# ELECTROMAGNETIC FIELD CALCULATIONS FOR MICROLENS OPTICAL SYSTEMS

by

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Microlenses are becoming more widely used in modern optical equipment. When the microlens diameter is comparable with the incident electromagnetic illumination wavelength, diffraction effects through the microlens aperture dominate and significantly affect the microlens optical properties leading to differences from that predicted by ordinary geometrical optics theory. In this work, the continuous-profile symmetrical biconvex microlens is selected for investigation. Its optical properties, with both monochromatic plane wave and TEM00 mode Gaussian beam illumination, are studied using the full-field Separation-of-Variables method (SVM) in the oblate spheroidal coordinate system by calculating the electromagnetic field distributions inside of and adjacent to the microlens. The microlens optical properties are also compared with the corresponding geometrical optics theory.

The investigations and discussions include the focusing properties of a single microlens with monochromatic plane wave illumination, the beam transformation properties of a single microlens with monochromatic TEM00 mode Gaussian beam illumination, the axial combination properties of dual microlenses with monochromatic plane wave and TEM00 mode Gaussian beam illumination, the interference properties

between dual parallel-arranged microlenses with monochromatic plane wave illumination, and the imaging properties of a single microlens with monochromatic plane wave and TEM00 mode Gaussian beam illumination. The optical properties of microlens optical systems are found to be similar to that given by the geometrical optics theory. The microlens actual focal length is measured for different profile and diameter microlenses and is compared with its corresponding geometrical focal length. It is shown that the microlens actual focal length is an important parameter and can be used to describe and approximately formulate the microlens optical properties. The transmitted beam waist position through a microlens calculated using the Rayleigh Range method (RRM) with the microlens actual focal length closely matches the exact value determined using the Separation-of-Variables method in the oblate spheroidal coordinate system. The axial combination properties of dual microlenses with monochromatic plane wave illumination and the imaging properties of a single microlens can also be described using the geometrical imaging formula with the microlens actual focal length.

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# Table of Contents

# List of Figures

# List of Tables

# Nomenclature

$\nabla$ del operator

$\nabla^2$ Laplace operator

$\beta$ spheroidal radial function type $=1,2,3$

$\lambda$ electromagnetic field wavelength

$\lambda_{l,m}$ eigenvalue that corresponds to the scalar eigenfunction $\prod_{l,m}$

$\varepsilon$ electric permittivity

$\varepsilon_{ext}$ dielectric constant of the external medium

$\varepsilon_r$ dielectric constant

$\tilde{\varepsilon}$ complex dielectric constant $=\varepsilon_r + j4\pi\sigma / \omega$

$\phi$ spheroidal azimuthal angle coordinate

$\hat{\phi}$ spheroidal azimuthal angle vector

$\eta$ spheroidal angle coordinate

$\eta_0$ spheroidal angle

$\hat{\eta}$ spheroidal angle vector

$\mu$ magnetic permeability

$\pi$ Pi

$\theta$ angle between the vector parallel to the microlens surface and that parallel to the spheroidal surface

$\theta_{inc}$ incident angle of the incident electromagnetic illumination

$\theta_{pol}$ electric field polarization angle of the incident electromagnetic illumination

$\sigma$ electric conductivity

$\omega$ angular speed of the incident electromagnetic field

$\xi$ spheroidal radial coordinate

$\xi_0$ spheroidal radius

$\hat{\xi}$ spheroidal radius vector

$\xi_l^{(1)} = \psi_l - j\chi_l$

$\prod$ scalar eigenfunction solution of the scalar Helmholtz equation

$\hat{\prod}$ vector eigenfunction solution of the vector Helmholtz equation

$\psi_l(kr)$, $\chi_l(kr)$ Ricatti-Bessel functions

**A** arbitrary vector on the microlens' surface

$A_{l,m}$ expansion coefficient

$A^s_{l',m}$, $A^\phi_{l',m}$ integrals that depend on both the microlens geometry and the incident illumination setting

$a$ semi-major axis length of the pure oblate spheroid and the microlens

$a_{l,m}$ expansion coefficient

$B_{l,m}$ expansion coefficient

$B^s_{l',m}$, $B^\phi_{l',m}$ integrals that depend on both the microlens geometry and the incident illumination setting

$b$ semi-minor axis length of the pure oblate spheroid

$b'$ semi-minor axis length of the microlens

$b_{l,m}$ expansion coefficient

$C_0$, $C_2$, $C_4$ coefficients

$c_{l,m}$ expansion coefficient

$d$ distance between dual microlenses, focus diameter

$d_{l,m}$ expansion coefficient

$D$ microlens diameter

$D_{sphere}$ sphere diameter

$e$ Euler's number

**E** electric field vector

$E_0$ the maximum electric field magnitude of the incident illumination

$E_{\max}$ the maximum electric field magnitude of the focus region

$e_{l,m}$ expansion coefficient

$f$ microlens focal length

$f_b$ microlens back focal length

$f_e$ microlens effective focal length

$f_{act}$ microlens actual focal length

$f_{geo}$ microlens geometrical focal length

$f_{semi}$ semi-focal length of the oblate spheroidal coordinate system

$\mathbf{H}$ magnetic field vector

$h$ spheroidal coordinate size parameter $= kf_{semi}$

$h_b$ distance between the microlens back principal plane and the microlens back spherical surface vertex

