STOCHASTIC DIFFERENTIAL EQUATION APPLIED TO HIGH FREQUENCY DATA ARISING IN GEOPHYSICS AND OTHER DISCIPLINES.

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To my

FATHER and MOTHER

with love

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by

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THESIS

Presented to the Faculty of the Graduate School of

The University of Texas at El Paso

in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE

Department of Mathematical Sciences

THE UNIVERSITY OF TEXAS AT EL PASO

August 2015

Acknowledgements

My sincere appreciation goes to the Almighty God for his love, mercy and abundant grace shown me throughout my education.

I would like to express my deep-felt gratitude to my advisor and chair of the Mathematical Sciences Department at The University of Texas at El Paso, Dr. Maria Christina Mariani for her mentoring, advice, encouragement, enduring patience and keen interest which brought my thesis to a successful completion.

I also wish to thank the other members of my committee, Dr. Granville Sewell of the Mathematical Sciences Department and Dr. Laura Serpa of the Geological Sciences Department, both at The University of Texas at El Paso. Their suggestions, comments and additional guidance were invaluable to the completion of this work.

I want to thank Dr. Hector Gonzalez-Huizar for his suggestions and providing me with the Chile geophysical data.

I also want to thank The University of Texas at El Paso Mathematical Sciences Department professors and staff for all their hard work and dedication, providing me the means to complete my degree.

My warmest gratitude also goes to my dear parents for their love, encouragement and constant support. I say God richly bless you.

And finally, I extend my sincere thanks to all friends and loved ones who contributed in various ways to make this work a success.

NOTE: This thesis was submitted to my Supervising Committee on the June 4, 2015.

Abstract

Estimating future seismic hazards of a region constitutes an important study many scholars have shown a renewed interest in the past few decades. A good estimation of the seismic hazard in a region requires predicting the location and magnitude of future seismic events. As the knowledge of the geophysical mechanisms that drive seismic events have increased, so have the corresponding mathematical model representations.

This thesis is devoted to the study of modeling geophysical data. We propose a stochastic differential equation arising on the superposition of independent Ornstein-Uhlenbeck processes driven by a $\Gamma(a, b)$ process. Superposition of independent $\Gamma(a, b)$ Ornstein-Uhlenbeck processes offers analytic flexibility and provides a class of continuous time processes capable of exhibiting long memory behavior. The stochastic differential equation is applied to geophysics by fitting the superposed $\Gamma(a, b)$ Ornstein-Uhlenbeck model to high frequency data series in California and Chile.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Estimating future seismic hazards of a region constitutes an important study many scholars have shown a renewed interest in the past few decades. A good estimation of the seismic hazard in a region requires the prediction of time, location and magnitude of future seismic events. As the knowledge of the geophysical mechanisms that drive seismic events has increased, so have the corresponding mathematical model representations.

Numerous deterministic and probabilistic models have been developed to depict various aspects of the mathematical modeling of seismic occurrence patterns and describing major stock indices. The most interesting in recent time has been the scale invariant functions and Lèvy models which have been used to estimate parameters related to some major events [1]. In this work, by looking at the preceding data collected before a major earthquake, the model estimated the parameters leading to these critical events. The modeling approach used was similar to [5], where they described the behavior of the financial market before the crash. Other probabilistic models such as the long term correlations have also been applied to the occurrences of seismic events [2].

It is now widely accepted that the time of occurrence and magnitudes of a sequence of earthquakes on a given source may be stochastically dependent, that is, there is a correlation between the number of events in successive time intervals [3]. Therefore in attempt to overcome the modeling problems associated with the memory-less property models described in previous literature, we propose a continuous-time stationary and non-negative stochastic differential equation that is useful in describing a unique type of dependence in a sequence of events.

Continuous-time stochastic volatility models are now popular ways to describe many "critical