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CONTROL OF NONLINEAR SYSTEMS

by

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TITLE

CONTROL OF NONLINEAR SYSTEMS

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CONTROL OF NONLINEAR SYSTEMS

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Most real world systems are at least somewhat nonlinear. Various methods have been proposed for controlling these nonlinear systems. A method, referred to as dynamic linearization, of controlling these systems is developed here. The method finds a feedback controller which forces the response of a given nonlinear system to follow the response of a predetermined linear system. This linear system will have been selected to meet some predetermined design criteria. A test is provided that gives sufficient conditions for the existence of such a control. Algorithms are also given that allow implementation on a digital computer, both for cases that satisfy the sufficient conditions and those that do not.

In addition to the deterministic case, the stochastic case is also presented. Separability is assumed and the dynamic linearization technique is coupled with an extended Kalman filter. To allow the control to have a dual effect, a term proportional to a function of the covariance matrix is added to the performance criterion. This case is also presented in a

form for implementation on a digital computer.

An overview of the current research in the control of deterministic nonlinear systems and in the stochastic control problem is also presented. Several illustrative examples and a more complex application to an aircraft landing system are also found here. In addition, some areas for further research are suggested.

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CHAPTER 1

INTRODUCTION

Most realistic control systems are at least somewhat nonlinear. Various techniques have been suggested for determining the control of these nonlinear systems. One of the most common is to model the nonlinear system over a particular range by an approximate linear model. Linear control theory, which has developed to a higher level of maturity than nonlinear theory, is then applied to determine the control. In general, however, nonlinear processes can only be adequately described by nonlinear models. It is the purpose of this dissertation to introduce a technique for controlling nonlinear systems.

Often it is sufficient to control a particular nonlinear system so that the system's response follows the response of a predetermined linear system. This linear system response will have been chosen to meet some predetermined design criteria. In this dissertation a method will be presented that finds a closed loop control law that causes the nonlinear system response to follow the response of a predetermined linear system. Thus the nonlinear system is made to mimic a linear system in a dynamic or continuous fashion. This can be referred to as dynamic linearization. An advantage of this method is that it uses the nonlinear system model directly.

The subject of this dissertation was motivated, in part, by work of Dr. James W. Cunningham of Sverdrup Technology, Inc., Arnold Airforce Station, Tennesse. He used a similar approach to control a high order system with time delays. By working backwards through the equations, he was able to determine the required control, to cause the output to respond like a second order linear system. However, the implementation details here are totally different. In addition, one of the goals here is to determine when such an approach will succeed. Also, the stochastic case is considered.

The organization of this dissertation is as follows. The basic theory will be presented including some sufficient conditions that guarantee the existence of such a control law. An algorithm will also be presented for implementation on a digital computer. Both the deterministic and stochastic cases will be examined. This method will then be applied to the design of an automatic aircraft landing system.

Chapter Two briefly overviews the main areas of deterministic nonlinear systems research in Section 2.1, then reviews the literature related to the control of deterministic nonlinear systems in Section 2.2. Some of the areas reviewed are transform or approximation methods, including linearization and series techniques

(Section 2.2.1), controllability studies (2.2.2), and stability approaches (2.2.3).

Chapter Three presents the basic method of this dissertation as applied to deterministic nonlinear systems. An overview of some of the problems encountered when controlling nonlinear systems is presented in Section 3.1. A review of the implicit function theorem and Pontryagin's minimum principle is presented in Section 3.2. In Section 3.3 the problem is stated and the method of solution is presented. Section 3.4 presents a method for implementation on a digital computer. A number of illustrative examples are given in Section 3.5.

The stochastic case is presented in Chapter Four. Section 4.1 defines the stochastic control problem and examines it's various properties. Section 4.2 reviews recent research related to the solution of the stochastic control problem. The stochastic dual adaptive control method (Section 4.2.1) and it's variations (Section 4.2.2), along with various heuristic methods (Section 4.2.4) are included in the discussion. In Section 4.3 the method presented in Chapter Three is extended to the stochastic case. Illustrative examples are given in Section 4.4.

In Chapter Five, the method presented in this dissertation is applied to determining the control law for an aircraft in the final phase of landing. A simple aircraft model is used which allows for random wind gusts of various intensities. Results show that this method could be used to reasonably control the aircraft. Finally, Chapter Six reviews the method presented and outlines some areas for further research.