$h_{ext}$ spheroidal coordinate size parameter for the external electromagnetic field $= k_{ext} f_{semi}$

$h_f$ distance between the microlens front principal plane and the microlens front spherical surface vertex

$h_{\text{int}}$ spheroidal coordinate size parameter for the internal electromagnetic field $= k_{\text{int}} f_{semi}$

$I_{l,m,l'}^{1-8}$ integral that depends only on the microlens geometry

$j$ imaginary unit, $j^2 = -1$

$\mathbf{k}_0$ propagation vector of the incident electromagnetic illumination

$k$ wave number of the electromagnetic field

$k_0$ wave number of the electromagnetic field in the vacuum

$k_1 = jk_0\tilde{\varepsilon}$

$k_2 = jk_0$

$L$ upper limit of $l$ index, distance between dual parallel-arranged microlenses

$l$ integer index

$l'$ integer index

$\mathbf{M}_{l,m}$ independent vector eigenfunction solution of the vector Helmholtz equation

$M$ upper limit of $m$ index, magnification coefficient

$m$ integer index

$\mathbf{N}_{l,m}$ independent vector eigenfunction solution of the vector Helmholtz equation

$n$ relative refractive index of the microlens

$\tilde{n}$ complex relative refractive index

$\hat{n}$ unit vector perpendicular to the microlens' surface

$\mathbf{P}$ time-averaged Poynting Vector

$R$ radius of curvature of the microlens spherical surface

$R_{l,m}^{(\beta)}(h,\xi)$ spheroidal radial function

RRM Rayleigh range method

$\mathbf{r}$ position vector

SVM Separation-of-Variables method

$S_{l,m}(h,\eta)$ spheroidal angle function

$s$ object distance

$s_e$ effective object distance

$s'$ image distance

$s_e'$ effective image distance

$\hat{s}$ unit vector parallel to the microlens' surface and perpendicular to $\hat{\phi}$

$t$ time

$w_0$ incident beam waist radius

$w_0'$ transmitted beam waist radius

$X_{sphere}$ X coordinate of the spherical particle center

$Y_{l,m}(\theta,\phi)$ spherical harmonic function

$Z$ Z coordinate of the incident beam waist

$Z'$ Z coordinate of the transmitted beam waist or the focused beam

$Z_2$ Z coordinate of the 2nd microlens

$Z_R$ Rayleigh range of the incident TEM00 mode Gaussian beam illumination

$Z_{sphere}$ Z coordinate of the spherical particle center

# Chapter 1. Introduction

Optical elements are widely used in the laboratory experiments and commercial applications. The regular-size optical element is most commonly used and its optical properties can be accurately described using classic geometrical optics theory [1]. Due to the relatively large size and weight, regular-size optical elements are prohibited from high capacity and minimized optical equipment applications. With the development of micro- and nano- fabrication technology [2] in the past few decades, a new type of optical element is fabricated in the micro scale, which is called the "micro optical element", with its characteristic size on the order of the optical wavelength. This new type of micro optical element has raised great research interests for its novel optical properties and minimized size and weight.

As the characteristic size of the micro optical element is comparable with the optical wavelength, based on the Huygens-Fresnel diffraction theory, the Fresnel diffraction effect [3] through the micro optical element aperture becomes relatively strong in comparison with that through the regular-size optical element and leads to optical properties quite different from that given by geometrical optics theory. Fundamentally, the investigation of the optical properties of micro optical element is equivalent to the study of the interaction of micro optical element with electromagnetic waves. The computational electromagnetics method can be applied to investigate this interaction by calculating the electromagnetic field distributions inside of and adjacent to the micro optical element. The corresponding optical properties of the micro optical element can then be deduced from the calculated electromagnetic field distributions. The

optical properties of different type and profile micro optical elements have been previously investigated using diverse analytical and computational methods. While the systematic research work on the optical properties of micro optical systems has not been conducted before. In this dissertation, the three-dimensional continuous profile symmetrical biconvex microlens is considered. The frequency domain full-field analytical oblate spheroidal coordinate system Separation-of-Variables method (SVM) [4] is applied to study the optical properties of microlens optical systems with monochromatic plane wave and TEM00 mode Gaussian beam illumination.

In Chapter 1, an overview of the analytical and numerical computational electromagnetics methods and a review of the research progress on the optical properties of the micro optical element are presented. In Chapter 2, the oblate spheroidal coordinate system and the geometry of the three-dimensional continuous profile symmetrical biconvex microlens are described first. Then brief introduction to the Separation-of-Variables method (SVM) in the oblate spheroidal coordinate system follows. The two types of illumination sources used in this work, monochromatic plane wave and TEM00 mode Gaussian beam, are also introduced. In Chapter 3, the focusing properties of a single microlens with arbitrary monochromatic plane wave illumination are studied. The beam transformation properties of a single microlens with monochromatic TEM00 mode Gaussian beam illumination are also discussed. In Chapter 4, the axial combination properties of dual microlenses with monochromatic plane wave illumination are investigated. In Chapter 5, the beam transformation properties through dual microlenses with monochromatic TEM00 mode Gaussian beam illumination are studied. In Chapter 6, the interference properties between dual parallel-arranged microlenses with