# COMPARISON OF THEORETICAL AND EXPERIMENTAL ELECTRON EMISSION

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ABSTRACT

In this work an attempt is made to compare some experimental electron emission data with theoretical formulas which give current density as a function of applied electric field, temperature and work function. The comparison is not conclusive but with some assumptions about the local microscopic field and local temperature, reasonable values are obtained for work function and emission area.

Computer programs are developed for further investigation and some experimental areas are suggested where the results will be more sensitive to comparison.

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# I. INTRODUCTION

The comparison of experimental electron emission with theoretical formulas has not been entirely successful. In this paper the theoretical work of Murphy and Good[1] is used as a basis for comparison and computer programs have been developed to facilitate this comparison.

Usually the theories of thermionic and field emission are studied separately by constructing appropriate expressions for the current in specific ranges of field and temperature.

At high temperatures and low field strength, emitted electrons are primarily those whose energy is greater than the maximum energy of the barrier. This is thermionic emission. In the theoretical model, temperature has no effect on the height or width of the barrier, but increases the number of electrons with energy greater than that of the barrier peak. The theoretical treatment of thermionic emission leads to the Richardson equation.[2] In an electric field which lowers the peak by an amount proportional to the square root of the applied field, the Richardson equation is modified by the Schottky dependence.[3]

At low temperatures and high field strength, electron emission is predominant through the barrier. The low temperature greatly

reduces the probability of electrons having energies above the Fermi level so that few electrons can escape the surface because of energies greater than the surface barrier. The high field has the effect of narrowing the barrier and, hence, increasing the transition probability that an electron with energy below the Fermi level can appear outside the surface. This is field emission and a theoretical treatment leads to the Fowler-Nordheim equation.[4,5]

Emission phenomenon was studied from a unified point of view by Murphy and Good.[1] They developed a general expression for the emitted current density as a function of field, temperature, and work function in the form of a definite integral. Each type of emission was then associated with a technique for approximating the integral. The approximation for low fields and high temperatures led to an extension of the Richardson-Schottky formula for the thermionic region. An analogous treatment for high fields and low temperature gave an extension to the Fowler-Nordheim equation for field emission and the region of field and temperature in which it is valid. A third approximation led to an expression for the emitted current density in a region intermediate between the thermionic and field regions.

## II. THEORETICAL BACKGROUND

The model used by Murphy and Good utilizes two forces, the image force and the force due to a constant electric field. The metal surface lies in the plane x = 0, and distance is measured outward. The image force is obtained by assuming that as the electron leaves the surface of the metal, it induces a positive charge in the metal whose distance from the surface is equal to that of the electron. The force is

$$F_{image} = -\frac{e^2}{(2x)^2} = -\frac{e^2}{4x^2} \qquad 1$$

The force due to the electric field is just

$$F = e\,E \qquad 2$$

Then

$$-\frac{\partial V(x)}{\partial x} = F + F_{image} = -\frac{e^2}{4x^2} + e\,E \qquad 3$$

Thus the potential

$$V(x) = -\int \left( \frac{e^2}{4x^2} + e\,E \right) dx$$

$$V(x) = -\frac{e^2}{4x} - e\,Ex + \text{constant}$$

where the constant is chosen so that V(x) is zero when E = 0.

or

$$V(x) = -\frac{e^2}{4x} - eEx \qquad 4$$

It is assumed that the potential inside the metal is constant ($= -W_B$) and that the image force goes to zero as x approaches zero.

Therefore equation 4 must be altered so that $V(+0) = -W_B$ and $V(x)$ is regular in the neighborhood of $x = 0$. The particular shape of the potential in this region is unimportant in this problem because most emission occurs at the Fermi level or above.

