POLE PLACEMENT AND FUZZY LOGIC CONTROL SCHEMES FOR AN INVERTED PENDULUM WITH DISTAL END CONTROL

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ABSTRACT

This thesis addresses the use of two different control schemes for an inverted pendulum with a freely hinged base and having a control torque applied at the distal end. The control torque is applied to achieve the objective of balancing the first link.

The first control approach involves the use of full spectral assignability to determine the proper torque value. The second control approach uses a fuzzy logic methodology which is based on the expert's knowledge about the system. The results obtained in each control scheme are analyzed and compared. The resulting control system could be used to stabilize a fireman's ladder or a child's bicycle.

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Chapter 1

INTRODUCTION

A common demonstration control problem is balancing an inverted pendulum [1]. The control system design presented in this thesis is not the conventional "rod-on-cart" inverted pendulum, but rather, an inverted pendulum with a hinged base and control torque applied at the distal end.

Chapter 2 presents the description of the dynamical model for the system. The system is modeled as a two link planar structure. The base is assumed to be a pinned (hinged) connection. A motor is placed at the revolute joint between the first and second links. The motor provides a control torque to accomplish the objective of balancing the first link. Two different control schemes are used to design the control system. The results obtained in each scheme are analyzed and compared to determine the best control approach to be used in the two-link structure. In a practical setting, the resulting control system could be used to stabilize a fireman's ladder or a child's bicycle.

The first control scheme involves the use of full spectral assignability to determine a state-feedback controller [2]. This approach requires the derivation of a

mathematical model describing the dynamical behavior of the system. The mathematical model of the two-link structure is obtained from the basic physical laws governing its motion. This involves the use of the concepts such as the kinetic and potential energies, as well as Lagrange's equations of motion [3]. By using state-variable representation, the mathematical model of the system can be described by a set of first-order nonlinear differential equations [4]. Upon using a moving linearization technique, an approximated linear time-varying model of the system can be obtained [5]. This model represents a linear time-invariant system over a sufficiently short sampling period. Hence, the eigenvalue assignment method can be used to compute the state feedback controller for the system over one sampling period at a time.

To verify the controllability of the system, a controllable canonical representation is obtained through a similarity transformation [4]. The controllable canonical representation is used to design a feedback gain matrix which properly assigns the eigenvalues of the system, to the desired locations. Using this feedback matrix, the control input torque is computed for the corresponding sampling time. The control approach using spectral assignability is discussed in chapter 3.

The presence of friction in the system is also considered when designing the state feedback controller. It is reasonable to assume that there exists friction in the system at the hinge and control points. This friction has a damping effect on the behavior of the system [6]. It will be shown that the presence of friction in the system is critical in obtaining an effective performance using state feedback control.

The second control approach discussed in this thesis applies the methodology of fuzzy logic to obtain the proper system controller. Fuzzy control is a technique that translates the expert's knowledge about the system into an actual control strategy [7]. Fuzzy logic has been succesfully applied to several control system problems such as congestion in computer netwoks, subway train control, and focussing of cameras. This method can be useful when traditional control theory is not directly applicable. The two-link planar structure fits in this category since the system behavior is described by a set of highly coupled nonlinear dynamical equations.

The basic step in the proposed fuzzy logic technique is to make a list of common-sense rules (formulated in natural language) that govern the control system [1]. After this, the fuzzy rules are translated into the actual control strategy by performing operations on the three main elements in a fuzzy control system. These elements are the fuzzified unit, the fuzzy controller, and the defuzzified unit [8]. The

function of each of these elements is discussed in detail in chapter 4.

In both control schemes, computer simulation programs are developed to verify the effectiveness of the control system. The results obtained for each case are essential in the determination of the best control approach for the two-link structure. By using an animation model, a better perception of the simulation of the control system is achieved.

Finally, an analysis and evaluation of the two control approaches discussed throughout this thesis is presented in chapter 5. Suggestions for future work are also provided.

