

HYBRID METHOD: ALGEBRAIC/INVERSE RADON TRANSFORM FOR REGION OF
INTEREST RECONSTRUCTION OF COMPUTED TOMOGRAPHY IMAGES.

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2009

DEDICATION

To all who has put faith on me and believed in me...

To my mother: I would like to say thank you because she has given me a good set of wings.

My brothers and my nephew: Because they are my arms to keep fighting.

My friends: Because we will laugh about all this work again!.

... *Sit Nomen Domini Benedictum*

PREVIEW

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ABSTRACT

Computed tomography is one of the most rapidly advancing method in modern medical imaging, because of is relatively low cost and outstanding quality. The main shortcoming of the method is the relatively high dose of radiation used. Consequently, considerable efforts have been made to obtain good image reconstructions from fewer projections or from using lower intensity beams (hence, noisier data). It has been known for decades that a scan with high intensity beams and large number of projections (typically, 180-360 for a planar image) will lead to a very good image reconstruction via an Inverse Radon Transform (IRT), but the method is not accurate when less than 30 projections are employed. On the other hand, algebraic methods are very efficient for small images; unfortunately, for an $N \times N$ image, the number of coefficients of the matrix corresponding to the linear system of equations is of the order of N^4 . Hence, even for a relatively small images with $N=256$, the matrix have around 4 billion coefficients. For almost all practical applications of the computed tomography, algebraic reconstruction are prohibitively time consuming.

The aim of this novel method is to combine the advantages of both IRT and ART to obtain a good image for a small region of the total image. Starting with the total projections of the large image, a Filtered Back Projections (FBP) algorithm is used to create an intermediate image; a small Region of Interest (ROI) is selected from the large image and the partial projections belonging to the ROI alone are calculated. These partial projections are subsequently used with an algebraic method to reconstruct the ROI. Assuming a region of interest (e.g., any abnormality suspected to be clinically important) is 10 times smaller than the whole image, the speed of the algebraic method is increased by a factor of roughly 10^6 (e.g, about one second

instead of 10 days); even more important, for three dimensional images, the factor is 10^9 (one second instead of 30 years).

Furthermore, for a small ROI, the system of equations to be solved algebraically become overdetermined even for a small number of available projections, while for the total image the system is severely under determined.

The influence of noise and of the reduced number of projections on the quality of the approximate projections of the ROI image is also investigated.

A new parameter δ is introduced and its defined as the difference of signal levels between the real image and the FBP reconstruction. The optimal value of the new parameter could be determined without using the original image. The quality of the reconstruction of the ROI depends on its size, with a better quality for larger sizes. Also, it was found that the method is robust even in the presence of noise.

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PREVIEW

INTRODUCTION

The use of the Computed Tomography (CT) has influenced enormously the imaging medicine for diagnostic or therapeutic purposes. The main advantages of Computed Tomography are: the imaging of the cross sectional parts of the anatomy, the differentiation between distinct tissues, and the fact that it is a non surgical invasive technique.

The quality of the CT reconstructed image, a cross sectional slice of the body, has been improved constantly during the four decades since its first introduction. The reconstruction time that was several hours for a small image has been reduced to less than a second for whole-body imaging. Most of those improvements were due to the progress of the electronic and the physical engineering of the CT scan machines and the algorithms for computerized reconstructions.

While initial CT image reconstructions have been performed with algebraic methods, the Inverse Radon Transform (IRT) is almost always preferred now. The reason is that in order to reconstruct a 2D image of $N \times N$ pixels, for any realistic value of N (e.g., $N > 30$) the number of variables exceeds largely the number of equations (the total number of available projections) and the system to be solved is prohibitively large, which makes the algebraic inversion methods unpractical. However, for very small images the algebraic methods performed very well and are in general more accurate than IRT. The aim of this work is to study an hybrid method for computerized tomography reconstruction using a mixture between the Algebraic Reconstruction Technique and the Inverse Radon Transform. The basic idea is to perform a fast algorithm reconstruction for the whole (large) image using an IRT method, Filtered Back Projection (FBP) and a high quality local reconstruction with of a much smaller Region of Interest image using a

powerful Algebraic Reconstruction Technique (ART). The implementation of ART is more involved and uses higher computer resources than the FBP; however it performs better for small images when a reduced number of projections are available. The hybrid method takes advantage of the best characteristics of the FBP and ART methods and it will help to improve local diagnostic and cancer treatment in the future.

The first chapter of this work gives a description of the physical, mathematical and image quality backgrounds. The second chapter talks about the computed tomography principles, tomographic acquisition, CT equipment and dosimetry. The third chapter states the mathematical approach for the tomography reconstruction, describing the mathematical acquisition of the tomography data, the FBP reconstruction algorithm and the ART algorithm. Chapter four describes the novel hybrid method and the procedure to implement it. Also, in this chapter is explained the computational set up of the numerical method. The fifth chapter shows the results of the implementation of the hybrid method; a noise correction of this method implementation that improves the reconstruction; and a study of the local image quality in function of the number of projections with noise and with a variation of the local reconstruction dimension. Finally, discusses the results, conclusions and future work proposed for this project.

CHAPTER 1: BACKGROUND PRINCIPLES

1.1 Physical Principles

While light has been present since the beginning of time, it was only in the 1860's when James C. Maxwell did a great synthesis between the electricity, magnetism and their role in explaining the light. After Maxwell's publication, *electromagnetic radiation* was pronounced as the new name of light and it not only referred to visible light but also to an amazingly wide spectrum of electromagnetic waves (*see figure 1.1*).

Although all electromagnetic radiations share common properties (e.g.: all of them can be described by electric and magnetic fields; all of them travel at the same velocity, the speed of light in vacuum $c = 300,000 \text{ kms}^{-1}$ all of them are carried by photons) each range of energy has quite different characteristics. The names that are given in the spectrum (*figure 1.1.1*) are related to the manner in which radiation is detected or produced; mathematically, those waves are expressed with the same set of equations.

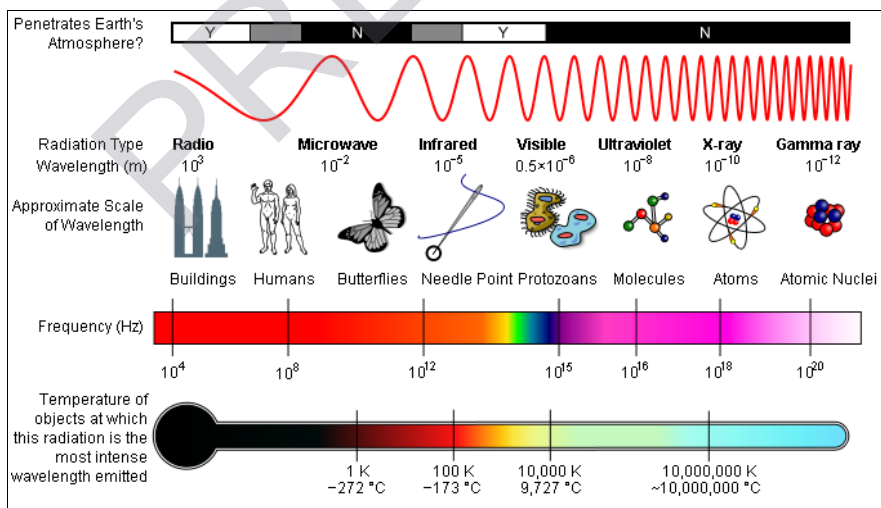


Figure 1.1.1 The Electromagnetic Spectrum.

We will try to explain some of those electromagnetic waves with more detail.