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DIAGRAM NORMAL FORMS AND THEIR APPLICATIONS TO THE THEORY OF MODELS.

by

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DIAGRAM NORMAL FORMS AND THEIR APPLICATIONS

TO THE THEORY OF MODELS.

BY

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K. S.

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INTRODUCTION

A set, Σ, of postulates determines a class, $\mathbb{K}(\Sigma)$ of mathematical systems; namely the class of all those systems in which every member of Σ is true. In the theory of models (*) we study mutual relations between the structural properties of sets of postulates and the properties of the classes of mathematical systems which they define.

We formulate the postulates in a formalized language, and refer to them as <u>sentences</u> - the sentences of that specified language. To the elements of the corresponding class of mathematical systems we refer to as <u>models</u>.

The present paper deals mainly with embeddability properties. It was in an attempt to solve a particular embeddability problem formulated by Hugo Ribeiro (§6), that the author introduced the notion of and proved the theorem on, the diagram normal form of an existential (or universal) sentence (§1): For every consistent existential sentence β of a specified theory, if the

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(*) See A. Tarski [9].

prenex normal form of β contains n quantifiers, then β is constructively equivalent to a disjunction of a finite number of "diagram sentences", each of length not exceeding n. This disjunction, called diagram normal form is unique up to the order of terms. A dual result for universal sentences holds.

The formulation of the concept of a diagram sentence is influenced by the following result of A. Tarski in [9]: if a class $\mathbb{K}$ of relational systems is the class of all systems of a given similarity class $\mathbb{R}$ in which a finite relational system of finite order belonging to $\mathbb{R}$ is not isomorphically embeddable, then $\mathbb{K}$ is a universal class. In the proof of this theorem a sentence is constructed whose structure is used here to define a diagram sentence. The results of §1 are used to the formulation and solution of several problems in the theory of models.

§2 is devoted to the derivation of immediate consequences of the theorem on diagram normal forms as applied to existential and universal classes; in particular to the establishment of a kind of converse of Tarski's theorem

above, a result which Fraïssé had already obtained, and was found here independently.

In § 3 we give a characterization of the finite embedding property in terms of finite reducibility. This concept has been studied by L. Henkin and other authors. The concept of finite reducibility had been used in connection with decision problems. We prove that a class $\mathbb{L}$ of relational systems (in particular algebras) has the finite embedding property if and only if the set of all universal sentences of the formalized theory, within which $\mathbb{L}$ is discussed, is finitely reducible with respect to $\mathbb{L}$. We prove further, that if a class $\mathbb{L}$ has the finite embedding property, if $\overline{\mathbb{L}}$ is the intersection of all similar universal classes containing $\mathbb{L}$ and if $\mathbb{N}$ is any class such that $\mathbb{L} \subseteq \mathbb{N}$ then the class $\mathbb{N} \cap \overline{\mathbb{L}}$ has the finite embedding property already with respect to the finite systems of $\mathbb{L}$. The result is applied to prove the existance of a class of groups with the finite embedding property which contains properly the class of all Abelian groups.

In § 4 - § 6 we discuss problems related to universal completeness. This notion has been introduced into the

theory of models and extensively studied by H. Ribeiro. Further results connected with this notion are proved in § 5. In particular, a sufficient condition is given for a set of sentences to be maximal universally complete. It is proved, also, that universally complete sets of sentences which have models of arbitrary large finite cardinality necessarily have the finite embedding property. The theorem-on diagram sentences is used in § 4 to give a characterization of N-universally complete sets of sentences in terms of embeddability properties of the classes of their models. In § 6 the sets of sentences which are universally complete relatively to simpler theories are similarly characterized. Examples are discussed.

The notation and terminology is modeled on that of A. Tarski in [9]; it was modified and supplemented to answer the needs of the particular problems considered.

§ 1. THE DIAGRAM NORMAL FORM.

Let the theory T be an applied predicate calculus with identity. We specify that T contains k predicate symbols $P_i^{r_i}(\ldots)$, $i = 1,\ldots,k$; each $P_i^{r_i}$ being of rank r_i.

1.1 DEFINITION. A sentence, $\delta(n)$ of the theory T, is a diagram sentence of length n if and only if $\delta(n)$ is an existential sentence $\bigvee x_1 \bigvee x_2 \ldots \bigvee x_n\ \alpha$ where α is a conjunction $\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_k \wedge \varepsilon$ of quantifier-free formulae $\alpha_1, \alpha_2, \ldots, \alpha_k, \varepsilon$ and

i) for every $i \leq k$, α_i is a conjunction of atomic formulae $P_i^{r_i}(\ldots)$ or their negations $\sim P_i^{r_i}(\ldots)$ such that for every sequence $\langle x_{j_1}, x_{j_2}, \ldots, x_{j_{r_i}} \rangle$ of elements of the set $X(n) = \{x_1, x_2, \ldots, x_n\}$ just one of $P_i^{r_i}(x_{j_1}, x_{j_2}, \ldots, x_{j_{r_i}})$ and $\sim P_i^{r_i}(x_{j_1}, x_{j_2}, \ldots, x_{j_{r_i}})$ appears in α_i;

ii) ε is the conjunction of the formulae $\sim(x_p = x_q)$ for every one of the subsets of two distinct elements from the set $X(n)$.

1.2 THEOREM. Every consistent existential sentence β of the theory T, whose prenex normal form contains n quantifiers is equivalent to a disjunction $\delta(n_1) \vee \delta(n_2) \vee \ldots \vee \delta(n_m)$