Chapter 2

DYNAMICAL MODEL

2.1 Description of the Dynamical System

To analyze and design a control system, it is necessary to use a mathematical model that describes the dynamical behavior of the system [2]. By definition a dynamical system is one whose output depends on previous input values. Such a system is generally changing with time due to external inputs and internal states. This chapter presents a derivation of the dynamical equations of motion for an inverted pendulum with a hinged base and control applied at the distal end. The preferred approach is the energy method, which is based on the analysis of the kinetic and potential energy of the system using Lagrange's equations of motion [3]. It is important to mention that in this case the dynamics of the electric or hydraulic motor driving the inverted pendulum are ignored.

The system is modeled as a two-link planar elbow arm structure as shown in Figure 2.1. The base is assumed to be a pinned i.e., hinged connection. A motor is modeled at the revolute joint (*joint A*) between the first and second links. It is assumed that the mass of the first rod is negligible compared to the mass of the motor

attached at the end of this rod. Thus, the mass of this rod is considered as a point mass m_1 at the distal end of the link. The distance from the origin *point 0,* or the pinned connection to the center of mass m_1 is l_1. For the second rod, m_2 is the center of mass of this rigid body, and l_2 is the distance from *joint A* to the center of mass m_2. The angle that describes the orientation of the first link is θ_1. This angle is measured from the horizontal line passing through *point 0* to the line passing through the center of mass m_1 (*line 0A*). The angle between *line 0A* and the line passing through the center of mass m_2 is θ_2. Finally, a control torque is supplied by the motor at *joint A* to achieve the objective of balancing the first link.

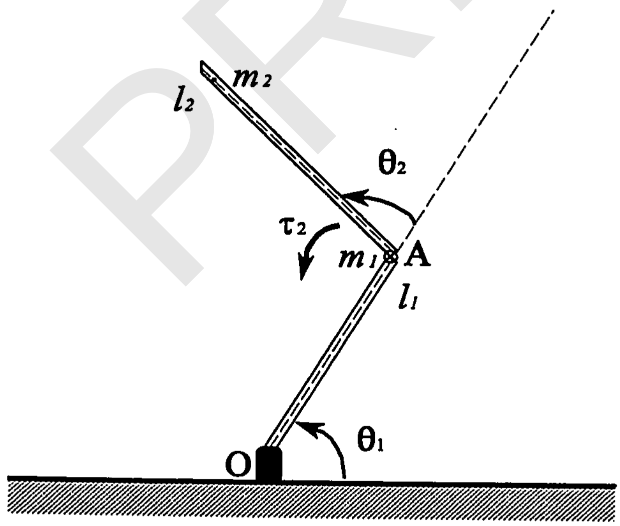

Figure 2.1 Two-link planar elbow arm structure.

2.2 The Dynamical Equations of Motion

In this section the dynamical equations of motion for the two-link planar structure are derived. These equations are necessary for obtaining a good mathematical model that describes the dynamical behavior of the system. Lagrange's equations of motion are used to derive the system dynamics [3].

2.2.1 Review of Lagrange's Equations of Motion

What follows is a brief review of some important concepts from physics that play a very important role in understanding the system dynamics.

Position Vector. The position vector of a mass can be described by the displacement vector **r** from the origin *point 0* to any *point P* [9]. The components of vector **r** are the coordinates x and y, so that

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} \qquad \textbf{(2.2.1)}$$

where **i** and **j** are the unit vectors in the x and the y directions respectively.

Linear Velocity. The linear velocity of a mass m orbiting at a distance r from the axis of rotation and angular velocity ω is given by [9]

$$v = \omega r \qquad \textbf{(2.2.2)}$$

The direction of this motion is always tangent to the circular path at each point of the trajectory.

Kinetic Energy. The kinetic energy of a mass m with linear velocity v is defined as [9]

$$T = \frac{1}{2} m v^2 \tag{2.2.3}$$

Potential Energy. In a gravitational field with constant g, the potential energy of a mass m at a height y above the reference level (the origin of coordinates) is given by [9]

$$U = mgy \tag{2.2.4}$$

Lagrange's Equations of Motion

In a conservative system, Lagrange's equation of motion is defined by [3]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{2.2.5}$$

$$(i = 1, 2, \ldots, n)$$

where

$\boldsymbol{q}$: n-vector of generalized coordinates

$\boldsymbol{Q}$: n-vector of generalized forces not derivable from a potential function.

