THE LEARNING OF MIXED STRATEGY SOLUTIONS IN TWO-PERSON, 2 X 2, ZERO-SUM GAMES: AN ARGUMENT FOR RATIONALITY

A Thesis

Presented to

the Faculty of the Department of Psychology

The University of Texas at El Paso

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

by

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August 19, 1974

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ACKNOWLEDGEMENTS

It has been the author's privilege to have associated with Drs. Edmund B. Coleman, William G. Lucker, Gerald R. Miller, and Timothy P. Roth in the development of this research. The foundations of this work rest upon the philosophies and refinements of experimental design and statistical analysis gained from association with Drs. Coleman and Miller. As a published researcher in this area of behavioral inquiry, Dr. Lucker's contribution to the method were highly valued. Appreciation is extended to Dr. Roth for his critical reading of the manuscript.

The design of the apparatus is credited to Mr. Thomas F. Linsley, a good friend and colleague. The funding for this research was provided through the office of the Graduate Dean.

ABSTRACT

The purpose of this research was to investigate competitive behavior in the context of two-person, 2 X 2, zero-sum games. Thirty-two university students were divided into Rational Competition (RC) and Programmed Competition (PC) groups. In the RC group, eight pairs of subjects competed across four game matricies for 200 plays. The linear trend of the response proportions was found to account for 94.79% of the variance ($p < .001$). Differences in trends were found across the game matrices ($p < .001$). Rational decision behavior predicted from the Minimax Theory (Von Neuman & Morgenstern, 1944) required an ordering of response proportions across the matrices. Significance was attained at $p < .0007$. Incentive was defined with respect to the value of random play. Incentive was found to interact with game value ($p < .02$). In the PC group, four subjects competed against programmed minimax mixtures in each matrix condition. These subjects were dichichotomized by the game values of the matrices. The learning of minimax mixtures was not evidenced. Game value was found to interact with matrices ($p < .02$). The variance in payoffs between the alternatives was proposed as an accounting for the effect. "Tracking" (Lieberman, 1962; Fox, 1972) was not evidenced. "Tracking" was interpreted as a function of the method of generating programmed sequences. In contrasting behaviors of the RC and PC groups, the "real world" validity of programmed interaction was rejected.

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THE LEARNING OF STRATEGIES IN 2 x 2, ZERO-SUM GAMES: AN ARGUMENT FOR RATIONALITY

Richard L. Ward

The Issue of Rationality

In 1929, John Von Neumann introduced the concept of gaming. The ideas which he had advanced later matured into a now classic text: Theory of Games and Economic Behavior (Von Neumann & Morgenstern, 1944). In reviewing the work upon its publication, Copeland (1945) declared that: "Posterity may regard this book as one of the major scientific achievements of the first half of the twentieth century." The declaration may have been prophetic.

The philosophical orientation of the work has been carefully (and harshly) reviewed by Daniel Ellsberg (1956). The review, Theory of the Reluctant Duelist, was perhaps the first to take issue with the determination of rational behavior advanced by Von Neumann and Morgenstern (1944). The prescriptive nature of the theory dictates that a game player's rational behavior requires him to minimize his maximum losses. The prescribed strategy is, from this view, essentially defensive. The player who would submit to a social situation wherein his rational behavior is merely a defensive position is assumed to do so only reluctantly (Ellsberg, 1956). Ellsberg (1956) concludes his review with the following critique: "A theory of reluctant duelists is not a small achievement. But it could not be reliable in predicting behavior in situations corresponding to the zero-sum two-person game; nor is it possible that players should be advised to conform to it against their inclinations. It is certainly not a theory of games. It is not a theory of rational behavior under conditions of uncertainty; that theory lies in the future (p. 923)."

Elements of the Prescriptive Theory

Games represent, in matrix (normal) form, the strategies available to people in social interactions and the payoffs yielded to each of the players when a play has been executed. The choices of the players are made under conditions of uncertainty, such that the outcome of the play is not determined until the strategic decisions of each of the players have been made.

The most elemental social interactions are represented in the 2 X 2 games. Two individuals (or groups) are required to decide between two alternatives. The combination of alternatives determines one of four outcomes.

