$, M<N

F is the DFT matrix

If we set $A=TF^H$ and ignore normalization then we have a general representation of an inverse ill posed Problem

Given by

$$A\underline{X}=\underline{b} \tag{2.7}$$