The Lagrangian function **L** is obtained by subtracting the potential energy from the kinetic energy [3],

$$L = T - U \tag{2.2.6}$$

2.2.2 Derivation of the Equations of Motion

The dynamical equations of motion for the two-link planar system are derived according to the geometry in Figure 2.1. The mass in each rod is considered as a point mass at the distal end of each link. Thus, there are two point masses m_1 and m_2. The kinetic energy of the system is given by

$$T = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2 \tag{2.2.7}$$

where V_1 and V_2 are the linear velocities of masses m_1 and m_2 respectively.

For the first link, the squared velocity of mass m_1 is given by

$$V_1^2 = l_1^2 \dot{\theta}_1^2 \tag{2.2.8}$$

The velocity of the second link is obtained by computing the position vectors to each point mass m_1 and m_2. As shown in Figure 2.2 these position vectors are $\mathbf{r}_2$ and $\mathbf{r}_1$, where $\mathbf{r}_{2/1}$ is the position vector of mass m_2 with respect to mass m_1.

Then,

$$\mathbf{r}_2 = \mathbf{r}_1 + \mathbf{r}_{2/1} \tag{2.2.9}$$

Rewriting vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_{2/1}$ using the corresponding x and y components yields

$$\mathbf{r}_1 = l_1\, cos\theta_1\, \mathbf{i} + l_1\, sin\theta_1\, \mathbf{j} \tag{2.2.10}$$

$$\mathbf{r}_{2/1} = l_2\, cos(\theta_1 + \theta_2)\, \mathbf{i} + l_2\, sin(\theta_1 + \theta_2)\, \mathbf{j} \tag{2.2.11}$$

$$\mathbf{r}_2 = [l_1\, cos\theta_1 + l_2\, cos(\theta_1 + \theta_2)]\, \mathbf{i} + [l_1\, sin\theta_1 + l_2\, sin(\theta_1 + \theta_2)]\, \mathbf{j} \tag{2.2.12}$$

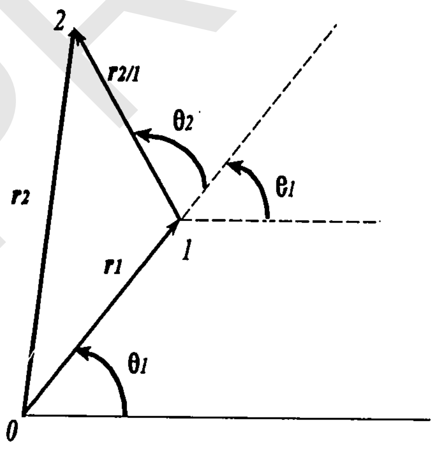

Figure 2.2 Position vector to point masses m_1 and m_2.

Now, to obtain velocity $\mathbf{V_2}$:

$$\mathbf{V_2} = \frac{dr_2}{dt} = [l_1 \sin\theta_1 \dot{\theta}_1 + l_2 \sin(\theta_1 + \theta_2)\ (\dot{\theta}_1 + \dot{\theta}_2)]\boldsymbol{i} + [l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos(\theta_1 + \theta_2)\ (\dot{\theta}_1 + \dot{\theta}_2)]\boldsymbol{j}$$

(2.2.13)

Then, the magnitude squared of the velocity is

$$\mathbf{V}_2^2 = l_1 \dot{\theta}_1^2 + l_2^2\,(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2 l_1 l_2 \dot{\theta}_1\,(\dot{\theta}_1 + \dot{\theta}_2) \cos\theta_2 \qquad \textbf{(2.2.14)}$$

Now the total kinetic energy of the system is,

$$T = \frac{1}{2} m_1\,(l_1^2 \dot{\theta}_1^2) + \frac{1}{2} m_2\,[l_1^2 \dot{\theta}_1^2 + l_2^2\,(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2 l_1 l_2 \dot{\theta}_1\,(\dot{\theta}_1 + \dot{\theta}_2) \cos\theta_2\,]$$