		Y: A_{y1}	Y: A_{y2}
X	A_{x1}	a, e	b, f
	A_{x2}	c, g	d, h

Table I. General Form of 2 X 2 Games.

All 2 X 2 games are represented by the matrix given in Table I. In this situation, if player X were to choose his first alternative (A_{x1}), and player Y were to choose his second alternative (A_{y2}), outcome b, f would obtain. This outcome yields a payoff of value b to the row player (X), and a payoff of f to the column player (Y). The payoffs may be in any form, depending upon the real world nature of the interaction, e.g., dead soldiers, girl slaves, money, revenge, happiness, etc. The payoffs numerically represent the subjective values of the outcomes to the players (Von Neumann & Morgenstern, 1944).

The games essentially fall into two categories, distinguished by the degree of conflict between the players concerning the desirability of the

various outcomes. In non-zero-sum games, the degree of conflict may vary. In zero-sum games, a state of pure conflict exists. The outcome most preferred by player X is least preferred by player Y, etc.

Given that the preferences for the various outcomes are strictly ordered, a population of three unique zero-sum games exists. These are the games numbered 11, 45, and 75 (Table II) in the taxonomic system developed by Rapoport and Guyer (1966).

2, 3	4, 1
1, 4	3, 2

Game 11

3, 2	4, 1
2, 3	1, 4

Game 45

2, 3	4, 1
3, 2	1, 4

Game 75

Table II. Population of 2 X 2 Zero-Sum Games with Payoffs as Preference Orderings.

The minimax theory (Von Neumann and Morgenstern, 1944) prescribes the rational strategy choices for these games. For games 11 and 45, the strategies A_{x1} and A_{y1} should be played consistantly. Both the row and column players receive their highest payoffs from these strategies in game 11. In game 45 the row player would have no reason to deviate from A_{x1}, thus the column player gains the highest payoff available to him from A_{y1}. In games 11 and 45, the row player's payoff, a, which results from this strategy combination, is both the minimum payoff in the row and the maximum payoff from the column. Thus, the minimax prescription is fulfilled. The payoff pair is known as the saddlepoint, the term being derived from the observation that if two perpendicular planes quarter an English saddle, their intersection determines coordinates with maximum and minimum values upon the surface of the saddle. This is illustrated in Figure I, where a saddlepoint is deter-

mined by the intersection of AB and CD. The point of intersection is the lowest point on AB, and the highest on CD.

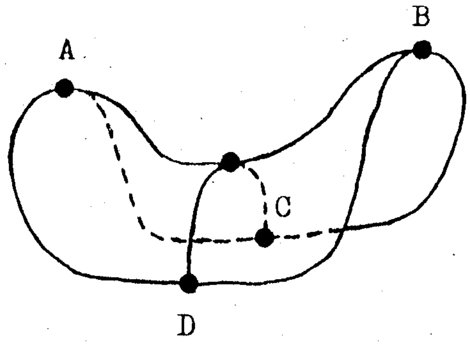

Figure I. Saddlepoint.

Game 75 is representative of the population of zero-sum games which require mixed strategy solutions. This is to say that the choice of alternatives by each of the players should be randomly mixed in a specified proportion, or chosen with probabilities equivalent to the specified proportion.

The appropriate mixture balances the payoffs resulting from various outcomes, such that the player obtains a security level of maximum value. A derrivation of the variance about this value has been advanced by Fox (1972).

Rapoport (1966) has detailed the algebraic logic in determining the appropriate mixtures. For game 75, it can be seen that if the row player mixes his alternatives with percentages of x and 1 -x, and player Y chooses A_{y1}, the payoff to X is given by $2x + 3(1 - x)$. When Y chooses A_{y2} the value is given by $4x + 1(1 - x)$. If the game value is to be independent of Y's strategies, and therefore secure, these quantities must be equal. Thus, $2x + 3(1 - x) = 4x + 1(1 - x)$, and $x = .50$. For the column player, who plays A_{y1} and A_{y2} with percentages of y and $1 - y$, the mixture is determined by $3y + 1(1 - y) = 2y + 4(1 - y)$, and $y = .75$. The row player should choose his alternatives in the proportion of 1:1. The column player should choose