The peak of the potential barrier occurs when the retarding image force equals the extracting field.

$$x_{Vmax} = + \frac{1}{2}\sqrt{e/E} \qquad 5$$

$$V_{max} = -\sqrt{e^3 E} \qquad 6$$

Table 1 lists $V_{max}$ and its distance from the surface for several values of electric field strength and a typical plot of $V(x)$ versus $x$ is shown in the right side of Figure 1. It is clear from the table and figure that the electric field both lowers and narrows the potential barrier. The width ($x_d$) of the potential barrier at the Fermi energy can be found by solving equation 4 for $V(x) = -\zeta$.

$$x_d = \sqrt{\frac{\zeta^2}{e^2E^2} - \frac{e}{E}} = \frac{1}{eE}\sqrt{\zeta^2 - V_{max}^2} \qquad 7$$

where $\zeta$ is the Fermi energy

TABLE 1

| E(volts/cm) | $V_{max}$ (ev) | $x_{Vmax}$ (cm) |
|---|---|---|
| $10^4$ | .038 | $1.9 \times 10^{-6}$ (= 190 Angstroms) |
| $10^5$ | .12 | $6.0 \times 10^{-7}$ (= 60 Angstroms) |
| $10^6$ | .38 | $1.9 \times 10^{-7}$ (= 19 Angstroms) |
| $10^7$ | 1.2 | $6.0 \times 10^{-7}$ (= 6 Angstroms) |
| $10^8$ | 3.8 | $1.9 \times 10^{-8}$ (= 1.9 Angstroms) |

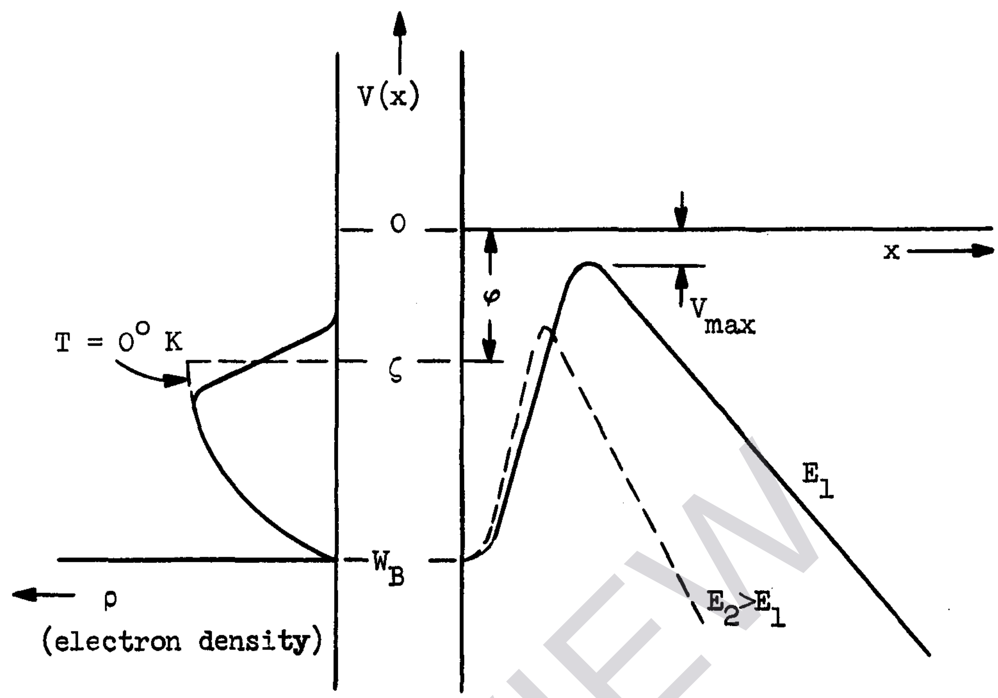


Figure 1

The left side of Figure 1 shows the electron density, (horizontal coordinate) as a function of electron energy for the free electron model using Fermi-Dirac statistics. The density of electrons with energy between W and W + dW is

$$d\rho = \frac{4\pi}{h^3} \quad (2m)^{3/2} \frac{\sqrt{W} \; dW}{1 + e^{(W - \zeta)/kT}} \qquad 8$$

Since the potential changes along only one coordinate, an electron will experience a change only in the component of velocity parallel to the x-coordinate. The density of electrons with velocity component $v_x$ can be obtained by integrating equation 8 over all values of $v_y$ and $v_z$. Then