(2.2.15)

Whereas the total potential energy of the system is,

$$U = U_{m1} + U_{m2} \qquad \textbf{(2.2.16)}$$

where U_{m1} = Potential energy due to mass m_1

U_{m2} = Potential energy due to mass m_2

Thus,

$$U = m_1 g l_1 \sin\theta_1 + m_2 g\,(\,l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)\,) \qquad \textbf{(2.2.17)}$$

The Lagrangian function is

$$L = T - U = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2 [l_1^2\dot{\theta}_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2l_1l_2\dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2]$$

$$- m_1 g l_1 \sin\theta_1 - m_2 g (l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)) \qquad \textbf{(2.2.18)}$$

Choosing generalized coordinates θ_1 and θ_2, expressions for $\frac{\partial L}{\partial \theta_1}, \frac{\partial L}{\partial \dot{\theta}_1}, \frac{\partial L}{\partial \theta_2}, \frac{\partial L}{\partial \dot{\theta}_2}$: are found:

$$\frac{\partial l}{\partial \theta_1} = -m_1 g l_1 \cos\theta_1 - m_2 g l_1 \cos\theta_1 - m_2 g l_2 \cos(\theta_1 + \theta_2) \qquad \textbf{(2.2.19a)}$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = \dot{\theta}_1 m_1 l_1^2 + m_2 l_1^2 \dot{\theta}_1 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)m_2 + l_1 l_2(\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2 m_2 + l_1 l_2 \dot{\theta}_1 \cos\theta_2 m_2 \qquad \textbf{(2.2.19b)}$$

$$\frac{\partial L}{\partial \theta_2} = -l_1 l_2 \dot{\theta}_1 m_2 (\dot{\theta}_1 + \dot{\theta}_2)\sin\theta_2 - m_2 g l_2 \cos(\theta_1 + \theta_2) \qquad \textbf{(2.2.19c)}$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = l_2^2(\dot{\theta}_1 + \dot{\theta}_2)m_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos\theta_2 \qquad \textbf{(2.2.19d)}$$

Therefore, the Lagrange equations of motion for this system are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = \tau_1 = 0 \qquad \textbf{(2.2.20a)}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = \tau_2 \tag{2.2.20b}$$

Equations (2.2.20) give the following expressions :

$$m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 \cos\theta_2 (2\ddot{\theta}_1 + \ddot{\theta}_2) + (m_1 + m_2) l_1^2 \ddot{\theta}_1 - m_2 l_1 l_2 \ddot{\theta}_2^2 \sin\theta_2 -$$
$$2 l_1 l_2 m_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2 + m_2 g l_2 \cos(\theta_1 + \theta_2) + (m_1 + m_2) g l_1 \cos\theta_1 = \tau_1$$

(2.2.21a)

$$m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 g l_2 \cos(\theta_1 + \theta_2) + m_2 l_1 l_2 \cos\theta_2 \ddot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_1^2 \sin\theta_2 = \tau_2$$

(2.2.21b)

Rearranging terms yields the following two coupled nonlinear differential equations:

$$\tau_1 = m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2 \cos\theta_2 (2\ddot{\theta}_1 + \ddot{\theta}_2) + (m_1 + m_2) l_1^2 \ddot{\theta}_1 - m_2 l_1 l_2 \dot{\theta}_2^2 \sin\theta_2 -$$
$$2 l_1 l_2 m_2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2 + m_2 g l_2 \cos(\theta_1 + \theta_2) + (m_1 + m_2) g l_1 \cos\theta_1 = 0 \tag{2.2.22a}$$

$$\tau_2 = m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 g l_2 \cos(\theta_1 + \theta_2) + m_2 l_1 l_2 \cos\theta_2 \ddot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_1^2 \sin\theta_2$$

(2.2.22b)

Equations (2.2.22) are the final dynamical equations of motion of the two-link planar system.