CHAPTER 2

CONTROL OF DETERMINISTIC NONLINEAR SYSTEMS: RECENT RESEARCH

Methods for controlling nonlinear systems are not very well developed when compared with those for linear systems. In recent years, there has been an increase in research in the study of nonlinear systems and their control. Most of this research has been concerned with the analysis of nonlinear systems, particularly in studying the response of a particular system. Less research has been done in the area of control of nonlinear systems. In this chapter a brief overview of nonlinear systems analysis techniques is given in Section 2.1. In Section 2.2, a survey of current research related to the control of deterministic nonlinear systems is presented. References [6, 57, 66, 93, 105, 137, 145] are good for further introductory material on nonlinear systems and their control.

Most work has been motivated by the interest in a particular application, such as nuclear reactor control [47, 150,], aircraft and missle guidance and control [48, 87, 128], control of chemical and industrial processes [11, 77, 129], overhead cranes [79, 111], automobile engines [103], and solar systems [125]. In addition most nonlinear research is concerned with a

particular class of systems, such as the bilinear case [67, 84, 85, 94], polynomial systems [112], second order systems [115], systems that are separable into linear and nonlinear parts [4], and systems of special forms such as the example given by Equation (2.1) [28, 62, 63, 64, 68, 119].

$$\dot{x} = f(x,t) + \sum_{i=1}^{m} u_i(t) g_i(x,t) \qquad (2.1)$$

The methods used for either analysis or control of these systems often tend to be based on the specific problem.

2.1 Nonlinear Systems Analysis

2.1.1 Techniques

Before reviewing some of the recent literature on the control of nonlinear systems, it would be worthwhile to overview some of the main techniques used in the study of nonlinear systems. Extensions of these are often used in the control of nonlinear systems.

2.1.1.1 Closed Form Analysis

In some cases a closed form solution of the problem is known or can be found. This known solution is then used in the analysis. However, it is normally extremely difficult or impossible to find closed form solutions. This method is applicable in only a very few special

cases.

2.1.1.2 Computer Simulations and Algorithms

The rapid advances in size and speed of computers, has caused an increase in the use of computer simulations in nonlinear systems analysis. This has stimulated the development of efficient algorithms, particularly for finding solutions to nonlinear initial value problems. Problems with split boundary conditions, however, are more involved and not always as easily adaptable to a computer solution.

2.1.1.3 Phase Plane

The phase plane concept has been used in the analysis of nonlinear systems for some time. The trajectories of the system are plotted in the state space. Since the representation of trajectories in three or more dimensions is not realistic, this method is really only useful for first or second order systems.

2.1.1.4 Transform or Approximation Methods

Perhaps the most frequent method used in the study of nonlinear systems is to transform or approximate the nonlinear system by a linear system. Then linear system theory is used to analyze the approximate equivalent linear system. A number of different methods of

approximation are used.

2.1.1.4.1 Transforms

One method that is used to change the nonlinear system to a linear one is to transform the states in the nonlinear equations in such a way that a linear system results. This is often done for a specific type of system and therefore the transform may not be useful for other types of nonlinear systems.

2.1.1.4.2 Direct Linearization

One of the most common methods used is to approximate the nonlinar system by linearizing about an optimal or nominal trajectory. Usually the linear terms of a Taylor series are used for the approximation. The linearization will depend only on the system and the choice of the optimal or nominal operating path. A disadvantage of this method is that the linearization must occur at each point.

2.1.1.4.3 Quasilinearization and Describing Functions

In quasilinearization [45], the operation performed by the nonlinear element on a specific finite input to the system is approximated by a linear operation. "These quasi-linear approximating functions, which describe approximately the transfer characteristic of the

nonlinearity, are termed describing functions" [45]. This linearization depends on the system and the input to the system. A sequence of these linear problems is solved. Each linear problem has an input that describes known deviations from the previous linear solution. This sequence of linear solutions converges to the nonlinear solution. The main use is for two point boundary value problems.

2.1.1.4.4 Series Expansion

The solution of the nonlinear system is assumed to be a series. The coefficients of the series are found which causes the series to satisfy the original nonlinear equation. The first terms of the series are usually linear, so this method works best when the system is only slightly nonlinear.

2.1.1.4.5 Volterra Series

The output of the system is represented in terms of the input by an infinite functional series of kernels, that is, a Volterra series [107].